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Discussion on "An Empirical Test of Bentham's Theory of the Persuasiveness of Evidence"

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INTRODUCTION

As a discussant, my role is to provide a constructive criticism of the article. On the good side, the article has tackled a difficult issue and an important one. Both practice and academia are struggling to get a grip on how to measure the persuasiveness of an item of evidence or an evidence set. As the authors identify, the audit process is nothing but evidential reasoning under uncertainty. Although not explicitly stated, the article deals with two issues: the persuasiveness of a single item of evidence, and the persuasiveness of an evidence set. Four out of six hypotheses (H1 -H3, and H6) deal with the persuasiveness of an evidence set.

The authors make a strong argument in favor of the need for a general theory of evidence and for a quantitative measure of the strength or persuasiveness of evidence to be used in decision aids and expert systems by practitioners. I fully concur with this sentiment. However, I do not entirely agree that: "Yet, to date, no general theory of audit evidence has been accepted in either the research or practice literature." I believe that general theories of evidence do exist in auditing and outside of auditing too. One such theory is based on the probability framework (e.g., Edwards 1984, Pearl 1990a-d, Dutta and Srivastava 1993, Kissinger 1977, Schum 1987, 1990, Stephens 1983, Toba 1975) and the other is based on the Dempster-Shafer theory of belief functions (e.g., Shafer and Srivastava 1990, Srivastava 1995a, 1995b, Srivastava and Johns 1992, Srivastava, et al. 1995, Srivastava and Shafer 1992). The probability framework is appropriate when we have detailed knowledge about various probabilities and conditional probabilities, whereas the belief-function framework is appropriate under situations where not all probabilities and/or conditional probabilities are known.

I feel that the authors have done a good job on the empirical work in the article. However, I am disappointed with the theory side of the paper. First, the authors are a bit carried away with Bentham's

theory. I believe that Bentham's theory is based on intuitive reasoning and is not based on well-founded theory. As I will show, all the hypotheses derived in the article from Bentham's theory can also be derived from the theory of evidence based on a probability framework.¹ What additional insights does Bentham's theory give auditors? Why do we need to go back over 150 years for a theory of evidence when currently we have well-founded theories based on the mathematical theory of probabilities?

Secondly, the authors seem to believe that they are determining the intrinsic characteristics of audit evidence that make the evidence persuasive. But, as stated earlier, four out of six hypotheses deal with the combination of evidence and not with the intrinsic properties of evidence that make it persuasive. Third, Bentham's theory does not consider the impact of the contextual situation on the strength of evidence (Twining 1985). For example, the same set of evidence would provide a different level of persuasiveness if the background story were changed. A suspect of a murder may be convicted for the crime based on the circumstantial evidence and his confession. However, it may later be determined that he only happened to pass by the murder scene and was threatened by the real murderer to admit the crime otherwise lose all his family members' lives.

In general, there are three important issues related to evidential reasoning in auditing: the structure of the audit evidence, the representation of the strength of evidence, and the combination of various items of evidence (e.g., Boritz and Jensen 1985, Boritz and Wensley 1990, Shafer and Srivastava 1990, and Srivastava and Shafer 1992). Bentham's theory of persuasiveness is not capable of dealing with any of the above issues in a comprehensive way. It provides no guidance for the persuasiveness of evidence in complex situations where one item of evidence supports several audit objectives or accounts. Also, it is incapable of dealing with situations where the evidence relates to partial ignorance, that is the evidence supports H_0 to some degree, contradicts H_0 to some other degree and maintains a certain level of support for both possibilities, $\{H_0, \sim H_0\}$, which represents ignorance. Although, the probability framework is not capable of dealing with such situations, the belief-function framework is.

PROBABILITY FRAMEWORK VERSUS BENTHAM'S THEORY

In this section, I will show how one can derive all the hypotheses developed by Caster and Pincus (1996) by using the probability framework. Dutta and Srivastava (1993) used the likelihood ratio, λ , (Edwards 1984) to represent the strength of audit evidence and showed how various items of evidence could be combined. The strength of evidence² E in support of hypothesis H is given by the likelihood ratio λ as:

$$\lambda = \frac{P(E|H)}{P(E|\sim H)}, \quad (1)$$

where $P(E|H)$ and $P(E|\sim H)$, respectively, are probabilities that E occurs given that H has occurred or H has not occurred ($\sim H$). For positive evidence, that is, for an item of evidence that supports H, the likelihood ratio, λ , will be greater than 1. As the value of λ increases, the strength of evidence increases. For positive evidence, E, the posterior probability is greater than the prior probability, that is, $P(H|E) > P(H)$. When $P(E|\sim H) = 0$ ($\lambda = \infty$), the posterior probability $P(H|E) = 1$, provided $P(E) > 0$, implying that H will occur whenever E will occur. For neutral evidence that adds no new information on H, $\lambda = 1$ and $P(H|E) = P(H)$. For a negative item of evidence, the likelihood ratio is less than 1 and positive; the lower the value, the stronger the negative evidence. For $\lambda = 0$, the negative evidence is infinitely strong; knowing that E has occurred will guarantee that $\sim H$ has occurred, that is, $P(\sim H|E) = 1$.

Dutta and Srivastava have shown that the combined strength, Λ , of n independent items of evidence bearing on one assertion is equal to the product of the individual strengths (λ s):

$$\Lambda = \prod_{i=1}^n \lambda_i. \quad (2)$$

Amount of Evidence: Number of Tests

This situation deals with independent items of evidence supporting one side of an issue as described by the authors. Proposition 1 below relates to the authors' hypothesis 1.

Proposition 1: As the amount of evidence (number of non-redundant tests) increase, the persuasiveness of the evidence set increases

It is clear from (2) that for positive evidence ($\lambda > 1$), the combined strength Λ which is the product of the individual strengths always increases as new items of evidence are included in the evidence set. For example, when there are two positive independent items of evidence with strengths λ_1 and λ_2 , the combined strength is $\lambda_1 \lambda_2$. Since $\lambda_1 > 1$, and $\lambda_2 > 1$ (positive evidence), the combined strength is always greater than the individual strengths. Consider a third positive item of evidence with strength λ_3 , the combined strength now will be $\lambda_1 \lambda_2 \lambda_3$ which is always higher than $\lambda_1 \lambda_2$. This result shows that as we increase the number of independent positive items of evidence in an evidence set, the combined strength of the evidence set increases. Similarly, we can show that for all negative items of evidence ($0 < \lambda < 1$), the combined strength of the evidence set increases as the number of negative items of evidence increases in the set. This discussion proves proposition 1.

Dispersion of Estimates

The authors have slightly modified Bentham's theory of persuasiveness of evidence to develop a hypothesis to suit the auditing situation. However, some important minor details seem to be omitted in the modification. Let me first focus on Bentham's theory for a situation where several witnesses are testifying about the same matter. According to the authors, "Bentham suggested a simple model for weighting the testimony of witnesses. When witnesses 'operated' strictly to prove a fact, the number of witnesses could be summed to determine the probative force of the evidence." From this discussion we can conclude that when two equally credible witnesses testify to the same side of the issue then the combined evidence is more persuasive than when two witnesses testify on opposite sides of the issue. This statement is similar to Hypothesis 3 and I will postpone further discussion until the next section.

The authors' modification of Bentham's theory for this section deals with making multiple estimates of the same quantity using several different independent audit procedures. This situation is very different from the first situation where witnesses testify on one side of an issue. In Bentham's case, there

are only two sides of the issue, guilty or not guilty, whereas in the auditing example there is a continuum of outcomes. Thus, we need to think carefully about the auditing situation. For example, in the authors' example of two estimates of the Allowance for Future Returns we may ask the following questions: (1) How close are the two estimates to the recorded value, that is, how strong is the evidence individually and collectively that the estimates support the recorded value? (2) Are the two estimates significantly different or the same, that is, how strong is the evidence that the two estimates are the same? The first question is of interest in auditing. However, it appears to me from hypothesis 2 that the authors are investigating the second question, which is of interest only if we are testing to determine whether two independent procedures yield the same result. There is no discussion on a third quantity, the recorded value. In the following paragraphs, I generate two testable propositions related to the above questions.

In order to answer the two questions raised above, I need to discuss some concepts that may not be familiar. We all know how to accept or reject a null hypothesis ($H_0: \mu = \mu_0$) in comparison with an alternative hypothesis ($H_a: \mu \neq \mu_0$) using a t-test. However, we do not know the level of support for or against the two hypotheses when the measured value is, say $\mu=b$. The likelihood ratio provides such a measure (see. e.g., Edwards 1984, and Pearl 1990c). Also, Dutta and Srivastava (1993) have argued that the likelihood ratio is a good measure of the strength of evidence. Swets et al. (1964) have used the likelihood ratio in signal detection theory. If f_0 and f_a represent, respectively, the probability density functions for H_0 and H_a then the likelihood ratio, $\lambda_{H_0}(b)$, representing the strength of evidence in favor of H_0 , when the measured value of $\mu = b$ is (Edwards 1984, Swets et al. 1964):

$$\lambda_{H_0} = \frac{f_0(\mu=b)}{f_a(\mu=b)}. \tag{3}$$

----- Figure 1 about here -----

----- Figure 2 about here -----

Figure 1 represents normal probability distribution functions for $H_0(\mu=100)$ and $H_a(\mu=150)$. Figure 2 shows the variation in the strength of evidence as a function of the measured value of μ . The

results are intuitive. We obtain the highest level of support for H_0 when the measured value is equal to the value of μ for H_0 and vice versa. Thus based on the above discussion, we can write the following proposition:

Proposition 2a: *Ceteris paribus*, as the difference between the estimate and the desired outcome decreases the persuasiveness of the evidence increases.

Let us consider the situation related to the second question asked earlier where we have two or more estimated values from independent procedures and we want to determine if they are all the same or significantly different. Such tests are performed using t-tests and ANOVA. However, our interest here is in terms of the strength of evidence that, for example, the two estimates are close to each other. Again representing the strength of evidence by the likelihood ratio, we can show that the likelihood ratio is the highest, that is, the level of support is the highest, when the difference between the two estimates is zero and decreases as the difference increases. This result can be written in terms of the following proposition:

Proposition 2b: *Ceteris paribus*, as the difference between two estimates increases the persuasiveness of the evidence in favor of the hypothesis that the two estimates are the same decreases.

The above proposition is similar to hypothesis 2 of the authors. Thus, hypothesis 2 is shown to be derived using probability framework. However, if the authors' contention is that hypothesis 2 deals with the assertion that the account balance is fairly stated then they should clearly state the reference point, the recorded mean of the account balance.

Composition of Evidence Set

This situation deals with a set of items of evidence with some in support of an assertion and some in support of the negation of the assertion. That is, some items of evidence are positive represented by likelihood ratios with values greater than one, and some are negative, supporting the

negation of the assertion, represented by likelihood ratios with values less than one. Consider the following two situations with three items of evidence.

Situation 1: All the three items of evidence are positive with strengths: λ_1 , λ_2 , and λ_3 (all λ s being greater than 1). The combined strength of the evidence set in favor of the assertion is equal to the product of all the likelihood ratios, $\lambda_C = \lambda_1\lambda_2\lambda_3$.

Situation 2: Two items of evidence are positive and one, say the third one, is negative (that is, $0 < \lambda_3 < 1$). The combined strength of the evidence set is $\lambda'_C = \lambda_1\lambda_2\lambda'_3$. Since $0 < \lambda'_3 < 1$, the combined strength under situation 2 is going to be always smaller than the combined strength under situation 1.

Thus, one can generalize the above result into the following proposition:

Proposition 3: *Ceteris paribus*, as the composition of the evidence set becomes more one-sided, the persuasiveness of the evidence set increases.

The above proposition is the same as hypothesis 3 of the authors. It should be noted that proposition 3 is true only when the common pieces of evidence on one side of the issue under the two situations have the same strengths. For example, the values of λ_1 and λ_2 under the two situations are constant.

Source Reliability

As Caster and Pincus point out, source reliability has been analytically modeled under the probability framework by Schum and DuCharme (1971) and empirically studied and validated by several researchers (Anderson et al. 1994, Bamber 1983, Rebele et al. 1988, Washington 1988). Based on the above studies, it is well accepted that as the source reliability of evidence increases, the evidence set becomes more persuasive. I do not really see the value this study adds to what we already know about it. I will not discuss the source reliability case any further.

Directness of Evidence

The authors contend without analytical proof that “direct evidence should be more persuasive than indirect evidence because alternative hypotheses can be ruled out.” I want to give a formal proof of their contention by providing a proof for the following proposition.

Proposition 5: *Ceteris paribus*, direct evidence is more persuasive than indirect evidence.

Consider the following example. Suppose you are staying in a dorm with two other roommates. You are still in your bed but want to know if it rained last night, so you ask your roommates whether it rained last night³. One of the roommates tells you that he was awake early in the morning and saw it raining. Let us say this evidence is E_1 which is direct. The other roommate looks outside the window and informs you that the lawn is wet. This evidence, E_2 , is indirect because there could be other reasons for the lawn to be wet. For example, the gardener might have turned on the sprinkler system last night. Let us assume for simplicity that the only other possibility for the lawn to be wet is the sprinkler system being on during the night. Let S represent the state that the sprinkler system is on and $\sim S$ represent the state opposite to S .

The strength of evidence E_1 in favor of hypothesis H_0 that it rained last night is given by:

$$\lambda_1 = \frac{P(E_1|H_0)}{P(E_1|H_a)},$$

where H_a is the alternative hypothesis that it did not rain. The above expression can be rewritten in terms of reliability R_1 of the roommate who said that he saw it rain last night as:

$$\lambda_1 = \frac{P(E_1|H_0)}{P(E_1|H_a)} = \frac{P(E_1|H_0)}{1 - P(\sim E_1|H_a)} = \frac{R_1}{1 - R_1}, \quad (4)$$

where the reliability of the roommate is assumed to be symmetric for simplicity. In other words, he says that it rained if indeed it did rain with the same reliability as he says that it did not rain if indeed it did not rain, that is, $R_1 = P(E_1|H_0) = P(\sim E_1|H_a)$. He tells the truth with reliability R_1 . The strength of evidence E_1 depends on reliability R_1 . The more reliable the source the more persuasive is the evidence. This situation is similar to the case of source reliability discussed earlier.

The strength of evidence E_2 can be expressed⁴ as:

$$\lambda_2 = \frac{R_2}{(1 - R_2) + (2R_2 - 1)P(S|H_a)}, \quad (5)$$

where R_2 is the reliability of the second roommate. λ_2 depends on R_2 and $P(S|H_a)$. For $P(S|H_a)=1$, $\lambda_2 = 1$, irrespective of the value of R_2 . This simply means that knowing the lawn is wet provides no evidence for H_0 that it rained last night if the sprinkler is always on when there is no rain.

Assuming the two roommates being equally reliable ($R_1 = R_2 = R$), we can show that the strength of evidence E_1 is greater than the strength of evidence E_2 , that is, $\lambda_1 > \lambda_2$, for $R > 0.5$ (see Table 1). That is, for reliable roommates ($R > 0.5$), the direct evidence is always more persuasive than the indirect evidence. This is what hypothesis 5 of the authors suggests. However, one can see from Table 1 that indirect evidence is stronger than direct evidence for unreliable sources ($R_1 < 0.5$, and $R_2 < 0.5$). Thus, proposition 5 is true only under certain conditions implying that hypothesis 5 can not be true in general.

----- Table 1 about here -----

For $P(S|H_a) = 0$, $\lambda_1 = \lambda_2$ when $R_1 = R_2 = R$, irrespective of the value of R . This means that the two items of evidence are identical since the sprinkler is not on when there is no rain. Thus, it is the same whether the first person tells that he saw it rain last night or the second person says after seeing outside that the lawn is wet; the strength of evidence in either case depends on the reliability of the two individuals. If they are perfectly reliable then, of course, it rained last night if one says that he saw it rain or the other tells that the lawn is wet.

Deviations from Expectations

Pincus (1991) has found that deviations from expectations reduce confidence in auditors' judgments, implying evidence is less persuasive when it is unanticipated than when it is anticipated. Caster and Pincus (1996) have related this finding to Bentham's theory of persuasiveness of evidence and proposed hypothesis 6. I want to show how one can derive this result using the probability framework. The basic premise is that the auditor expects one kind of outcome, but after conducting the

audit, he or she obtains just the opposite. As an example, consider that the prior year working papers suggest that there is no material misstatement expected in the account balance. This means the auditor has positive evidence towards the account balance being fairly stated based on the prior year working papers. Let us represent the strength of this evidence by λ_1 ($\lambda_1 > 1$). However, after conducting the audit the auditor finds that the account balance is materially misstated and would require an adjustment. This evidence is negative and its strength can be represented by λ_2 with λ_2 being less than 1. If we combine the two items of evidence then the resulting strength will be less than λ_1 . If the situation were just reversed, that is, the prior year information was negative and the current year evidence was positive, then the combined strength would still be less than the strength of the positive evidence. However, if both were positive or both were negative then the combined strength in favor of or against the assertion would be stronger than the individual strengths. This discussion proves the following proposition:

Proposition 6: *Ceteris paribus*, when evidence is unanticipated, the evidence set becomes less persuasive.

The above proposition is hypothesis 6 of the authors.

Interaction of Characteristics

We do expect to see interaction between various characteristics of persuasiveness of evidence since they all jointly determine the overall strength of the evidence set. Using the likelihood approach to represent strength of evidence, one can easily analyze the pair-wise interaction and how it would affect the overall strength of the evidence set. The authors' findings about the interactions were significant, as expected. However, in one situation, they contend that they obtained an unexpected result: "the subjects were insensitive to dispersion in estimates when all evidence was negative." However, it seems that if we consider the conservative nature of the auditor then the above finding is not really unexpected; it is quite normal for the auditor to adjust the account if all the evidence is negative irrespective of the amount of dispersion in estimates.

OTHER ISSUES AND CONCLUSIONS

In this section, I summarize my remaining concerns on the article. The authors refer to the work of Toba (1975), Kissinger (1977), and Stephens (1983), and also recognize that evidence aggregation might be viewed as a problem of combining probabilities. However, they seem to have missed the work on combining evidence by Dutta and Srivastava (1993) who have done just that; they have developed a theoretical framework for aggregating evidence by combining their strengths (based on likelihood ratios).

Bentham's theory of persuasiveness is unduly emphasized in the summary section. Formal theories of evidential reasoning do exist outside of auditing and even in auditing (e.g., for probability framework, see, Edwards 1984, Pearl 1990a-1990d, and Dutta and Srivastava 1993, and for the Dempster-Shafer theory of belief functions, see Shafer and Srivastava 1990, Srivastava and Shafer 1992, Srivastava 1995a, 1995b). Further, the authors state that:

The results of this study are promising in that Bentham's theory does appear to capture important characteristics of evidence sets that affect persuasiveness in an audit setting. Further research to more fully develop and test this model would be helpful in establishing *its appropriateness as a general theory of persuasiveness of audit evidence* (page 24, emphasis added).

I feel this statement is too strong given that we have formal theories of evidence such as probability theory and the Dempster-Shafer theory of belief functions. As shown in previous sections, there is nothing special about Bentham's theory. All the results have been derived using probability framework. One can derive the same results under the belief-function framework.

Bentham's theory is not a comprehensive theory of evidence as claimed by the authors. It does not provide any guidance as to how or what level of persuasiveness one would obtain when combining evidence in a complex network of audit objectives as discussed by Srivastava (1995a, 1995b). Also, Bentham's theory provides no guidance as to how one can determine the persuasiveness of evidence when a certain level of ignorance is present with only partial knowledge that the null hypothesis is true or

an alternative hypothesis is true. Even the traditional probability theory has problems dealing with such items of evidence. However, one can use the Dempster-Shafer theory of belief functions to deal with such items of evidence (Shafer 1976).

I believe the whole audit process is nothing but evidential reasoning under uncertainty. Currently, we have several well founded frameworks such as the Bayesian framework and the Dempster-Shafer theory of belief functions that should provide us with ample opportunities to deal with the real world issues related to audit decisions. There are many unanswered theoretical as well as empirical questions related to such issues as (1) representation of the strength of evidence, (2) measurement of the strength of evidence, (3) combination of the strength of various items of evidence, (4) persuasive evidence versus convincing evidence, (5) integration of statistical and non-statistical evidence (for the belief-function case see, Srivastava and Shafer 1994), and so on. The authors' work is just the beginning.

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Figure 1

Normal Probability Distribution Density Functions, $f(x|H_0)$, and $f(x|H_a)$, for $H_0(\mu = 100)$ and $H_a(\mu = 150)$ with $\sigma = 20$.

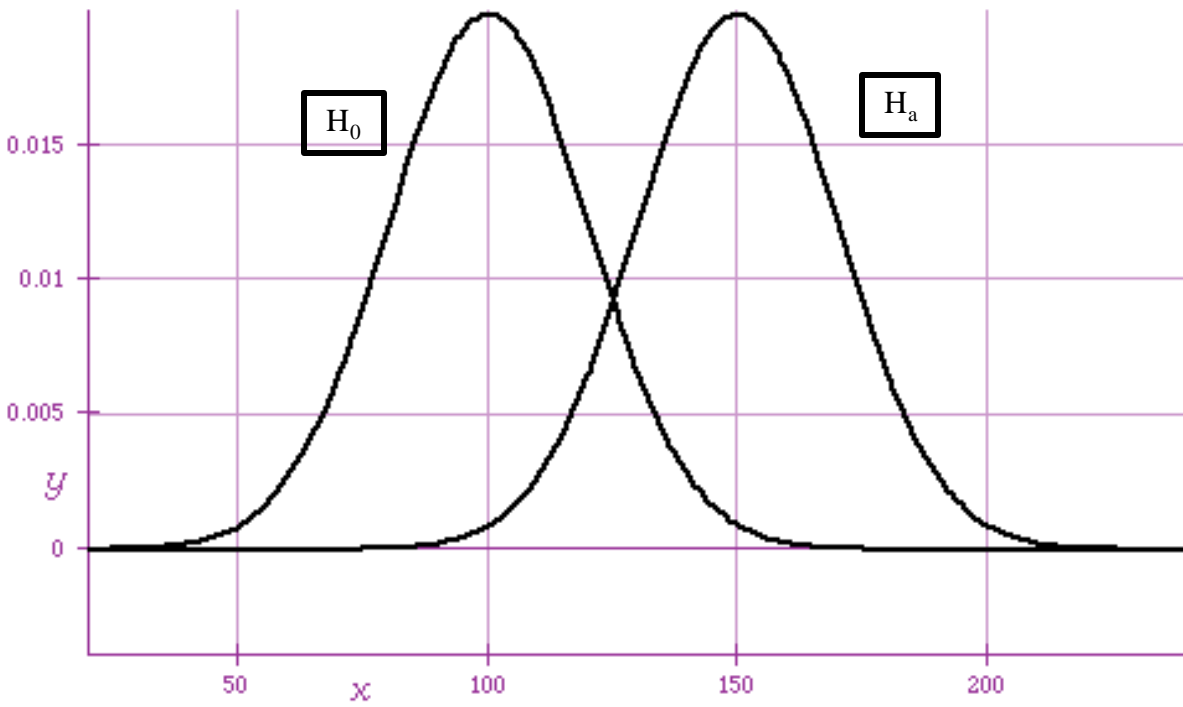


Figure 2

The Likelihood Ratios, λ_{H_0} and λ_{H_a} , in Favor of $H_0(\mu = \$100)$ and $H_a(\mu = \$150)$, respectively, as a function of the observed values of mean μ . The Likelihood Ratios are defined as:

$$\lambda_{H_0} = f(\mu=b | H_0) / f(\mu=b | H_a), \text{ and } \lambda_{H_a} = 1 / \lambda_{H_0}.$$

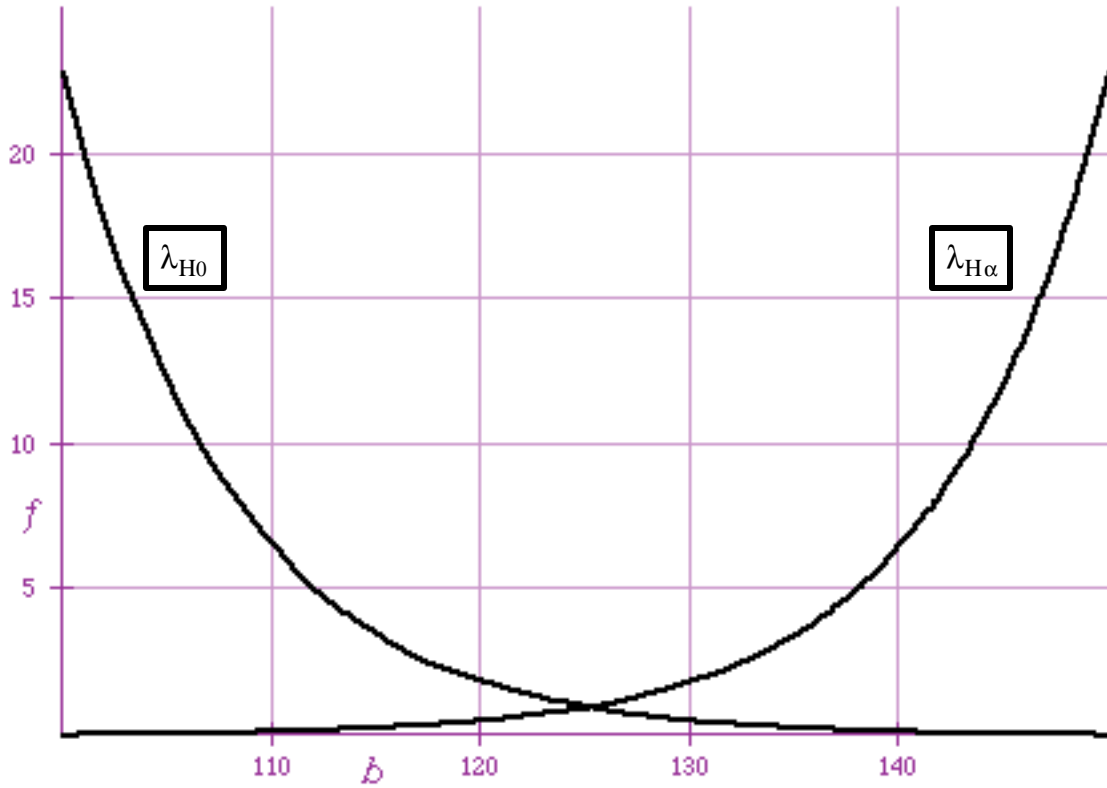


Table 1

Directness of Evidence

Probability that sprinkler is on given that it is not raining $P(S H_a)$	Reliability (R)	Strength of Evidence E_1 (λ_1)	Strength of Evidence E_2 (λ_2)
1	1.0	∞	1
	0.8	4.00	1
	0.6	1.50	1
	0.5	1.00	1
	0.4	0.67	1
	0.2	0.25	1
	0.0	0.00	1
0.8	1.0	∞	1.25
	0.8	4.00	1.18
	0.6	1.50	1.07
	0.5	1.00	1.00
	0.4	0.67	0.91
	0.2	0.25	0.63
	0.0	0.00	0.00
0.5	1.0	∞	2.00
	0.8	4.00	1.60
	0.6	1.50	1.20
	0.5	1.00	1.00
	0.4	0.67	0.80
	0.2	0.25	0.40
	0.0	0.00	0.00
0.4	1.0	∞	2.50
	0.8	4.00	1.82
	0.6	1.50	1.25
	0.5	1.00	1.00
	0.4	0.67	0.77
	0.2	0.25	0.36
	0.0	0.00	0.00
0.0	1.0	∞	∞
	0.8	4.00	4.00
	0.6	1.50	1.50
	0.5	1.00	1.00
	0.4	0.67	0.67

	0.2	0.25	0.25
	0.0	0.00	0.00

¹ One can also derive these hypotheses using belief functions.

² This definition of the strength of evidence is limited to situations where no ignorance is present. In fact, we can not define the strength of evidence under situations of partial ignorance using the probability framework. The Dempster-Shafer theory of belief functions provides a better framework for such situations (Shafer 1976).

³ Pearl (1988) has discussed a similar example.

⁴ The likelihood ratio, λ_2 , can be written as:

$$\begin{aligned} \lambda_2 &= \frac{P(E_2|H_0)}{P(E_2|H_a)} = \frac{P(E_2 \cap S|H_0) + P(E_2 \cap \sim S|H_0)}{P(E_2 \cap S|H_a) + P(E_2 \cap \sim S|H_a)}, \\ &= \frac{P(E_2|S \cap H_0)P(S|H_0) + P(E_2|\sim S \cap H_0)P(\sim S|H_0)}{P(E_2|S \cap H_a)P(S|H_a) + P(E_2|\sim S \cap H_a)P(\sim S|H_a)}, \\ &= \frac{R_2 P(S|H_0) + R_2 P(\sim S|H_0)}{R_2 P(S|H_a) + (1 - R_2) P(\sim S|H_a)}, \\ \text{or} \quad \lambda_2 &= \frac{R_2}{(1 - R_2) + (2R_2 - 1)P(S|H_a)}, \end{aligned}$$

where R_2 is the reliability of the second roommate. It is also assumed to be symmetric:

$$R_2 = P(E_2|S \cap H_0) = P(E_2|\sim S \cap H_0) = P(E_2|S \cap H_a) = P(\sim E_2|\sim S \cap H_a).$$

That is, the second roommate tells the truth with reliability R_2 .