

The Dempster-Shafer Theory of Belief Functions for Managing Uncertainties: An Introduction and Fraud Risk Assessment Illustration

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ABSTRACT

The main purpose of this paper is to introduce the Dempster-Shafer theory (“DS” theory) of belief functions for managing uncertainties, specifically in the auditing and information systems domains. We illustrate the use of DS theory by deriving a fraud risk assessment formula for a simplified version of a model developed by Srivastava, Mock, and Turner (2007). In our formulation, fraud risk is the normalized product of four risks: risk that management has incentives to commit fraud, risk that management has opportunities to commit fraud, risk that management has an attitude to rationalize committing fraud, and the risk that auditor’s special procedures will fail to detect fraud.

We demonstrate how to use such a model to plan for a financial audit where management fraud risk is assessed to be high. In addition, we discuss whether audit planning is better served by an integrated audit/fraud risk assessment as now suggested in SAS 107 (AICPA 2006a, see also ASA 200 in AUASB 2007) or by the approach illustrated in this paper where a parallel, but separate, assessment is made of audit risk and fraud risk.

Key Words: Dempster-Shafer Theory of Belief Functions, Fraud Risk, Audit Risk

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1. INTRODUCTION

This article first introduces the basics of the Dempster-Shafer (DS) theory of belief functions and shows how this framework can be used for managing uncertainties using a fraud risk assessment illustration. We also discuss the issue of whether audit planning is better served by an integrated audit/fraud risk assessment as now suggested in audit standards such as SAS 107 (AICPA 2006a, see also ASA 200 in AUASB 2007) or by the approach illustrated in this paper where a parallel, but separate, assessment is made of audit risk and fraud risk.

DS theory has been applied to many business problems to help manage uncertainties related to audit risk, information security risk, information quality assessment, mergers and acquisitions, and portfolio management. Examples include Srivastava and Shafer (1992), Sun, Srivastava and Mock (2006), Srivastava and Li (2008), Bovee, Srivastava and Mak (2003), Srivastava and Datta (2002), and Shenoy and Shenoy (2002). Additional references are provided in the review article by Srivastava and Liu (2003) and the book *Belief Functions in Business Decisions* edited by Srivastava and Mock (2002).

DS theory has been argued to be a better framework than probability theory for modeling uncertainties in real world problems. For example, the shortcomings of probability theory in modeling uncertainties in medical diagnostics evidence are discussed by Gordon and Shortliffe (1984, p. 529):

We believe that the advantage of the Dempster-Shafer theory over previous approaches is its ability to model the narrowing of the hypothesis set with the accumulation of evidence, a process that characterizes diagnostic reasoning in medicine and expert reasoning in general. An expert uses evidence that, instead of bearing on a single hypothesis in the original hypothesis set, often bears on a

larger subset of this set. The functions and combining rule of the Dempster-Shafer theory are well suited to represent this type of evidence and its aggregation.

In an auditing context, Srivastava and Shafer (1992, p252) argue:

... the usefulness of the Bayesian approach is limited by divergences between the intuitive and Bayesian interpretations of audit risk. For example, according to SAS No. 47 (AICPA 1983), if an auditor decides not to consider inherent factors, then the inherent risk is set equal to 1. Since a probability of 1 means certainty, this seems to be saying that it is certain that the account is materially in error. But this is not what the auditor has in mind when deciding not to depend on inherent factors. The auditor's intention is represented better by a belief-function plausibility of 1 for material error, which says only that the auditor lacks evidence based on inherent factors.

Srivastava and Jones (2008) discuss several additional problems with using probability theory to model uncertainties. For example, criticizing the use of probability theory for expressing the strength of evidence they state: "... all items of evidence modeled under probability theory will always be mixed. However, it is quite common in the real world to find pure positive evidence or pure negative evidence."

In auditing, an example of pure positive evidence is applying analytical procedures and observing that the current year's account balances are completely in line with the auditor's projections. This evidence may be assessed as being positive in that it provides support, say a level of support of 0.2 on a scale of 0-1, that the account balance is fairly stated and also assessed as not providing any evidence in support of the assertion that the account balance is materially misstated. If we express the above evidence in terms of probability as $P(a) = 0.2$ that the account balance is fairly stated, then by definition a 0.8 probability should be assigned to the state that the account is materially misstated ($\sim a$). Inferring that $P(\sim a) = 0.8$ from the analytical review evidence implies that the evidence is mixed which contradicts the assumption that the auditor did not observe any evidence that suggests that the account is materially misstated. Under DS theory we can model purely positive, purely negative and also mixed evidence, whereas in probability

theory it is not possible to model purely positive or purely negative evidence in any context except certainty.

Another problem that is highlighted by Srivastava and Jones (2008) is the difficulty of modeling ignorance using probability theory. As a result, it is difficult to distinguish between a situation where one has full knowledge of the situation and another where one does not have any knowledge. We show how one can model this situation under DS theory in a following section.

In addition to theoretical criticisms, there is empirical evidence showing the value of DS theory in modeling how decision makers think of uncertainties. For example, Curley and Golden (1994) found that subjects, in an experiment to determine the most likely suspect of a murder mystery based on multiple items of evidence which pertained to multiple suspects, were mapping their judgments consistent with DS theory. In an auditing context, Harrison, Srivastava, and Plumlee (2002) found that only 19% of auditors' judgments about strength of evidence could be modeled under probability theory whereas 100% of the judgments could be modeled if one uses the DS theory.

Fukukawa and Mock (2011) show that DS theory provides a richer set of risk concepts that an auditor may wish to consider. For example, whereas probability theory only encompasses a single notion of risk, the probability of material misstatement, DS theory suggests several including the plausibility of misstatement (Srivastava and Shafer, 1992) and the belief of misstatement. The latter two measures facilitate the explicit assessment of uncertainties the auditor must confront and thus the consideration of the auditor's risk preferences. These risk concepts and the basics of DS theory are elaborated and illustrated in the following sections.

The remainder of the paper is organized as follows. We introduce the Dempster-Shafer theory of belief functions in Section 2. In Section 3, we discuss whether audit planning is better

served by an integrated audit/fraud risk assessment or by the parallel assessment approach illustrated in this paper. In Section 4 we discuss and illustrate a fraud risk assessment formula which is derived mathematically in Appendix B. In Section 5, we present a summary and conclusion.

2. DEMPSTER-SHAFER THEORY OF BELIEF FUNCTIONS

In this section we introduce the basics of the Dempster-Shafer (DS) theory of belief functions. DS theory is based on the work of Arthur Dempster during the 1960's and in particular by Glenn Shafer's treatise *A Mathematical Theory of Evidence* (Shafer 1976). This particular theory is especially relevant to auditing and assurance as it focuses on evidence and evidential reasoning.

There are three basic functions that are important to understanding and applying DS theory: the *basic belief mass function* which specifies the belief mass distribution (m-values) over all possible sub-sets of a frame of discernment, the *Belief function*, and the *Plausibility function*. Similar to Bayes' rule in probability theory, Dempster's rule is used in DS theory to combine multiple independent items of evidence pertaining to a variable (i.e., assertion) as discussed in Appendix A.

Basic Belief Mass Functions

The *basic belief mass function* is similar to the probability distribution function with one very important difference. Under probability theory, the probability distribution function assigns probability mass to each element of a frame, say $\Theta = \{a_1, a_2, a_3, \dots, a_n\}$, consisting of a mutually exclusive and exhaustive set of elements $\{a_1, a_2, a_3, \dots, a_n\}$. Suppose the probability mass assigned to an element a_i is represented by $P(a_i)$ which represents the probability that a_i is true.

Under probability theory $P(a_i)$ takes a value between 0 and 1 such that sum of all such probability masses add to one, that is $\sum_i P(a_i) = 1$.

The principal difference in the two theories is that under DS Theory the *basic belief mass* is assigned not only to single elements of the frame $\Theta = \{a_1, a_2, a_3, \dots a_n\}$ but also to all the subsets of the frame consisting of two elements, three elements, and so on, such as $a_1a_2, a_1a_2a_3, \dots a_1 \dots a_n$, to all the elements of the frame. Let us express the basic *belief mass* assigned to a set of elements, say A , by $m(A)$, which takes a value between 0 and 1 such that the sum of all the m -values is equal to one, that is $\sum_{A \subseteq \Theta} m(A) = 1$, similar to probability mass. The belief mass assigned to the empty set is zero, $m(\emptyset) = 0$.

Srivastava and Shafer (1992) point out that the m -values can be assigned by the decision maker (the auditor) on the basis of subjective judgment or can be derived from a compatibility relationship between a frame with known probabilities and the frame of interest. Using the financial statement audit as an example, let us suppose that an auditor performs a review of sales documents for a significant sales transaction and finds no discrepancies among the documents. Based upon this evidence, the auditor assigns a medium level of support, say 0.6 on a scale of 0-1, to the assertion, 's', that the sales transaction actually occurred. At the same time, the auditor notices that several documents have been manually prepared rather than being prepared by the company's computerized accounting system, which may indicate a risk of fictitious revenue. Thus the auditor assigns a low level of support, say 0.2, to the assertion '~s' that the sale did not actually occur. Using the basic belief mass function, the auditor can represent the overall evidence as follows:

$$m(s) = 0.6, m(\sim s) = 0.2, \text{ and } m(\{s, \sim s\}) = 0.2.$$

The above m-values represent the level of support obtained from the evidence described above. A value of $m(s) = 0.6$ represents 0.6 degree of belief, on a scale of 0-1, that 's' is true, while $m(\sim s) = 0.2$ represents the belief that ' $\sim s$ ' is true based on the evidence, and $m(\{s, \sim s\}) = 0.2$ represents the belief not assigned to any particular state, but assigned to the entire frame $\{s, \sim s\}$, which represents ignorance.

The above m-values represent mixed evidence; some support in favor of the assertion, and some support against the assertion. Pure positive evidence can be expressed as $m(s) > 0$, and $m(\sim s) = 0$, and pure negative evidence as $m(s) = 0$, and $m(\sim s) > 0$.

Belief Function

Belief in a set of elements, say A, of a frame Θ , represents the total belief that one has based on the evidence obtained. It is the sum of all the belief masses assigned to elements that are contained in the set A and the belief mass assigned to the set A. Mathematically, one can express the total belief in the set A as $Bel(A) = \sum_{B \subseteq A} m(B)$. Unlike probability theory, $Bel(A) = 0$ represents lack of evidence about A, while $P(A) = 0$ represents the impossibility of A. However, $Bel(A) = 1$ represents certainty, that is A is certain to occur, similar to $P(A) = 1$, which also represents the certainty that A is true.

Continuing the previous audit example, let us suppose that the auditor does not have other audit evidence to support or negate the assertion that sales have occurred, then the belief in 's' that sales have occurred is 0.6, i.e., $Bel(s) = m(s) = 0.6$, and the belief that sales have not occurred is 0.2, i.e., $Bel(\sim s) = m(\sim s) = 0.2$. And by definition, $Bel(\{s, \sim s\}) = m(s) + m(\sim s) + m(\{s, \sim s\}) = 0.6 + 0.2 + 0.2 = 1.0$, a belief that either s or $\sim s$ is true. The auditors job is to decide which state is true, in our example, whether the reported sales actually have occurred (s) or not ($\sim s$).

Plausibility Function

Plausibility in a set, say A of a frame Θ consisting of a mutually exclusive and exhaustive set of elements, represents the maximum possibility that a set A is true given all the evidence. Mathematically, it is equal to the sum of the belief masses over all the subsets of Θ that have non-zero intersection with the set A . One can express the plausibility that A is true as:

$$Pl(A) = \sum_{A \cap C \neq \emptyset} m(C).$$

The plausibility of A can also be expressed as the complement of the belief in ‘not A ’, that is $Pl(A) = 1 - Bel(\sim A)$. $Pl(A) = 1$ implies that A is possible and at the same time that we do not have any evidence that ‘not A ’ is true, that is $Bel(\sim A) = 0$. However, $Pl(A) = 0$ implies that A is impossible, similar to $P(A)$ being zero. Also, $Pl(A) = 0$ implies that the $Bel(\sim A) = 1$, that is if A is not plausible, then ‘not A ’ is true for sure.

In the previous audit example, the plausibility of the assertion that sales have and have not occurred can be expressed as:

$$Pl(s) = m(s) + m(\{s, \sim s\}) = 0.6 + 0.2 = 0.8 = 1 - Bel(\sim s) = 1 - 0.2 = 0.8,$$

$$Pl(\sim s) = m(\sim s) + m(\{s, \sim s\}) = 0.2 + 0.2 = 0.4 = 1 - Bel(s) = 1 - 0.6 = 0.4,$$

$Pl(A)$ is the maximum possible belief that can be assigned to the set A and thus is the most conservative assessment of risk given available evidence. From this perspective, plausibility plays an important role in defining various risks. For example, Srivastava and Shafer (1992) define plausibility of material misstatements being present in the financial statements as the audit risk. Sun, Srivastava and Mock (2006) use the plausibility that information is not secure to be the information security risk. In the following section we use the plausibility that financial statements are misrepresented due to management fraud to represent fraud risk.

3. COMBINED OR SEPARATE FRAUD RISK ASSESSMENT?

The auditing profession has taken the position that auditors do not need a separate fraud risk assessment model as derived in this paper. Following the massive fraudulent financial reporting cases that occurred at the beginning of this decade, the profession decided that the original audit risk model of SAS 47 (AICPA 1983), later superseded by SAS 107 (AICPA 2006a), could be used in a combined assessment of fraud risk along with the risk of errors and misappropriation of assets. For example, SAS 109 (AICPA 2006b, see also ASA 315 in AUASB 2006b) states (emphasis added):

.01 This section establishes standards and provides guidance about implementing the second standard of field work, ...

This section should be applied in conjunction with the standards and guidance provided in other sections. In particular, the auditor's responsibility to consider fraud in an audit of financial statements is discussed in section 316, *Consideration of Fraud in a Financial Statement Audit*.

.05 Obtaining an understanding of the entity and its environment, including its internal control, is a continuous, dynamic process of gathering, updating, and analyzing information throughout the audit. Throughout this process, the auditor should also follow the guidance in section 316.

Similarly, *International Standard on Auditing 240* (2009) Paragraph 13(1) suggests that “professional skepticism” may be relied upon to help detect management fraud. According to this standard, “professional skepticism” is defined as “an attitude that includes a questioning mind, being alert to conditions which may indicate possible misstatement due to error or fraud, and a critical assessment of audit evidence.”

In other words, the profession has redefined the misstatement term in the definition of inherent risk (IR) to include misstatements due to errors, misappropriation of assets, and management fraud without providing rigorous guidance as to how the combined risk can be assessed or how audit evidence should be assessed and aggregated to assess the combined risk.

Although SAS 107 (AICPA 2006a, see also ASA 200 in AUASB 2007) does provide detailed qualitative guidance on how to assess fraud risk based on the three fraud risk factors, the audit risk model clearly does not capture the logic of assessing fraud risk and subsequently planning the audit to detect fraud. SAS 107 (AICPA 2006a, see also ASA 200 in AUASB 2007) provides the following guidance on how to use the audit risk model (paragraph 26):

The model, $AR = RMM \times DR$, expresses the general relationship of audit risk and the risks associated with the auditor's assessments of risk of material misstatement (inherent and control risks); of the risk that substantive tests of details and substantive analytical procedures would fail to detect a material misstatement that could occur in a relevant assertion, *given that such misstatements occur and are not detected by the entity's controls*; and of the allowable risk that material error will not be detected by the test of details, given that a material misstatement might occur in a relevant assertion and *not be detected by internal control* or substantive analytical procedures and other relevant substantive procedures (emphasis added).

Our approach is based on focusing on fraud cues and finding the source of fraud (i.e., the presence of incentives, opportunities and attitude) and the corresponding controls such as corporate governance and then implementing special forensic procedures that would be expected to detect the fraud at the calculated level of risk. Thus our approach is similar to Zimbelman (1997) who examined the effectiveness of requiring auditors to decompose inherent and control risks in the Audit Risk Model to separately assess fraud risk. However, Zimbelman (1997) did not study an explicit fraud risk assessment model such as what we have proposed in Equation 2. He hypothesized that the decomposed judgment process would help auditors focus on fraud cues and thus reach improved audit decisions and found that decomposition of a separate fraud risk assessment did lead to greater attention to fraud red flags and greater budgeted hours than auditors using the Audit Risk Model. We believe similarly, that a separate fraud risk assessment model as derived in this paper will have similar benefits.

4. FRAUD RISK ASSESSMENT MODEL

In this section we discuss a fraud risk assessment formula as derived in Appendix B based on DS theory. Figure 1 is a schematic diagram of the variables and items of evidence that need to be considered in assessing fraud risk. Such a diagram is known as an evidential diagram or evidential network. This illustration is based on a simplified version of the fraud risk assessment model discussed by Srivastava, Mock and Turner (2007).

----- Figure 1 about here -----

The illustration permits the auditor to assess the belief and plausibility that management has committed financial statement fraud (F) based on assessments of three “fraud triangle” factors (SAS 99, AICPA 2002, see also ASA 240 in AUASB 2006a):

1. The Incentives (I) that management has to commit fraud such as obtaining a bonus
2. Opportunities (O) that management has to commit fraud such as overriding controls, and
3. Attitude (A) or propensity that management has which allows them to rationalize committing fraud.

Within Figure 1 the relationship among these three factors is expressed as a logical “AND” relationship between the variable F and the three fraud factors as depicted by the hexagonal box.

Srivastava, Mock and Turner (2007) have considered the possibility that the three fraud factors are interrelated, for example that attitude may be influenced by incentives and vice versa. Similarly, opportunity may be influenced by incentives and attitude. In Figure 1 we simplify the Srivastava et al model by assuming no relationships among the three fraud triangle factors. This simplification helps us derive the more tractable fraud risk model which is illustrated in this

paper. Note that in the illustration we consider fraud risk at an assertion level, that is, the variable F stands for the possibility of fraud related to a particular account-level assertion such as occurrence, existence or valuation.

In Figure 1, the rounded boxes represent the variables F, I, A, and O as defined earlier. We assume these variables are binary in nature. For example, the two states for variable F, {f, ~f}, are f = the state where management fraud is present, and ~f = the state where management fraud is not present.

Similarly, the two states for I, {i, ~i}, are i = Incentives are present which may motivate management to commit fraud and ~i = Incentives are not present for management to commit fraud. One can use similar definitions for the binary states of A, {a, ~a}, and of O, {o, ~o}.

In terms of these values, the ‘AND’ relationship can be expressed in terms of set notation as: $f = i \wedge a \wedge o$. In words this expression says that ‘f’ is true if and only if ‘i’, ‘a’, and ‘o’ are true at the same time. This relationship can also be written in terms of the logical ‘OR’ relationship for the negation of the variables, i.e.,

$$\sim f = (\sim i \wedge a \wedge o) \vee (i \wedge \sim a \wedge o) \vee (i \wedge a \wedge \sim o) \vee (\sim i \wedge \sim a \wedge o) \vee (\sim i \wedge a \wedge \sim o) \vee (i \wedge \sim a \wedge \sim o) \vee (\sim i \wedge \sim a \wedge \sim o).$$

This relationship implies that fraud will not occur if any one of the fraud risk factors is not present, or if any two of the fraud risk factors are not present, or if all the three factors are not present.

The rectangular boxes in Figure 1 represent items of audit evidence pertaining to various variables. For example, Evidence E_I represents evidence pertaining to variable I (Incentives) such as the existence of a management bonus package based on financial performance. The more complete model presented in Srivastava, Mock and Turner (2007) explicitly considers the presence of related threat factors and safeguards in the accounting system or in the management

control system. These are not considered in the Figure 1 illustration. As derived in Appendix B, the belief, $Bel(f)$, and plausibility, $Pl(f)$, that management has committed financial statement fraud is given by (See Equations B8, and B9 in Appendix B):

$$Bel(f) = 1 - [1 - m_I(i) m_A(a) m_O(o)][1 - m_S(f)]/K, \quad (1)$$

$$Pl(f) = Pl_I(i)Pl_A(a)Pl_O(o)Pl_S(f)/K, \quad (2)$$

where K is the renormalization constant in Dempster's rule and is defined in Equation B7. The plausibility functions, $Pl_I(i)$, $Pl_A(a)$, $Pl_O(o)$, and $Pl_S(f)$, respectively, represent the plausibility that incentive is present (i.e. 'i' is true), the plausibility that attitude is present (i.e., 'a' is true), the plausibility that opportunity is present (i.e., 'o' is true), and the plausibility that fraud could be present (i.e., 'f' is true).

The above formulas for $Bel(f)$ and $Pl(f)$ in (1) and (2) are general in the sense that they include positive, negative or mixed items of evidence but do not consider any interrelationships among the three fraud factors as considered by Srivastava, Mock, and Turner (2007). $Bel(f)$ represents the belief that fraud is present based on the evidence gathered at the assertion level and the evidence that incentives, opportunities, and attitude to commit fraud exist. Whereas the plausibility in fraud, $Pl(f)$, represents the risk of fraud, FR , (i.e., $FR = Pl(f)$) being present in the assertion of an account. By definition, $Pl(f)$ is the complement of the belief that there is no fraud, i.e., $Pl(f) = 1 - Bel(\sim f)$. Thus, if we have evidence to support that there is no fraud with a belief of say 0.95, then the plausibility of fraud, i.e., the fraud risk would be 0.05. Given the high cost to the audit firm of not discovering material fraud, the plausibility of fraud should be kept at a low level and belief in fraud at a very low level.

Note that fraud risk plausibility, $Pl(f)$, in Equation (2) is the product of four plausibilities, $Pl_I(i) \times Pl_A(a) \times Pl_O(o) \times Pl_S(f)$. Each plausibility term represents the risk associated with the

corresponding variable. For example, $Pl_I(i)$ represents the plausibility that significant incentives to commit fraud are present. Let us denote this risk by RI, that is $RI = Pl_I(i)$. Similarly, let us use $RA = Pl_A(a)$ to represent the risk of management exhibiting an attitude which may rationalize committing fraud; $RO = Pl_O(o)$, the risk of opportunity being present to commit fraud, and $RS = Pl_S(f)$, the risk that auditor's special procedures (forensic procedures) will fail to detect fraud. Thus, in terms of these symbols, we have the following fraud risk formula:

$$FR = RI \times RA \times RO \times RS/K \quad (3)$$

The above fraud risk formula makes logical sense in that it implies that fraud will go undetected if fraud exists as a result of incentives, attitude, and opportunity being present and also if the auditor's special procedures fail to detect fraud (RS). There are cases in the planning phase where one can assume the renormalization constant K to be unity. This would be appropriate if the auditor has no direct evidence at F to support that there is no fraud, i.e., $m_S(\sim f) = 0$, and there is no direct evidence to support that there is no incentive and there is no attitude and there is no opportunity to commit fraud, i.e., $m_I(\sim i) = m_A(\sim a) = m_O(\sim o) = 0$. In such a situation, the fraud risk model becomes very simple:

$$FR = RI \times RA \times RO \times RS. \quad (4)$$

Assessing Audit Risk, Fraud Risk and Audit Planning

To illustrate the use of the above formulation, consider next the steps needed to assess both audit risk based on a SAS 107 (AICPA 2006a, see also ASA 200 in AUASB 2007) definition and then fraud risk based on the above formula. For each of the assessments and based on available client information, each risk assessment would need to be made using a scale from 0.00 to 1.00, where 0.00 = no chance of occurrence, 0.50 = a 50% chance of occurrence or similar to a coin flip, and 1.00 = a 100% chance of occurrence.

First the auditor would assess the risk of a material misstatement resulting from unintentional accounting systems errors including material weaknesses in internal control over financial reporting (PCAOB Auditing Standard No. 5, 2007, see also ASA 315 in AUASB 2006b). This would involve deciding on Acceptable Audit Risk (AAR). Typically acceptable audit risk is set at a low level between .05 and 0.10.

Next, the auditor would assess Inherent Risk (IR), Control Risk (CR), Analytical Procedure Risk (APR), and Test of Details Risk (TD). Risk of material misstatements due to errors and misappropriation of assets then may be calculated as $IR \times CR \times APR \times TD$ which is the usual algebraic specification of audit risk. If AAR is set at say 0.05, then the needed level of TD risk, TD', may be derived from this formula: $TD' = AAR / (IR \times CR)$

As step two, fraud risk (the risk of management committing fraud) would be assessed and the acceptable risk for special forensic audit procedures (RS) may be determined. These determinations would be based on the above simplified fraud risk formula as expressed in Equation (5):

$$FR (\text{Fraud risk}) = RI \times RA \times RO \times RS \quad (5)$$

Based on the information the auditor would have on the client, each of the factors would need to be assessed. These would also be made using the 0.00 to 1.00 scale defined above.

If the auditor is in the phase in an audit where special forensic auditing procedures are being considered, but none have been implemented, then $RS = 1.0$ and thus

$$\text{Fraud Risk} = RI \times RA \times RO$$

Also, if the audit firm plans to achieve a target level of fraud risk, for example a very low risk of Acceptable Fraud Risk (AFR) = 0.005 in order to give a clean opinion, the prior

assessments may be used to compute the level of risk of special forensic procedures (RS') needed in the audit program:

$$RS' = AFR / (RI \times RA \times RO) = 0.005 / (RI \times RA \times RO)$$

These two steps allows the auditor to determine the target detection risk associated with the special forensic procedures, that is to set the risk guidelines for both standard test of details audit procedures (TD') and special forensic audit procedures (RS'). Two cases are of particular interest:

1. No special procures are required: In many cases, the auditor's assessment of RI, RA, and RO are such that the product of these risk are less than or equal to the target acceptable fraud risk (AFR). In such cases, no special forensic audit procedures would be needed. This would be the case in an audit if, for example, the client has implemented a system of management controls and corporate governance such that RI or RO is very low. For example, assume incentives are managed such that management is judged to have very minimal incentive to comment fraud, say assessed RI = .005. Then, even if RA and RO are assessed at 1.0, RS' = 1.0! This means that the audit plans could be set with 100% risk that special forensic audit procedures would not detect fraud. In this case, no special procedures would need to be conducted.
2. Special procures are required: A second very critical case would be when the assessments imply that special forensic procedures are needed. Assume the same very low risk of Acceptable Fraud Risk (AFR) = 0.005 is the firm's maximum fraud risk in order to give a clean opinion, but assume the audit team, following their SAS 99 (AICPA 2002, see also ASA 240 in AUASB 2006a) assessments of fraud risks, assess RI = 0.5 and RO =0.4. Given limited evidence about RA, it is assessed at 1.0. This means that the auditor believes there is a 50% chance of significant incentives that could motivate management to commit financial statement fraud and a 40% chance that there are opportunities, perhaps because of a weak system of internal control over financial reporting, to perpetuate fraud.

Thus $RS' = 0.005 / (RI \times RA \times RO) = 0.005 / (0.5 \times 1.0 \times 0.4) = 0.025$. This implies that quite strong special forensic procedures are required such that the combined detection risk of these procedures is only 2.5%.

These two illustration show how the derived fraud risk formula could be implemented using a two-step approach. Mock, Srivastava and Wright (2010) conducted an experimental study to investigate the impact of using the above fraud risk model on the assessed value of the

fraud risk. They used two cases, one with high fraud risk scenario and the other with a low fraud risk scenario based on a real fraud case. One of their interesting findings suggests that under the high fraud risk situation the auditors who used the above fraud risk model to assess the fraud risk along with using the traditional audit risk model for planning the audit were better able to distinguish between the high and low fraud risk treatments. However, similar to prior research (Zimbelman 1997), they were not able to translate this ability to distinguish level of fraud risk into a set of special forensic procedures that were judged to be more effective.

4. SUMMARY AND CONCLUSION

In this paper we introduce the Dempster-Shafer theory of belief functions for managing uncertainties and demonstrate its use by deriving a fraud risk assessment formula for a simplified version of the Srivastava, Mock and Turner model (2007). In addition, we have discussed the use of the fraud risk assessment model for planning a financial audit with the risk of the presence of not only material misstatements due to errors and irregularities, but also due to management fraud. And finally, we argue against the use of a single audit risk model as proposed by the AICPA through SAS 107 (AICPA 2006a, see also ASA 200 in AUASB 2007), and suggest an alternative developed in this paper that auditors use two separate risk assessments models, one for errors and irregularities and the other for assessing management fraud.

Since this paper is an introductory paper, we have simplified the derivation of the management fraud risk assessment formula by not considering the interrelationships among the fraud triangle factors. Srivastava, Mock, and Turner (2007) do consider the interrelationships among the fraud triangle factors in their model. One can further modify the formula derived in this paper by decomposing the risk related to each fraud factor into two components, one for the

existence of the corresponding threat factors and the other for the existence of the corresponding control factors such as corporate governance, compensating committee, and audit committee.

APPENDIX A

DEMPSTER'S RULE OF COMBINATION

Like Bayes rule in probability theory, Dempster's rule is used in DS Theory to combine independent items of evidence. Let us consider two items of evidence E_1 and E_2 pertaining to a frame Θ and the corresponding belief masses as represented by m_1 and m_2 . The combined belief masses (m-values) for a subset A of the frame Θ using Dempster's rule are given by (Shafer 1976):

$$m(A) = (1/K) \sum \{m_1(B_1)m_2(B_2) | B_1 \cap B_2 = A, A \subseteq \Theta\},$$

where K is a "renormalization" constant given by:

$$K = 1 - \sum \{m_1(B_1)m_2(B_2) | B_1 \cap B_2 = \emptyset\}.$$

In order to illustrate the combination of evidence, we will continue to use the evidence obtained in the audit example discussed in Section 2. In the first example, the auditor obtained evidence in support of the assertion that sales had occurred from the review of sales documents, providing the following m-values:

$$m_1(s) = 0.6, m_1(\sim s) = 0.2, m_1(\{s, \sim s\}) = 0.2$$

Suppose that the auditor performs an additional audit procedure by confirming with a sample of customers the occurrence (existence) of sales transactions and obtains confirmations from all customers in the sample that the sales did occur. Assume further that the strength of this evidence is assessed as 0.7 that it confirms s ; 0.0 that s is disconfirmed and 0.3 unassigned, i.e.,

$$m_2(s) = 0.7, m_2(\sim s) = 0, m_2(\{s, \sim s\}) = 0.3$$

Using Dempster's rule of combination, one can combine the above independent items of evidence as follows:

$$K = 1 - [m_1(s) \times m_2(\sim s) + m_1(\sim s) \times m_2(s)] = 1 - (0.6 \times 0.0 + 0.2 \times 0.7) = 0.86,$$

$$\begin{aligned} m(s) &= [m_1(s) \times m_2(s) + m_1(s) \times m_2(\{s, \sim s\}) + m_1(\{s, \sim s\}) \times m_2(s)]/K, \\ &= [0.6 \times 0.7 + 0.6 \times 0.3 + 0.2 \times 0.7]/0.86 = 0.74/0.86 = 0.86, \end{aligned}$$

$$\begin{aligned} m(\sim s) &= [m_1(\sim s) \times m_2(\sim s) + m_1(\sim s) \times m_2(\{s_2, \sim s_2\}) + m_1(\{s, \sim s\}) \times m_2(\sim s)]/K, \\ &= [0.2 \times 0 + 0.2 \times 0.3 + 0.2 \times 0]/0.86 = 0.06/0.86 = 0.07, \end{aligned}$$

$$m(\{s, \sim s\}) = [m_1(\{s, \sim s\}) \times m_2(\{s, \sim s\})]/K = [0.2 \times 0.3]/0.86 = 0.07.$$

Based on the combined m-values for the assertion that sales have occurred, the auditor can conclude that, according to the evidence collected, he/she has 0.86 level of support that the sales have occurred, i.e., $Bel(s) = 0.86$, and 0.07 level of support that the sales have not occurred, i.e., $Bel(\sim s) = 0.07$. The plausibility that the sales actually have occurred is $Pl(s) = 1 - Bel(\sim s) = 1 - 0.07 = 0.93$, and the plausibility that [a material amount of] stated sales have not occurred, representing fictitious sales, is $Pl(\sim s) = 1 - Bel(s) = 1 - 0.86 = 0.14$. As can be seen, even with the strong confirmation test results, the plausibility of the sales being misstated is still higher than the normal acceptable risk level, e.g. 0.05, and further testing of this assertion would be needed.

In the following appendix, we provide a formulation that will aid the audit team in assessing such risks, particularly those related to financial statement fraud.

APPENDIX B

DERIVATION OF FRAUD RISK ASSESSMENT FORMULA

In order to derive a fraud risk assessment formula for the evidential diagram in Figure 1, we complete the following three steps. First, we need to express the assessed strengths of all the evidence depicted in Figure 1 in terms of belief masses. These assessments indicate whether the evidence supports the presence and/or absence of each of the variables. These belief masses are given in the corresponding evidence boxes in Figure 1. For example, the belief masses related to the evidence pertaining to incentives I are $\{m_I(i), m_I(\sim i), m_I(\{i, \sim i\})\}$

Second, we combine the belief masses at I, A, and O, and propagate the resulting belief masses through the ‘AND’ relationship to the variable F. Third, we combine the two sets of belief masses at the variable F; one set of belief masses from the evidence E_S directly pertaining to F, and the other set that was obtained as a result of propagation of belief masses from the three fraud risk factors.

Propagation of Belief Masses from I, A, and O, to F

Since the diagram in Figure 1 is an ‘AND tree, we use Proposition 1 of Srivastava, Shenoy and Shafer (1995) to propagate belief masses from I, A, and O to F and obtain the following belief masses.

$$m_{F \leftarrow IAO}(f) = m_I(i)m_A(a)m_O(o) \quad (B1)$$

$$m_{F \leftarrow IAO}(\sim f) = 1 - [1 - m_I(\sim i)][1 - m_A(\sim a)][1 - m_O(\sim o)] \quad (B2)$$

$$m_{F \leftarrow IAO}(\{f, \sim f\}) = [1 - m_I(\sim i)][1 - m_A(\sim a)][1 - m_O(\sim o)] - m_I(i)m_A(a)m_O(o) \quad (B3)$$

Combination of Belief Masses at variable F

In Figure 1, we have two sets of m-values at variable F, one from Evidence E_S as defined in Equation 4, and the other set is the result of propagating belief masses from the variables I, A,

and O to F, as given in Equations (B1-B3). We use Dempster's rule to combine the two sets of belief masses. However, since all the variables in the present case are binary variables, we use Srivastava (2005, Equations 10-13) to determine the combined belief masses:

$$m_F(f) = 1 - [1 - m_{F \leftarrow IAO}(f)][1 - m_S(f)]/K, \quad (B4)$$

$$m_F(\sim f) = 1 - [1 - m_{F \leftarrow IAO}(\sim f)][1 - m_S(\sim f)]/K, \quad (B5)$$

$$m_F(\{f, \sim f\}) = m_{F \leftarrow IAO}(\{f, \sim f\}) m_S(\{f, \sim f\})/K, \quad (B6)$$

where K is renormalization constant defined as

$$K = 1 - [m_{F \leftarrow IAO}(\sim f)m_S(f) + m_{F \leftarrow IAO}(f)m_S(\sim f)]. \quad (B7)$$

Belief and Plausibility of Fraud

From Equations (B4 & B5) and Equations (B2 & B3), we obtain the following belief and plausibility in fraud:

$$\begin{aligned} \text{Bel}(f) &= 1 - [1 - m_{F \leftarrow IAO}(f)][1 - m_S(f)]/K, \\ &= 1 - [1 - m_I(i) m_A(a) m_O(o)][1 - m_S(f)]/K \end{aligned} \quad (B8)$$

$$\begin{aligned} \text{Pl}(f) &= [1 - m_I(\sim i)][1 - m_A(\sim a)][1 - m_O(\sim o)][1 - m_S(\sim f)]/K, \\ &= \text{Pl}_I(i)\text{Pl}_A(a)\text{Pl}_O(o)\text{Pl}_S(f)/K \end{aligned} \quad (B9)$$

The above formulas for the belief in fraud, $\text{Bel}(f)$, and the plausibility in fraud, $\text{Pl}(f)$, are general in the sense that they include positive, negative or mixed items of evidence. However as noted, we do not consider any interrelationships among the three fraud factors as considered by Srivastava, Mock, and Turner (2007).

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Figure 1. An Evidential Diagram for Assessing Fraud Risk in a Financial Statement Audit which includes Fraud Triangle Factors assuming no interrelationships.

[A rounded box represents a variable, a rectangle represents an item of evidence, and a hexagonal box represents a relationship].

