I would suggest that concern about conflicting duties be limited to the person performing a task and that person's immediate supervisor. An immediate supervisor normally could be expected to be able to perform the tasks assigned to a subordinate. However, it is unlikely that any person in a supervisory level above the immediate supervisor would perform a task regularly performed by someone two organizational levels below them.

The authors also indicate that the hierarchical relationships incorporated into the model enable one to pinpoint separate roles that may lead to collusion. Normally auditors do not extend audit testing because of the possibility of collusion. Collusion is always a possibility. To extend tests when collusion is a possibility would mean that all tests would always be extended. Thus, there would be no need to evaluate internal control for purposes of determining the extent of tests. This is not to say that tests wouldn't be extended if collusion was suspected. However, in normal circumstances, the auditor must rely on his/her minimum tests to detect errors perpetrated and concealed by collusive efforts. Focusing on potential collusion appears to be an unnecessary complication of the internal control evaluation process.

The practitioner's view of internal control is changing. Conflicting duties are now given less attention than in years past. There has been a migration of concern from potential defalcations to other potential financial statement errors, specifically fraudulent financial reporting that is not necessarily the result of the misappropriation of company assets. That migration is reflected in SAS No. 55.

Unfortunately the model does not yet reflect this change. It does not attempt to evaluate the internal control environment—an area where most help is needed by practitioners. Internal control evaluation needs to focus more on the Charles Keatings of the world than on the potential conflicting duties of those that do their bidding.

Evaluating the control environment may require a far less structured approach. Perhaps what is needed is an expert system developed based on the collective knowledge of past management frauds. The interrelationship of accounting systems and control procedures is important; this is the focus of the model. However, the interrelationship of management attitudes and the control environment is critical. If I had one suggestion to make, I would encourage the authors to expand their model in that direction.

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Aggregation of Evidence in Auditing: A Likelihood Perspective

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SUMMARY

We have discussed a general scheme for aggregating audit evidence under the probability theory framework. We have argued that audit evidence forms a network of variables, variables being the accounts constituting the financial statements, the audit objectives of the accounts, and the financial statement as a whole. We have derived general expressions for combining evidence for two situations: (1) various items of evidence bearing on one variable, and (2) various items of evidence bearing on a cluster of variables. These general results are used to demonstrate how audit evidence can be aggregated on an actual audit to obtain posterior probability that the account or the financial statements are fairly presented. We have also discussed various types of dependencies encountered by the auditor and how these dependencies can be treated in the framework. Several limitations of the present study have been discussed and suggestions for further research are presented.

INTRODUCTION

The purpose of this paper is to develop a scheme for aggregating various items of evidence in an audit for planning and evaluation. It is well known in the auditing literature that some items of evidence bear on the whole financial statement, while some bear only at the account level or at the audit objective level of the account (Srivastava and Shafer 1992). Also, some items of evidence bear on more than one audit objective of an account (e.g., see AICPA 1988; Boritz and Jensen 1985; Boritz and Wensley 1990; Graham 1985a-1985e; Srivastava 1991). The auditor must aggregate all such items of evidence to form an opinion on the fairness of the financial statements taken as a whole.

Currently, auditors use their judgment (Graham 1985a, 14) to aggregate various items of evidence bearing at different levels of the account. This paper attempts to develop an objective approach under the probability theory framework to aggregate such items of evidence. Such an approach should provide an effective and efficient audit. Efficiency will increase because the auditor will be able to see the impact of evidence at all the levels. Effectiveness will increase from using the structured approach.

The audit risk model of SAS No. 47 (AICPA 1983) and the Bayesian models of Leslie (1984), Kinney (1984, 1989), and Sennetti (1991) all ignore the structure of evidence in auditing. Graham (1985a-1985e) points out that SAS No. 47 does not consider the structure of audit evidence. Boritz and Jensen (1985) first made an attempt to incorporate the structure of audit evidence in the aggregation process. They, however, considered only a tree-type structure; they did not consider situations where one item of evidence may support two or more objectives. Leslie et al. (1986) recognize the importance of the structure of audit evidence and em-

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phasize the point that assurances from various items of evidence need to be assessed at the management-assertion level of the account and then be combined. They also consider the relationship between various accounts (e.g., accounts receivable depends on sales and cash receipts) in the aggregation process. However, they do not develop a mechanism to combine the evidence at all the levels: at the financial statement level, the account level, and the audit objective level.

Recently, the AICPA has recognized the importance of the structure of evidence and its relation to various audit objectives in assessing control risk. In SAS No. 55, "The Auditor's Responsibility for Assessing Control Risk" (AICPA 1988, para. 3), we find the following statement:

> After obtaining this understanding, the auditor assesses control risk for the assertions embodied in the account balance, transaction class, and disclosure components of the financial statements.

Although the AICPA has not yet required auditors to consider the individual audit objectives when assessing other risks (e.g., inherent risk and detection risk), it appears from Graham's discussion that many are already doing so (Graham 1985a-1985e). Boritz and Wensley (1990) have developed a computer system that does consider the structure of audit evidence, but they use a heuristic approach to aggregate the strength of evidence. Such heuristics have been seen to fail in complex systems (Buchanan and Shortliffe 1984).

In this paper we employ the techniques of probability propagation and local computations developed in the artificial intelligence (AI) literature (Pearl 1986, 1988; Lauritzen and Spiegelhalter 1988; Shafer and Shenoy 1990; Shenoy and Shafer 1988, 1990) to describe the mechanism for aggregating audit evidence under the probability theory framework. However, we modify the approach by propagating the likelihood ratios instead of probabilities. A scheme based on the likelihood ratios should be of special interest in auditing for two reasons: (1) the likelihood ratio measures the strength of evidence as discussed later, and (2) the auditor should be able to determine the strength of evidence, that is, the likelihood ratios, more easily than the conditional probabilities using either subjective judgment or statistical results. It should be emphasized that our objective in the present work is to show how various items of evidence can be aggregated in the probability theory framework if we had knowledge of the corresponding strengths in terms of the likelihood ratios. We will not discuss how the likelihood ratios can be obtained, or the underlying probability models of various audit procedures. These issues, however, are important and future research efforts should be directed in this area.

An audit evidence aggregation scheme under the belief-function framework has been developed by Shafer and Srivastava (1990), Srivastava and Shafer (1992), Srivastava et al. (1990), and Srivastava (1991). However, since the Bayesian formalism remains the dominant theory of belief revision and evidence aggregation, the scheme developed in the current study using the likelihood ratios should have wider applicability.

The aggregation of audit evidence in the probability theory framework is a three-step process. The first step involves constructing a network that represents the relationships and dependencies between various accounts and objectives. The second step involves quantifying the strength of evidence in terms of likelihood ratios.[1] The third step involves aggregation of all the items of evidence to determine whether (1) a given audit objective of an account has been met, (2) a given account on the balance sheet is not materially misstated, or (3) the financial statements are fairly presented. This paper deals with all three steps of the process.

[1]Under the Bayesian framework, the likelihood ratio is a consistent measure of the strength of evidence. The likelihood ratio, λ, is the ratio of the two conditional probabilities, $P(\mathcal{E}|a)$ and $P(\mathcal{E}|\sim a)$, i.e., $\lambda = P(\mathcal{E}|a) / P(\mathcal{E}|\sim a)$ where $\mathcal{E}$ stands for the evidence and a stands for the assertion that it is met. An item of evidence is positive, i.e., the evidence supports the assertion when the likelihood ratio is greater than one (λ>1) and the evidence is negative, i.e., it supports the negation of the assertion when λ<1. When λ=1, the evidence is neutral, i.e., the evidence has no information in support of or against the assertion. The higher the value of the likelihood ratio for an item of evidence, the stronger the support in favor of the assertion. If we assume that we have no prior knowledge about the assertion, then the prior odds is one and the posterior odds is equal to the likelihood ratio according to Bayes' theorem. Thus, for λ=4 and prior odds equal to one, the posterior probability for the assertion to be met is 0.8 (see A-12 in the Appendix), while for λ=19, the posterior probability will be 0.95. For more details see Edwards (1984) and Dutta (1991).

STEP ONE: CONSTRUCTION OF AN EVIDENTIAL NETWORK

The objective here is to construct a network of audit evidence. Figure 1 represents a commonly used structure of evidence for an audit of the accounts receivable (Arens and Loebbecke 1991). A rounded box in the network represents a variable; variables are the individual accounts or the audit objectives of an account or the financial statement as a whole. In figure 1, for simplicity, we assume that the accounts receivable account has only three audit objectives: Completeness, Existence, and Valuation. We further assume that the variables in figure 1 are binary; either the account is fairly stated or not fairly stated, either the audit objective is met or not met, either the financial statement is fairly presented or not fairly presented. A circle with an "&" symbol implies that the variable on the left-hand side is related to the variables on the right-hand side through an "and" relationship. For example, the accounts receivable balance is fairly stated only when all three audit objectives on the right have been met.

A rectangular box in figure 1 represents an item of evidence, which may consist of one or more audit procedures. Table 1 presents all the procedures performed in figure 1. If an item of evidence directly bears on one or more variables, then it is connected by a line to those variables. It should be mentioned that in Procedures 4, 12, and 16 we have assumed that each of these procedures bears on only one objective. This is done to simplify the calculations and show step-by-step how various items of evidence are aggregated. In general, we do not need to make this assumption and the computations will involve a few more steps. According to Arens and Loebbecke (1991, 391-393), Procedure 4 should be connected to "Cash Receipts Valid" and "Cash Receipts Properly Valued," Procedure 12 should be connected to "Non-fictitious Sales" and "Sales Properly Valued," and Procedure 16 should be connected to three variables: "Cash Receipts Valid," "Cash Receipts Complete," and "Cash Receipts Properly Valued."

As shown in Step Three: Aggregation of Evidence, the evidential network must be used to guide and simplify computations involved. The structure of the network allows us to deal separately with pairs or clusters of variables that are connected closely. We evaluate the evidence bearing on an assertion, say X; we then analyze how our revised opinion about X affects our belief about another assertion, say Y; we then evaluate how the revised opinion about Y affects our opinion about Z, and so on. As discussed below, use of an evidential network for representing various items of evidence in an audit makes it more intuitive to treat various types of dependencies, such as the dependencies among the evidence and the dependencies among the accounts.

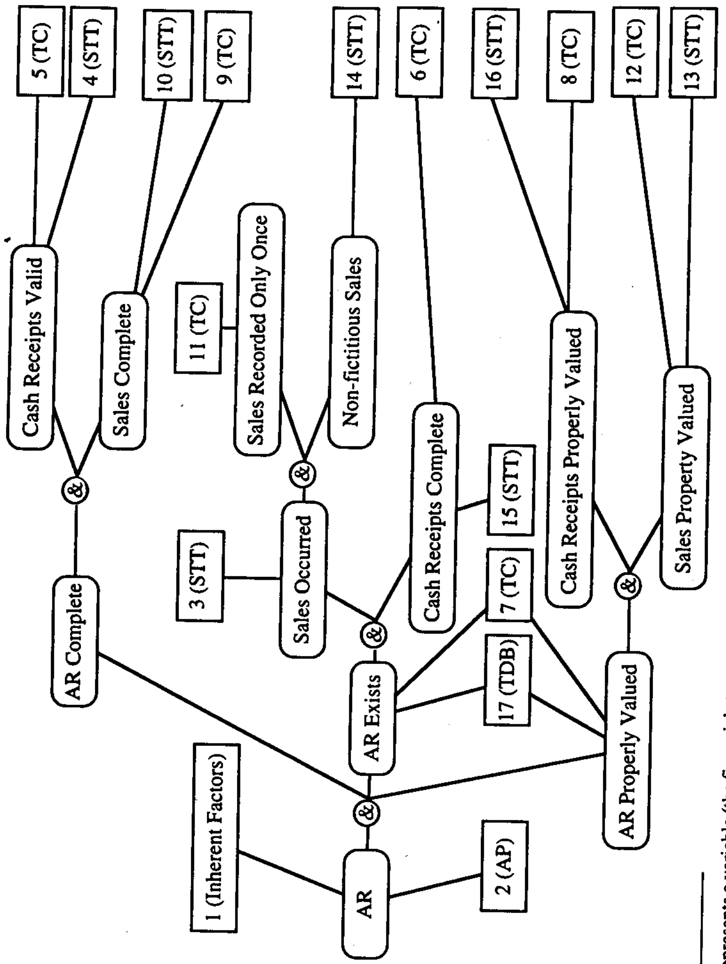
Treatment of Dependence Between Evidence

There are several audit procedures that, when conducted, simultaneously provide evidence bearing on two or more objectives or accounts. For example, confirmations of accounts receivables provide evidence that bears on both the existence and the valuation objectives of the accounts receivable. In the network, such relationships are represented by connecting the evidence to the related variables. Procedure 17 in figure 1 denotes such an item of evidence.

Treatment of Dependence Between Accounts

There are various inter-relationships and dependencies between various accounts and objectives. In such cases, obtaining evidence for an objective of an account indirectly bears on some other objective of some other account. For example, if the sales are complete and the cash receipts are valid, then the completeness objective of the accounts receivable is met. This is represented in figure 1 by connecting the completeness objective of the sales and the validity objective of cash to a node denoting the "and"

FIGURE 1
Evidential Network in an Audit of Accounts Receivable Account (Arens and Loebbecke 1991)*



*A rounded box represents a variable (the financial statements as a whole, the accounts in the financial statements, audit objectives) and a rectangular box represents an item of evidence. The line joining an item of evidence to a variable implies that the evidence directly bears on the variable. A circle with "&" implies that the variable on its left is related to the variables on its right with an "and" relationship. For example, accounts receivable's completeness objective is met only when cash receipts are valid and sales are complete (See table 1 for the description of the test procedures).

TABLE 1
The Audit Procedures used in Figures 1, and 6 - 11 (Arens and Loebbecke 1991: 391-393)

1 (Inherent Factors) - Prior years' experience with the account, related accounting system, and the control environment. Also, the knowledge about the competence and trustworthiness of accounting personnels working in the sales and collection cycle, and other relevant inherent factors.

2 (AP) - (i) Review accounts receivable trial balance for large and unusual receivables. (ii) Calculate ratios indicated in carry-forward working papers (not included here) and follow up any significant changes from prior years.

3 (STT) - Review the sales journal and ledger for unusual transactions and amounts.

4 (STT) - (i) Review the cash receipts journal and the ledgers for unusual transactions and amounts. (ii) Review the subsidiary ledger for miscellaneous credits.

5 (TC) - Observe for segregation of duties between receipt and recording of cash and also preparation of independent bank reconciliation statement.

6 (TC) - Observe whether a restrictive endorsement is used on cash receipts.

7 (TC) - Observe whether monthly statements are mailed.

8 (TC) - Observe whether the accountant reconciles bank account.

9 (TC) - Account for a sequence of shipping documents.

10 (STT) - Trace selected shipping documents to duplicate sales invoice and the sales journal for assurance that each one has been billed and included in the journal.

11 (TC) - Account for a sequence of sales invoices in the sales journal.

12 (TC) - For selected duplicate invoice numbers from the sales journal, examine underlying documents for indication of internal certification that the total amount recorded in the journal, date, customer name, pricing, extension, and footings have been checked.

13 (STT) - Trace selected duplicate invoice numbers from the sales journal to (a) Duplicate sales invoice, and test for the total amount recorded in the journal, date, customer name. Check the pricing, extensions, and footings. (b) Bill of lading, and test for customer name, product description, quantity, and date. (c) Duplicate sales order, and test for customer name, product description, quantity, date, and internal approval. (d) Customer order, and test for customer name, product description, quantity, date, and credit approval by the credit manager.

14 (STT) - Trace recorded sales from the sales journal to the file of supporting documents, which include a duplicate sales invoice, bill of lading, sales order, and customer order.

15 (STT) - Obtain the prelisting of cash receipts, and trace amounts to the cash receipts journal, testing for name, amount, and date.

16 (STT) - Compare the prelisting of cash receipts with the duplicate deposit slip, testing for names, amounts, and dates. Trace the total from the cash receipts journal to the bank statement, testing for dates, amounts of deposit, and delay in deposit.

17 (TDB) - Confirm accounts receivable using positive confirmations above a given amount and perform alternative procedures for all confirmations not returned on the first and second request.

relationship and connecting this "and" node to the completeness objective of the accounts receivable. Thus, obtaining evidence for cash validity or sales completeness has a bearing on the completeness objective of the accounts receivable.

STEP TWO: QUANTIFICATION OF AUDIT EVIDENCE

The quantification of evidence is vital to the aggregation of audit evidence in the network. There are various measures of the strength of evidence (e.g., see Edwards 1984; Friedman 1986; Jeffrey 1983; Good 1983). However, as discussed in Dutta (1991), some of these measures are inconsistent and lead to anomalies. It has been demonstrated in Dutta (1991) that a measure based on the likelihoods provides a consistent measure of the strength of evidence and captures the intrinsic property of the evidence (see also footnote 1).

Likelihood is defined as the probability of obtaining the evidence given the assertion. That is, $P(\mathcal{E}|a)$ and $P(\mathcal{E}|\sim a)$, are the likelihoods, where $\mathcal{E}$ is the evidence and a is the assertion. Fisher introduced the name likelihood (Fisher 1922). Since then, several researchers have canvassed a "likelihood principle," which states that in assessing a hypothesis in light of an item of evidence, only the likelihoods count (e.g., see Edwards 1984; Savage 1961). Commenting on the importance of likelihood, Savage (1961) states "Given the likelihood function in which the experiment resulted, everything else about the experiment ... is irrelevant." Good (1983) also asserts that the likelihoods have "sharp uncontroversial values." Since the likelihoods are intrinsic to the evidence, a measure based on the likelihood ratio will also depend on the intrinsic properties of the evidence.

According to the likelihood measure, the strength of evidence equals the likelihood ratio. The likelihood ratio is obtained by computing the ratio of the conditional probability of the evidence given the assertion to the conditional probability given the negation of the assertion. This definition has a simple multiplicative property. If $\mathcal{E}_1$ and $\mathcal{E}_2$ denote two independent items of evidence, then the combined strength of $\mathcal{E}_1$ and $\mathcal{E}_2$ is the product of the individual strengths.[2]

STEP THREE: AGGREGATION OF EVIDENCE

The overall aggregation of evidence becomes a process of combining (propagating) the likelihoods through the network. The variables in our formulation are assumed to be binary variables: variables having only two possible values, such as, the accounts receivable balance is either fairly stated or materially misstated. Binary variables allow us to represent the strength of evidence in terms of the likelihood ratios. The scheme developed in this section for combining the likelihoods will be useful in combining various item of evidence in an audit in order to determine the posterior odds or the posterior probability that (1) a given audit objective of an account is met, (2) a given account is not materially misstated, or (3) the financial statements are fairly stated.[3]

We will represent the strength of individual evidence as λ_i^j, where λ denotes the likelihood ratio, the subscript denotes the variable or variables that the evidence bears on, and the superscript stands for different items of evidence bearing on the same variable. For example, $\lambda_{1,4}^2$ represents the strength of the second item of evidence bearing jointly on the objectives 1 and 4. Similarly, λ_A^1 represents the strength of the first item of evidence bearing directly on account "A." We will represent the aggregated or combined strength of all the evidence that directly bears on a variable "i" by Λ_i. For example, $\Lambda_{2,3}$ represents the combined strength of all the direct evidence bearing on the objectives 2 and 3.

[2] The likelihood ratios for $\mathcal{E}_1$, $\mathcal{E}_2$, and $\mathcal{E}_1\&\mathcal{E}_2$ are given by $P(\mathcal{E}_1|a)/P(\mathcal{E}_1|\sim a)$, $P(\mathcal{E}_2|a)/P(\mathcal{E}_2|\sim a)$, and $P(\mathcal{E}_1\&\mathcal{E}_2|a)/P(\mathcal{E}_1\&\mathcal{E}_2|\sim a)$, respectively. Since $P(\mathcal{E}_1\&\mathcal{E}_2|a) = P(\mathcal{E}_1|a).P(\mathcal{E}_2|a)$ and $P(\mathcal{E}_1\&\mathcal{E}_2|\sim a) = P(\mathcal{E}_1|\sim a).P(\mathcal{E}_2|\sim a)$, as the two items of evidence are assumed to be independent, the likelihood ratio for $\mathcal{E}_1\&\mathcal{E}_2$ becomes the product of the likelihood ratios for $\mathcal{E}_1$ and $\mathcal{E}_2$. When $\mathcal{E}_1$ and $\mathcal{E}_2$ are not independent, the likelihood ratio for $\mathcal{E}_1\&\mathcal{E}_2$ can be written as:

$$\frac{P(\mathcal{E}_1\&\mathcal{E}_2|a)}{P(\mathcal{E}_1\&\mathcal{E}_2|\sim a)} = \frac{P(\mathcal{E}_1|a)}{P(\mathcal{E}_1|\sim a)} \cdot \frac{P(\mathcal{E}_2|\mathcal{E}_1\&a)}{P(\mathcal{E}_2|\mathcal{E}_1\&\sim a)}$$

which is the product of the likelihood ratio for $\mathcal{E}_1$ and the likelihood ratio for $\mathcal{E}_2$ conditioned on $\mathcal{E}_1$ (see Edwards 1984 for details).

[3] It can be argued that the auditors are interested in the aggregation of evidence only at the audit objective level. Therefore, aggregation beyond the audit objective level is superfluous. However, the level at which the evidence should be aggregated, and opinion formed, is a moot issue and is not addressed in this paper. Moreover, our purpose here is to provide a general mechanism for aggregating evidence, and we do so for all the levels of the financial statement (e.g., audit objective, group of audit objectives, account balance, and the entire financial statement). Thus, we provide flexibility to the user who can choose the level at which he or she wishes to combine all the items of evidence and form his or her opinion.

Aggregation of Evidence Bearing on One Variable

First, let us describe the process of aggregating evidence bearing on one objective. As shown in Footnote 2, if there are two items of evidence bearing on one variable, then the combined strength is simply the product of the two individual strengths. This result can be generalized for n items of evidence bearing on one variable O_i yielding the strength of the combined evidence, Λ_i, to be

$$\Lambda_i = \prod_{j=1}^{n} \lambda_i^j \quad (1)$$

where j represents the individual item of evidence bearing directly on objective O_i. When the items of evidence are not independent, the combined strength (that is, the combined likelihood ratio) would be given as the product of the conditional likelihood ratios (see the discussion in footnote 2).

Let us assume that we are auditing account "A" with six audit objectives, $O_1, O_2, \ldots O_6$. Further, assume that the evidential network for this audit is given in figure 2. The auditor has either performed the procedures, evaluated the findings, and concluded about the strength of evidence for each procedure, or has planned an audit with the level of strengths given by various λ's as shown in the rectangular boxes in figure 2. The network in figure 2 represents the original problem without any computations. Figure 3 to figure 5 illustrate the sequence of aggregation of the evidence. Figure 3 is the result of aggregating evidence that bears on only one objective. The aggregation is accomplished by using Equation (1) for each objective.

Table 2 shows the result of aggregation for figure 3. For each objective we identify the items of evidence that bear solely on the objective. We then combine, for each objective, all the evidence bearing on the objective using Equation (1). As discussed earlier, the combined strength of all the evidence on an objective is simply the product of the strengths of the individual items of evidence bearing on the objective.

Next we combine the evidence that bears on two or more objectives. In order to do so, we form a cluster of the objectives which are inter-dependent. As discussed earlier, an item of evidence simultaneously bearing on two or more objectives makes the objectives inter-dependent. For example, in figure 2, there is an item of evidence that bears on the objectives 1 and 4, thus making these objectives inter-dependent. Figure 4 represents clustering of the objectives 1 and 4, and the objectives 2, 5, and 6. The strength of the combined evidence in such a situation is somewhat complex and is discussed below.

Aggregation of Evidence in a Cluster of Independent Variables

In the Appendix, we have derived general expressions for the combined strength of evidence for a cluster of two and three independent variables [see Equations (A-11), (A-12), (A-14), and (A-15)]. These expressions are derived in terms of the strength of various items of evidence bearing on individual and clusters of variables and in terms of the prior odds of the individual variables. Since we are clustering objectives O_1 and O_4 in figure 4, we will use Equation (A-12) to obtain the combined strength of evidence bearing on objectives O_1 and O_4. The result is:

$$\Lambda_{1,4} = \frac{\Lambda_1\Lambda_4\lambda_{1,4}^1}{1 + \Lambda_1\pi_1 + \Lambda_4\pi_4}. \quad (2)$$

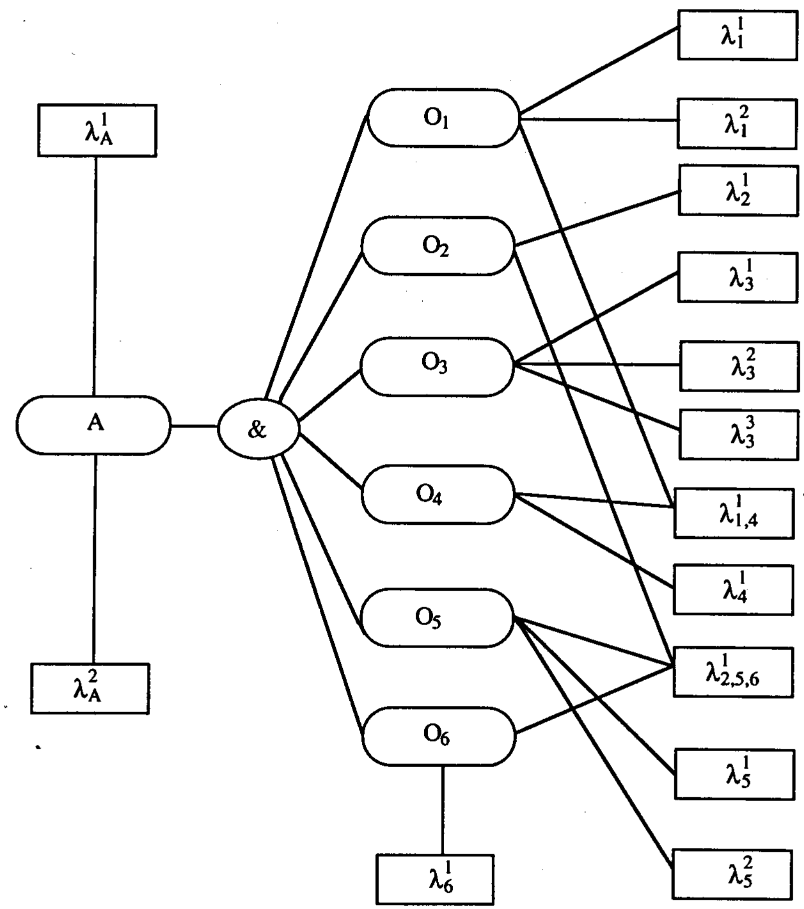
where π represents the prior odds. The aggregated evidence bearing on the cluster of objectives O_2, O_5, and O_6 in figure 4 can be obtained from Equation (A-15):

$$\Lambda_{2,5,6} = \frac{\Lambda_2\Lambda_5\Lambda_6\lambda_{2,5,6}^1}{1+\Lambda_2\pi_2+\Lambda_5\pi_5+\Lambda_6\pi_6+\Lambda_2\Lambda_5\pi_2\pi_5+\Lambda_2\Lambda_6\pi_2\pi_6+\Lambda_5\Lambda_6\pi_5\pi_6}. \quad (3)$$

Combination of Evidence on Account

We now compute the strength of the combined evidence bearing on the account. As shown in figure 4, all the items of evidence bearing on the objectives have been aggregated and the

FIGURE 2
A Generic Network of Audit Evidence*

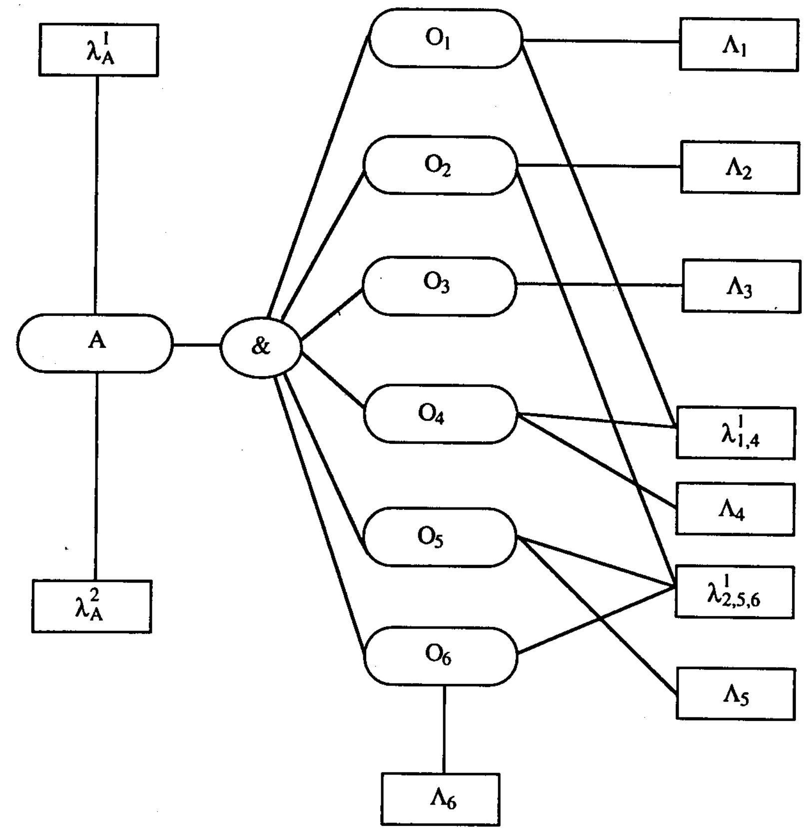


* "A" denotes the account, O denotes the objectives and λ denotes the strength of evidence bearing on the objective(s) or on the account.

combined strength bears on the respective clusters. In figure 4, we have three clusters, $\{\{O_3\}, \{O_1, O_4\}, \{O_2, O_5, O_6\}\}$. We now form another cluster of all the objectives comprised of these three clusters. The combined evidence on the new cluster bears on the account as a whole as shown in figure 5. The aggregation process is similar to the case of three variables as discussed earlier. We use equation (A-15) to obtain the resultant strength:

$$\Lambda_{1,\ldots,6} = \frac{\Lambda_{1,4}\Lambda_3\Lambda_{2,5,6}}{1+\Lambda_{1,4}\pi_1\pi_4+\Lambda_3\pi_3+\Lambda_{2,5,6}\pi_2\pi_5\pi_6+\Lambda_{1,4}\Lambda_3\pi_1\pi_3\pi_4+\Lambda_3\Lambda_{2,5,6}\pi_2\pi_3\pi_5\pi_6+\Lambda_{1,4}\Lambda_{2,5,6}\pi_1\pi_2\pi_4\pi_5\pi_6}. \quad (4)$$

FIGURE 3
A Diagrammatic Representation of Aggregation of Evidence Bearing Solely on One Variable (Objective or Account)*



* Λ denotes the combined evidence bearing on one variable. Other notations are explained in figure 2.

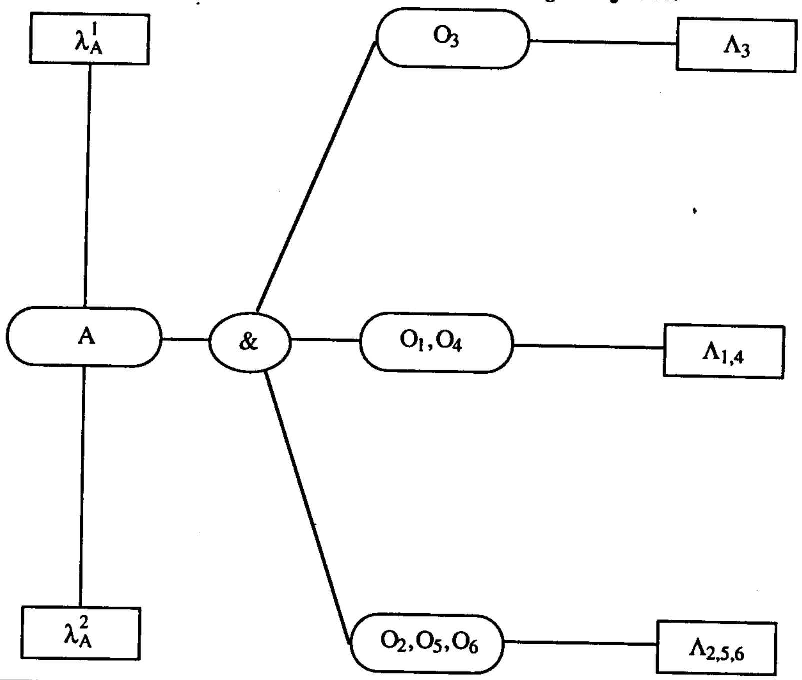
We now compute the strength of evidence bearing on the account. We know that the account is fairly stated if all the objectives are met. As seen in figure 5, we have two direct items of evidence bearing on the account and one indirect evidence as a result of the combined evidence bearing on all the objectives. The strength of evidence on the assertion that all the objectives are met is computed in Equation (4). This is the indirect evidence bearing on the account. Thus, we combine the three items of evidence as depicted in figure 5. Using Equation (1), the overall strength of evidence on account "A" is:

$$\Lambda_A = \Lambda_{1,\ldots,6}\lambda_A^1\lambda_A^2 \quad (5)$$

TABLE 2
Aggregation of Evidence Bearing On Only One Objective (see figures 2 & 3)

Objective	Evidence Bearing Solely on One Objective λ_i^j	Combined Evidence Λ_i
O_1	λ_1^1, λ_1^2	$\Lambda_1 = \lambda_1^1 \times \lambda_1^2$
O_2	λ_2^1	$\Lambda_2 = \lambda_2^1$
O_3	$\lambda_3^1, \lambda_3^2, \lambda_3^3$	$\Lambda_3 = \lambda_3^1 \times \lambda_3^2 \times \lambda_3^3$
O_4	λ_4^1	$\Lambda_4 = \lambda_4^1$
O_5	λ_5^1, λ_5^2	$\Lambda_5 = \lambda_5^1 \times \lambda_5^2$
O_6	λ_6^1	$\Lambda_6 = \lambda_6^1$

FIGURE 4
A Diagrammatic Representation of Clustering of Objectives



The posterior odds can be given by

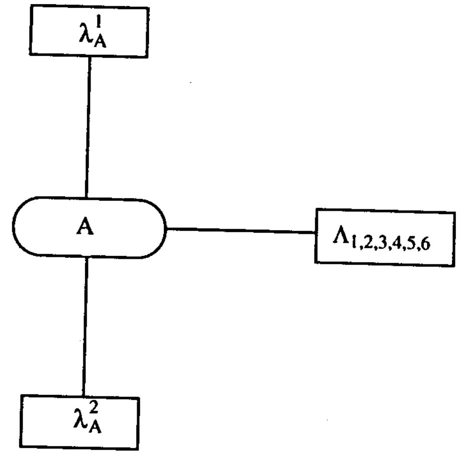
$$\text{Posterior Odds} = \Lambda_{1,\ldots,6}\lambda_A^1\lambda_A^2\pi_1\pi_2\pi_3\pi_4\pi_5\pi_6 \quad (6)$$

where π stands for the prior odds. The auditor is interested in the posterior probability, which can be obtained from the posterior odds:

$$P(A \text{ is fairly stated} \mid \forall E) = \frac{\text{Posterior Odds}}{1 + \text{Posterior Odds}}. \quad (7)$$

So far we have demonstrated how different items of evidence at various levels of an account can be aggregated, under the probability theory framework, in order to determine whether the account is fairly stated. We can extend the same approach one step further and combine the strengths at each account to determine the overall strength at the financial statement level to determine whether the financial statement is fairly presented.[4]

FIGURE 5
A Diagrammatic Representation of Aggregation of Evidence at the Account Level



The strength of evidence can propagate backward from the financial statement level and the account level to the level of an audit objective of an account. In the following paragraphs we will discuss how to determine the impact of various items of evidence (evidence at the financial statement level, at the account level, and the audit objective level) on the audit objective level of an account. In other words, we want to determine the posterior probability that the audit objective of an account is met given that the auditor has collected evidence bearing at various levels of the account.

Assessing the Total Strength of Evidence on Audit Objectives

Here, we want to describe a scheme that will aggregate items of evidence at the financial statement level, at the account level, and at the audit objective level to determine whether an objective is met. The combination scheme can also be used to determine the total strength of evidence on a set of objectives. The items of evidence that bear solely on an objective support the objective. In addition, the items of evidence that bear on any set that contains the objective also support the objective. Thus, the total strength of evidence bearing on an objective i, denoted as Ω_i, is the product of the strength of evidence bearing on any super-set of the objective. Mathematically, this can be represented as:

$$\Omega_i = \prod_{B \supseteq i} \lambda_B \quad (8)$$

Similarly, the total strength of evidence bearing on a set of objectives is the product of the combined evidence on the set [obtained from Equation (A-12) or (A-15)] and the strength of evidence bearing on all its super-sets. Math-

[4]Here we assume that the financial statement is fairly stated if and only if each account is fairly stated. Under this assumption we have no tradeoffs between the accounts; the only aggregation needed is the trivial "AND" node. We multiply the posterior probabilities on each account and obtain the posterior on the financial statement or use the approach of combining the likelihood ratios as discussed in the Appendix. There are some anomalies in such trivial aggregation; however, discussion of these is beyond the scope of this paper.

ematically, if σ is the set of objectives, the total evidence bearing on σ, Ω_σ, is given by:

$$\Omega_\sigma = \Lambda_\sigma \cdot \prod_{B \supset \sigma} \lambda_B \qquad (9)$$

Thus, the mechanism of combining evidence, developed in this section, is versatile and can accomplish propagation of evidence from the objectives to the accounts and from the accounts to the financial statement, as well as from the financial statement and the accounts to the audit objectives.

A NUMERICAL EXAMPLE OF AGGREGATION OF EVIDENCE

In this section, we consider the evidential network shown in figure 1 for the illustration of the evidence aggregation process. The network in figure 6 is the same as in figure 1. The only difference is that in figure 6, the auditor either has planned the desired strength of evidence to be obtained from various procedures or has already performed the procedures and has evaluated the corresponding strengths.[5] These strengths are expressed in terms of the likelihood ratios, λ, and they are given in the corresponding rectangular boxes in figure 6 (see Edwards 1984 for more details on the likelihood ratio). It should be noted that the network in figure 6 was obtained by mapping the procedures given by Arens and Loebbecke (1991, 391-393). However, figure 6 has been somewhat simplified for computational simplicity as discussed earlier.

In order to compute the aggregated evidence bearing on the account, we first aggregate the items of evidence bearing directly on the sub-objectives. We use Equation (1) to accomplish this purpose. The results are given in table 3 and figure 7.

The next step is to obtain the strength of evidence bearing on the audit objectives of the account. This is achieved by combining the strength of evidence bearing on the sub- objectives. For simplicity, we assume throughout our numerical example that the prior odds equal one[6] for all variables. We know from table 3 that the strength of evidence bearing on the validity of cash receipts and the completeness of sales equals 15 for both. From figure 6, we also know that AR is complete if both cash receipts are valid and sales are complete. Therefore, the strength of evidence bearing on the complete-

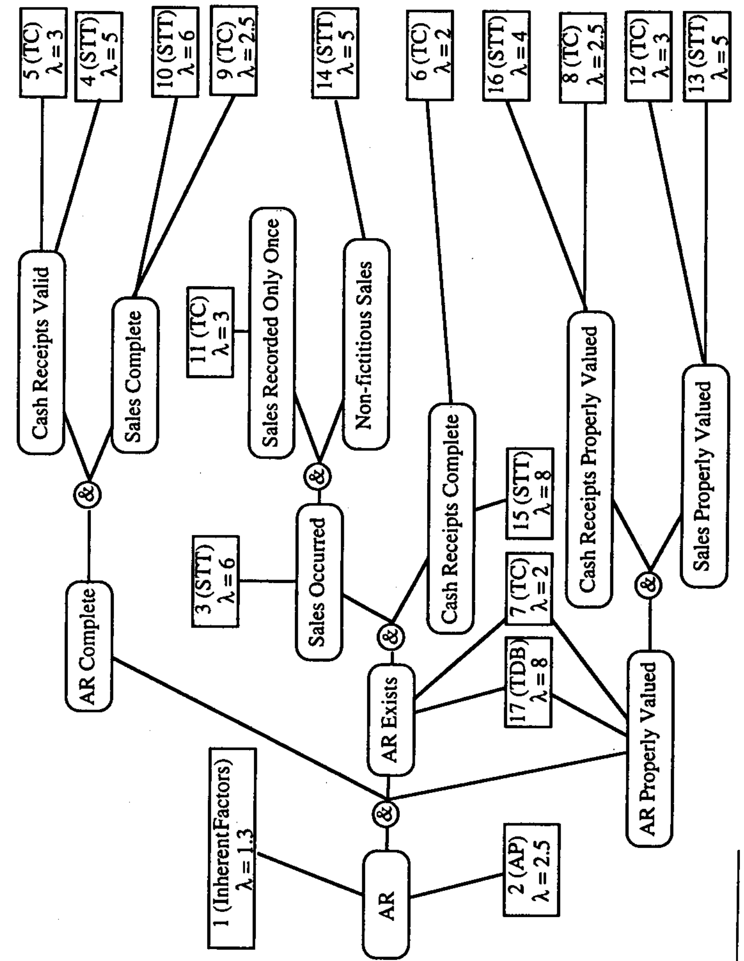
[5] In order to solve the numerical example, we are assuming that all the evidence has been obtained and the strength of evidence is known. However, in a real audit the evidence is obtained sequentially, and this can be easily incorporated in our scheme. The auditor can evaluate the network with the evidence that he or she has obtained and substitute a value of one for the strength of evidence for those procedures which have not yet been conducted. Once the procedure is conducted, and the strength of evidence assessed, the auditor can re-evaluate the network with the appropriate value for the strength of evidence. This way the auditor can assess the impact of the new item of evidence on various audit objectives and the accounts.

[6] In other words, we are assuming that the auditor has no prior knowledge of the fairness of the audit assertions. In the Bayesian framework, no knowledge is represented by assigning equal probabilities to the assertion and to its negation. In terms of odds, it implies an odd of one.

TABLE 3
Aggregation of Evidence Bearing on Only One Objective (see figure 6)

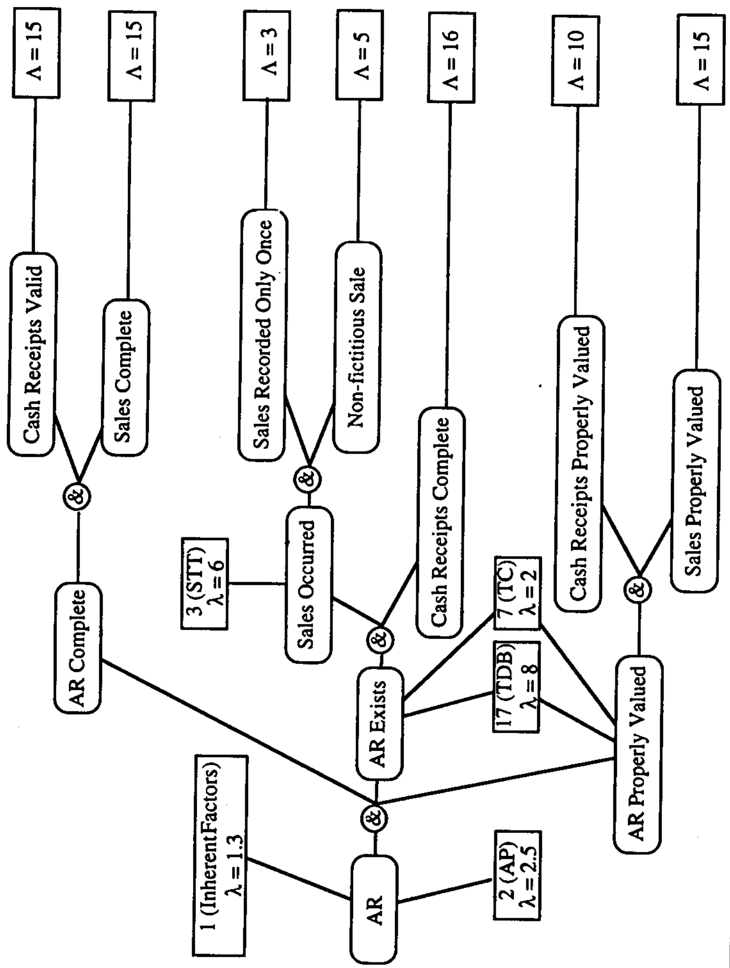
Objective O_i	Procedures	Combined Evidence Λ_i
Cash Receipts Valid (CRV)	P4, P5	$\Lambda_{CRV} = 5 \times 3 = 15$
Sales Complete (SC)	P10, P9	$\Lambda_{SC} = 6 \times 2.5 = 15$
Sales Recorded Only Once (SR)	P11	$\Lambda_{SR} = 3$
Non-Fictitious Sales (NFS)	P14	$\Lambda_{NFS} = 5$
Cash Receipts Complete (CRC)	P6, P15	$\Lambda_{CRC} = 2 \times 8$
Cash Receipts Properly Valued (CRV)	P16, P8	$\Lambda_{CRV} = 2.5 \times 4 = 10$
Sales Properly Valued (SV)	P12, P13	$\Lambda_{SV} = 3 \times 5 = 15$

FIGURE 6
A Network of Evidence for Accounts Receivable*



*See table 1 for the description of the procedures given in the rectangular boxes.

FIGURE 7
Aggregation of Evidence at the Sub-Objective Level*



*See table 1 for the description of the procedures given in the rectangular boxes.

ness objective of AR can be obtained by substituting Λ_{CRV} and Λ_{SC} in Equation (A-12). In this case, the strength of evidence on "AR Complete" can be computed as (15).(15)/(1+15+15), which is equal to 7.26.

Next we compute the combined strength of evidence bearing on the occurrence of sales. From figure 6, the occurrence objective of sales is met if all sales are recorded only once and there are no fictitious sales. Thus, the evidence directly bearing on these two sub-objectives has to be aggregated in order to determine the combined strength of evidence on occurrence of sales. From table 3, we know that the evidence obtained from Procedure 11 has a strength of 3 and bears on the assertion that sales were recorded only once. Also, the evidence obtained from Procedure 14 bears on the assertion of "Non-Fictitious Sales" and has a strength of 5. Furthermore, an item of evidence obtained through Procedure 3 directly bears on the occurrence of sales objective and has a strength of 6. Therefore, the combined evidence on "Sales Occurred" can be computed using Equation (A-12) as (3).(5)(6)/(1+3+5), which is equal to 10.

Similarly, the combined evidence on the valuation objective of AR can be computed by aggregating evidence bearing on the valuation objectives of sales and cash receipts. The assumption here is that AR is properly valued if the sales are properly valued and the cash receipts also are properly valued. From table 3 we know that the strength of evidence that cash receipts are properly valued equals 10 and that sales are properly valued equals 15. Thus, the combined strength of evidence on the valuation objective of AR can be computed as (10).(15)/(1+10+15), equals 5.77. These results are given in figure 8.

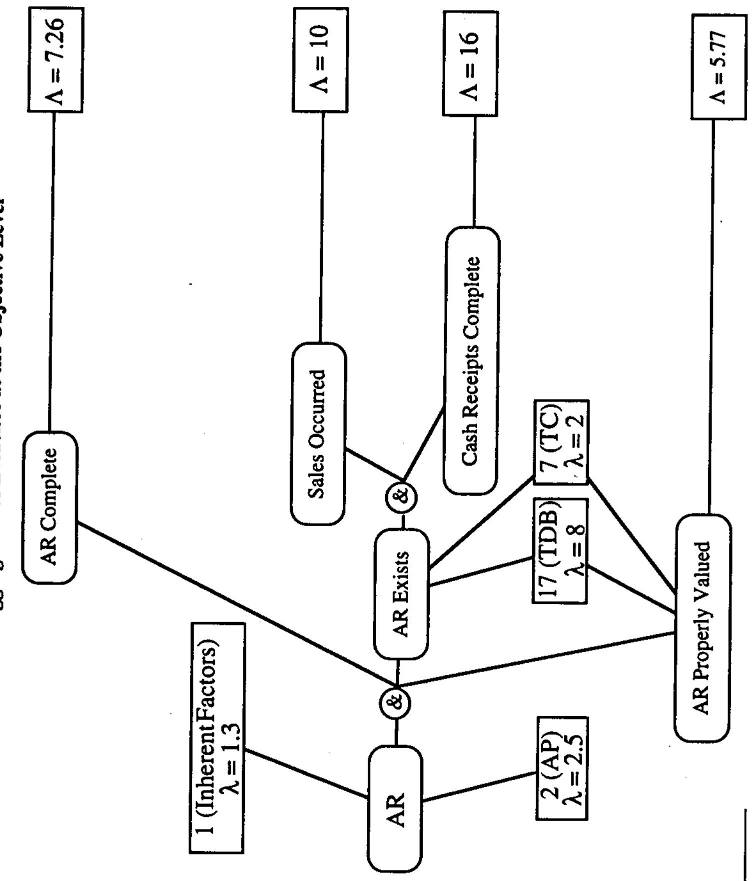
We now combine the evidence bearing on variable "AR Exists." We know from Figure 6 that if sales occur and cash receipts are complete, then AR exists. Further, the evidence bearing on the completeness objective of cash receipts equals 16 (from table 3) and that on the occurrence of sales has been computed above as 10. Thus, the combined evidence on the existence objective of AR can be computed using (A-12) as (10).(16)/(1+10+16), which is 5.93. This is shown in figure 9.

Next, we cluster the valuation and existence objectives of AR. There are two procedures (7 and 17) which provide evidence that simultaneously bears on the two objectives. In order to incorporate the evidence obtained from these procedures, we have to cluster the objectives. We know from the discussion above that indirect evidence bearing solely on the valuation objective of AR has a strength of 5.77 and that on the existence objective of AR has a strength of 5.93. The evidence obtained from Procedures 7 and 17 has strengths of 2 and 8, respectively. The combined evidence bearing on the cluster can be computed using Equation (A-12) as (5.93)(5.77)(2)(8)/(1+5.93+5.77), which equals 43.11 (see figure 10).

We next form a cluster of all the three objectives. From the above discussion we know that the strength of evidence on the completeness objective of AR equals 7.26, and that on the cluster of existence and valuation objective equals 43.11 (see figure 10). Therefore, the combined evidence on all the three objectives can be obtained using Equation (A-12) as (43.11)(7.26)/(1+43.11+7.26), which equals 6.09 (see figure 11).

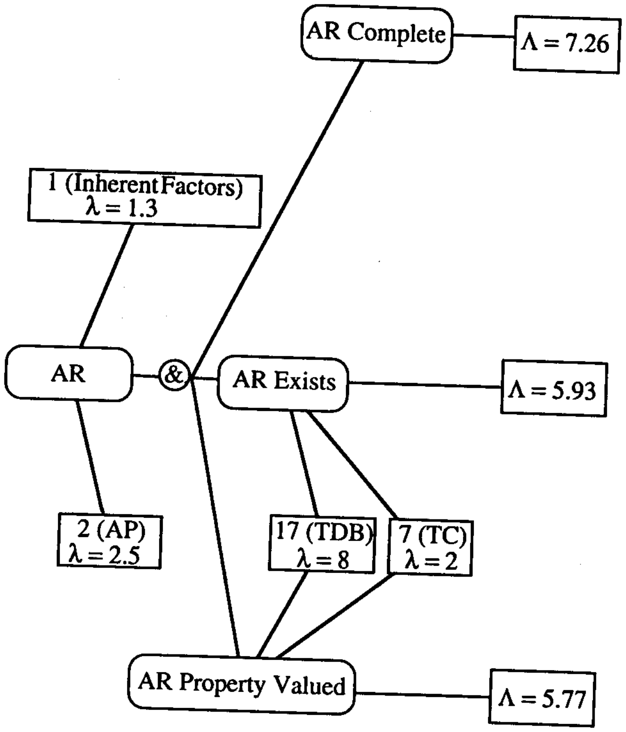
The combined evidence on the fairness of AR can be obtained by aggregating the direct evidence on AR with the combined indirect evidence on AR (see figure 11). The combined indirect evidence bearing on AR, as computed above has a strength of 6.09. There are two items of direct evidence bearing on AR, inherent factors and evidence obtained through analytical procedures, with strength of evidence 1.3 and 2.5, respectively. These items of evidence are combined using Equation (1). The result is 19.79 (1.3x2.5x6.09). Assuming the prior odds to be one for all the variables, the posterior probability can be computed using Equations (6) and (7) as 0.95. This result implies that the auditor, after having evaluated all evidence gathered in the audit process as shown in figure 6, has obtained a posterior probability of 0.95 that the account is fairly stated and all the objectives have been met.

FIGURE 8
Aggregation of Evidence at the Objective Level*



*See table 1 for the description of the procedures given in the rectangular boxes.

FIGURE 9
Aggregation of Evidence at the Objective Level*



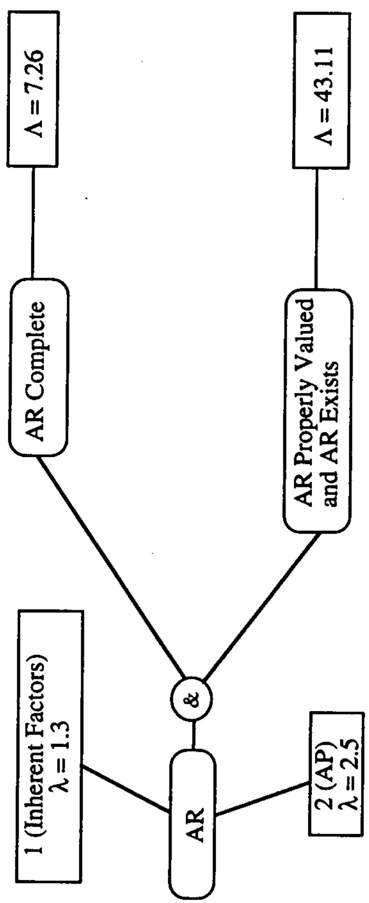
*See table 1 for the description of the procedures given in the rectangular boxes.

CONCLUSION

The main purpose of this paper is to show how various items of audit evidence can be aggregated in the probability theory framework. We argued that the evidence in auditing bears on different levels of the account and at times is interrelated. We have discussed a general approach for (1) combining various items of evidence bearing on one variable, and (2) combining items of evidence bearing on a cluster of variables. These results are used to demonstrate how audit evidence can be aggregated on an actual audit to obtain the posterior probability that the account or the financial statements are fairly presented. We have also discussed various types of dependencies encountered by the auditor and how these dependencies can be treated in the probability theory framework.

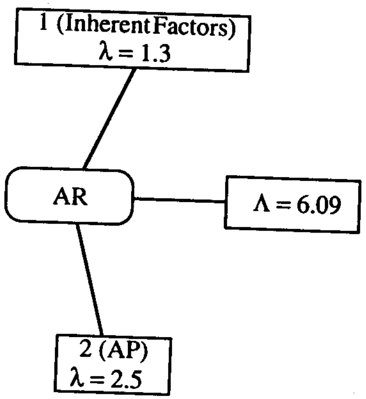
Besides the advantage of being able to objectively aggregate evidence at various levels of the financial statement that should lead to an efficient and effective audit, there are several limitations with the approach that need to be discussed. Some of these limitations are in common with the SAS No. 47 approach. First, we did not discuss how the likelihood ratios of au-

FIGURE 10
Clustering of Valuation and Existence Objectives*



*See table 1 for the description of the procedures given in the rectangular boxes.

FIGURE 11
Aggregation of Evidence at the Account Level



dit evidence can be assessed. Second, we did not incorporate materiality judgments into our model. Materiality is a crucial notion in auditing; however, we know no aggregation model which incorporates it while ascertaining the total evidence obtained. Third, we did not consider the cost factor in our evidence aggregation model. It is important to the auditor that the total costs of an audit be minimized while achieving a desired level of posterior probability. Fourth, we have assumed that each audit objective is equally important. This will bring inefficiency in the audit process because the auditor may have to do a lot more work for an objective that may not be as important as others. Fifth, we have not distinguished between the overstatement and the understatement material errors. All these issues need further research.

The framework developed here for aggregating evidence is not only useful for evaluating an audit, but can also be used in planning an audit. The scheme can be used for performing sensitivity analysis as to which procedure is more important and thus the auditor can identify areas which require further audit work. The SAS No. 47 approach does not provide an objective way to combine evidence at various levels of the account. Furthermore, our aggregation scheme is developed using the theory of probability and thus has a sound theoretical foundation. With computers such schemes can be automated and decision support systems can be constructed for audit evaluation and planning.

APPENDIX
AGGREGATION OF EVIDENCE IN A CLUSTER OF VARIABLES

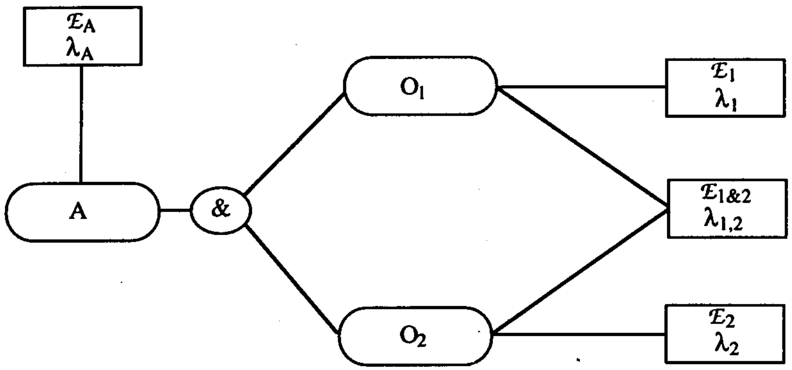
In this Appendix we want to derive a general expression for the aggregated strength of evidence on a cluster of variables. We will consider two cases: (1) where the variables in the cluster are inter-dependent, and (2) where the variables in the cluster are independent. For each case, we will derive the expression for the aggregated strength of evidence on a cluster of two variables. We will extend the result for the independent case to a cluster of three variables since we need that result more frequently in our discussion. One can easily extend the formulas to a cluster of n variables (the authors have derived the general expressions; one can obtain a copy of it by writing to the authors).

We have already shown that if there were several independent items of evidence bearing on a single variable, then the combined evidence is simply the product of the strengths of the individual items of evidence. That is, if the strength of an item of evidence is represented by the likelihood ratio, λ, then the combined strength, Λ, of all the independent items of evidence bearing on a single variable is: $\Lambda = \prod_j \lambda^j$, where λ^j represents the strength of jth evidence on the variable (for dependent items of evidence see footnote 2). However, when we want to combine evidence bearing on a cluster of variables, then the combination rule is not so straightforward.

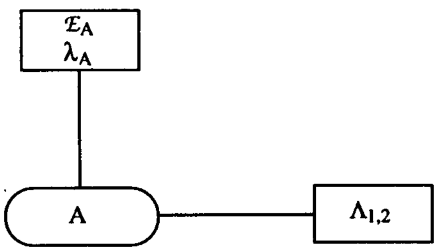
Assume that the auditor is auditing an account with two objectives. The account balance is fairly stated (*a*) only when the two objectives have been met, that is, "*a*" is true if and only if "o_1," "o_2" are true (we will represent the names of variables by capital letters and the values by small letters). Further, assume that there are four items of evidence $\mathcal{E}_1$, $\mathcal{E}_2$, $\mathcal{E}_{1\&2}$, and $\mathcal{E}_A$. The evidence denoted as $\mathcal{E}_1$ bears only on O_1, whereas the item of evidence denoted as $\mathcal{E}_{1\&2}$ bears jointly on O_1 and O_2 (see Figure A-1, panel (a)). The auditor would like to aggregate all these items of evidence to determine whether the account is fairly stated and the objectives are met. In other words, the auditor would like to determine the posterior probability $P(a\&o_1\&o_2|\forall\mathcal{E})$, where $\mathcal{E}$ consists of $\mathcal{E}_1$, $\mathcal{E}_2$, $\mathcal{E}_{1\&2}$, and $\mathcal{E}_A$.

Our objective is to express the above posterior in terms of various likelihood ratios, that is, the strength of evidence. In order to achieve this objective, we complete the following steps.

FIGURE A-1
Evidential Network for Account "A" with Two Objectives*



(a)



(b)

*The account is fairly stated only when the two objectives have been met.

Let us define

$$\mathcal{A} = a\&o_1\&o_2 \tag{A-1}$$

and

$$\sim\mathcal{A} = (\sim a\&\sim o_1\&o_2)\cup(\sim a\&o_1\&\sim o_2)\cup(\sim a\&\sim o_1\&\sim o_2). \tag{A-2}$$

Applying Bayes' rule, we obtain:

$$P(\mathcal{A}|\forall\mathcal{E}) = P(a\&o_1\&o_2|\forall\mathcal{E}) = \frac{P(\forall\mathcal{E}|a\&o_1\&o_2)P(a\&o_1\&o_2)}{P(\forall\mathcal{E})}. \tag{A-3}$$

We can assume that each item of evidence is conditionally independent of every thing else given the variable(s) it bears on (Edwards 1984). These conditions yield the following expression for (A-3):

$$P(\mathcal{A}|\forall\mathcal{E}) = \frac{P(\mathcal{E}_A|a)P(\mathcal{E}_1|o_1)P(\mathcal{E}_2|o_2)P(\mathcal{E}_{1\&2}|o_1\&o_2)P(a\&o_1\&o_2)}{P(\forall\mathcal{E})}. \tag{A-4}$$

Also, we know that

$$P(\sim\mathcal{A}|\forall\mathcal{E}) = P(\sim a\&\sim o_1\&o_2|\forall\mathcal{E}) + P(\sim a\&o_1\&\sim o_2|\forall\mathcal{E}) + P(\sim a\&\sim o_1\&\sim o_2|\forall\mathcal{E}). \tag{A-5}$$

With a similar argument that we have used in obtaining (A-4) from (A-3), we can write each term on the right hand side of (A-5) in a similar fashion. The first term can be written as:

$$P(\sim a\&\sim o_1\&o_2|\forall\mathcal{E}) = \frac{P(\mathcal{E}_A|\sim a)P(\mathcal{E}_1|\sim o_1)P(\mathcal{E}_2|o_2)P(\mathcal{E}_{1\&2}|\sim o_1\&o_2)P(\sim a\&\sim o_1\&o_2)}{P(\forall\mathcal{E})}. \tag{A-6}$$

The posterior odds can be written as (A-3) divided by (A-5):

$$\text{Posterior Odds} = \frac{P(\mathcal{A}|\mathcal{E}_A\&\mathcal{E}_1\&\mathcal{E}_2\&\mathcal{E}_{1\&2})}{P(\sim\mathcal{A}|\mathcal{E}_A\&\mathcal{E}_1\&\mathcal{E}_2\&\mathcal{E}_{1\&2})}. \tag{A-7}$$

We will use (A-3), (A-5), and (A-7) to derive the desired expressions for the two cases as given below.

Independent Variables:

We assume here that the two objectives "O_1" and "O_2" are independent, that is, $P(o_1\&o_2) = P(o_1)P(o_2)$. Also, we know that "$a$" is true if and only if "o_1" and "o_2" are true, implying that . These conditions imply that: $P(a|o_1\&o_2) = P(\sim a|\sim o_1\&o_2) = P(\sim a|o_1\&\sim o_2) = P(\sim a|\sim o_1\&\sim o_2) = 1$

$$\begin{aligned} P(a\&o_1\&o_2) &= P(a|o_1\&o_2)P(o_1)P(o_2) = P(o_1)P(o_2),\\ P(\sim a\&\sim o_1\&o_2) &= P(\sim a|\sim o_1\&o_2)P(\sim o_1)P(o_2) = P(\sim o_1)P(o_2),\\ P(\sim a\&o_1\&\sim o_2) &= P(\sim a|o_1\&\sim o_2)P(o_1)P(\sim o_2) = P(o_1)P(\sim o_2),\\ P(\sim a\&\sim o_1\&\sim o_2) &= P(\sim a|\sim o_1\&\sim o_2)P(\sim o_1)P(\sim o_2) = P(\sim o_1)P(\sim o_2). \end{aligned} \tag{A-8}$$

Substituting (A-4) and (A-5) in (A-7) and using for each term in (A-5) an expression similar to (A-6), using the properties in (A-8), dividing the numerator and denominator of the resulting expression by $P(\sim o_1)P(\sim o_2)P(\mathcal{E}_A|\sim a)P(\mathcal{E}_1|\sim o_1)P(\mathcal{E}_2|\sim o_2)P(\mathcal{E}_{1\&2}|\sim o_1\&\sim o_2)$, and remembering that

$$P(\mathcal{E}_{1\&2}|\sim o_1\&\sim o_2) = P(\mathcal{E}_{1\&2}|\sim o_1\&o_2) = P(\mathcal{E}_{1\&2}|o_1\&\sim o_2), \tag{A-9}$$

we obtain the desired expression for the posterior odds:

$$\text{Posterior Odds} = \frac{\lambda_A\lambda_1\lambda_2\lambda_{1,2}\pi_1\pi_2}{1+\lambda_1\pi_1+\lambda_2\pi_2}. \tag{A-10}$$

The symbol π stands for the prior odds and λ for the strength of evidence or the likelihood ratios:

$$\pi_A = \frac{P(a)}{P(\sim a)},\ \pi_1 = \frac{P(o_1)}{P(\sim o_1)},\ \text{and}\ \pi_2 = \frac{P(o_2)}{P(\sim o_2)},$$

and

$$\lambda_A = \frac{P(\mathcal{E}_A \mid a)}{P(\mathcal{E}_A \mid \sim a)},\ \lambda_1 = \frac{P(\mathcal{E}_1 \mid o_1)}{P(\mathcal{E}_1 \mid \sim o_1)},\ \lambda_2 = \frac{P(\mathcal{E}_2 \mid o_2)}{P(\mathcal{E}_2 \mid \sim o_2)},\ \lambda_{1,2} = \frac{P(\mathcal{E}_{1\&2} \mid o_1\&o_2)}{P(\mathcal{E}_{1\&2} \mid \sim o_1\&\sim o_2)}.$$

Equation (A-10) is an important result. It provides the combined strength of evidence. We can write (A-10) as

$$\text{Posterior Odds} = \Lambda_{1,2}\lambda_A\pi_1\pi_2, \quad \text{(A-11)}$$

where

$$\Lambda_{1,2} = \frac{\lambda_1\lambda_2\lambda_{1,2}}{1+\lambda_1\pi_1+\lambda_2\pi_2}. \quad \text{(A-12)}$$

Equation (A-12) represents the combined strength on the cluster $\{O_1,O_2\}$, and (A-11) represents the effects of all the evidence gathered at various levels: the evidence at the account level, A; at the two objective levels, O_1 and O_2; and at the cluster level, $\{O_1,O_2\}$. From Equation (A-11), it appears that the evidence coming from the individual objectives and the cluster through "and" relationship works as an item of evidence for the main variable "A" (see figure A-1, panel b). This result makes intuitive sense because if we did not have any evidence at the account level but had strong evidence that each objective is met, then we would have evidence that the account is fairly stated.

The posterior probability can be obtained by:

$$P(\mathcal{A} \mid \forall\mathcal{E}) = \frac{\text{Posterior Odds}}{1 + \text{Posterior Odds}}. \quad \text{(A-13)}$$

where the posterior odds is defined by (A-11).

In order to test the validity of Equation (A-12) let us assume that we have λ_1=4 and λ_2=4 for the two objectives. The resulting strength of the combined evidence for the account would be 1.78 (from Equation (A-12)). This would yield a posterior probability of 0.64 using Equation (A-13). In fact, if we use the posterior probability (which is 0.8 for λ_1=4) for each objective that it is met given the evidence, one would obtain exactly the same result.

In the case of a cluster of three independent objectives, one can write (with similar steps as above) the posterior odds in terms of the prior odds and the likelihood ratios for various items of evidence as:

$$\text{Posterior Odds} = \Lambda_{1,2,3}\lambda_A\pi_1\pi_2\pi_3, \quad \text{(A-14)}$$

where

$$\Lambda_{1,2,3} = \frac{\lambda_1\lambda_2\lambda_3\lambda_{1,2}\lambda_{1,3}\lambda_{2,3}\lambda_{1,2,3}}{1+\lambda_1\pi_1+\lambda_2\pi_2+\lambda_3\pi_3+\lambda_1\lambda_2\lambda_{1,2}\pi_1\pi_2+\lambda_1\lambda_3\lambda_{1,3}\pi_1\pi_3+\lambda_2\lambda_3\lambda_{2,3}\pi_2\pi_3}. \quad \text{(A-15)}$$

Equations (A-11) and (A-12), and (A-14) and (A-15) are the desired results for the independent case.

Dependent Variables:

We assume here that the two objectives "O_1" and "O_2" are not independent, that is, $P(o_1\&o_2) = P(o_1)P(o_2 \mid o_1)$. Also, we know that "$a$" is true if and only if "o_1" and "o_2" are true, implying that $P(a \mid o_1\&o_2) = P(\sim a \mid \sim o_1\&o_2) = P(\sim a \mid o_1\&\sim o_2) = P(\sim a \mid \sim o_1\&\sim o_2) = 1$. These conditions imply that:

$$\begin{aligned} P(a\&o_1\&o_2) &= P(a \mid o_1\&o_2)P(o_1)P(o_2 \mid o_1) = P(o_1)P(o_2 \mid o_1), \\ P(\sim a\&\sim o_1\&o_2) &= P(\sim a \mid \sim o_1\&o_2)P(\sim o_1)P(o_2 \mid \sim o_1) = P(\sim o_1)P(o_2 \mid \sim o_1), \\ P(\sim a\&o_1\&\sim o_2) &= P(\sim a \mid o_1\&\sim o_2)P(o_1)P(\sim o_2 \mid o_1) = P(o_1)P(\sim o_2 \mid o_1), \\ P(\sim a\&\sim o_1\&\sim o_2) &= P(\sim a \mid \sim o_1\&\sim o_2)P(\sim o_1)P(\sim o_2 \mid \sim o_1) = P(\sim o_1)P(\sim o_2 \mid \sim o_1). \end{aligned} \quad \text{(A-16)}$$

Substituting (A-4) and (A-5) in (A-7) and using for each term in (A-5) an expression similar to (A-6), using the properties in (A-16), dividing the numerator and denominator of the resulting expression by $P(\sim o_1)P(\sim o_2 \mid \sim o_1)P(\mathcal{E}_A \mid \sim a)P(\mathcal{E}_1 \mid \sim o_1)P(\mathcal{E}_2 \mid \sim o_2)P(\mathcal{E}_{1\&2} \mid \sim o_1\&\sim o_2)$, and remembering that

$$P(\mathcal{E}_{1\&2} \mid \sim o_1\&\sim o_2) = P(\mathcal{E}_{1\&2} \mid \sim o_1\&o_2) = P(\mathcal{E}_{1\&2} \mid o_1\&\sim o_2), \quad \text{(A-17)}$$

we obtain the desired expression for the posterior odds:

$$\text{Posterior Odds} = \frac{\lambda_1\lambda_2\lambda_{1,2}\lambda_A\pi_1 \dfrac{P(o_2 \mid o_1)}{P(\sim o_2 \mid \sim o_1)}}{1 + \lambda_1\pi_1 \dfrac{P(\sim o_2 \mid o_1)}{P(\sim o_2 \mid \sim o_1)} + \lambda_2 \dfrac{P(o_2 \mid \sim o_1)}{P(\sim o_2 \mid \sim o_1)}}. \quad \text{(A-18)}$$

The symbols π and λ stand for the prior odds and the likelihood ratios (i.e., the strength of evidence), respectively, and were defined in the previous section.

Equation (A-18) provides the posterior odds for $\mathcal{A} = a\&o_1\&o_2$ given all the evidence in terms of various likelihood ratios, the prior odds π_1 for the objective O_1, and the three ratios of the conditional probabilities involving the two objectives. It appears from (A-18) that the auditor needs to estimate only the ratios of various probabilities instead of absolute probabilities. This is in accordance with the "likelihood principle" (Edwards 1984).

The validity of (A-18) can be tested by considering the following two special cases. In case 1, let us assume that the two objectives are independent, that is, $P(o_2 \mid o_1) = P(o_2 \mid \sim o_1) = P(o_2)$, and $P(\sim o_2 \mid o_1) = P(\sim o_2 \mid \sim o_1) = P(\sim o_2)$. This condition reduces the expression in (A-18) to (A-10), which is what one would expect under the independent assumption. In case 2, let us assume that the two objectives are identical, that is, $P(o_2 \mid o_1) = P(\sim o_2 \mid \sim o_1) = 1$, and $P(o_2 \mid \sim o_1) = P(\sim o_2 \mid o_1) = 0$. This assumption reduces (A-18) to a result that is valid for a single objective, which is the expected result.

REFERENCES

American Institute of Certified Public Accountants. 1983. *Audit Risk and Materiality in Conducting an Audit.* Statement on Auditing Standards, No, 47. New York, NY: AICPA.

———. 1988. *Consideration of the Internal Control Structure in a Financial Statement Audit.* Statement on Auditing Standards, No. 55. New York, NY: AICPA.

Arens, A. A., and J. K. Loebbecke. 1991. *AUDITING: An Integrated Approach.* Englewood Cliffs, NJ: Prentice Hall, Inc.

Boritz, J. E., and R. E. Jensen. 1985. An hierarchical, assertion-oriented approach to planning audit evidence-gathering procedures. Symposium on Audit Judgment and Evidence Evaluation. University of Southern California.

———, and A. K. P. Wensley. 1990. Structuring the assessment of audit evidence: An expert system approach. *Auditing: A Journal of Practice & Theory* (Supplement): 49–109.

Buchanan, B. G., and E. H. Shortliffe. 1984. *Rule-Based Expert Systems: The MYCIN Experiments of the Stanford Heuristic Programming Project.* Reading, MA: Addison-Wesley.

Dutta, S. K. 1991. Evidence aggregation for planning and evaluation of audit: A theoretical study. Doctoral dissertation, University of Kansas.

Edwards, A. W. F. 1984. *Likelihood: An Account of the Statistical Concept of Likelihood and its Application to Scientific Inferences.* Cambridge: Cambridge University Press.

Fisher, R. A. 1922. On the mathematical foundations of theoretical statistics. *Philosophical Transactions of the Royal Society* CCXXII: 310.

Friedman, R. D. 1986. A close look at probative value. *Boston University Law Review* (May): 733–759.

Good, I. J. 1983. *Good Thinking: The Foundation of Probability and its Applications.* Minneapolis, MN: University of Minnesota Press.

Graham, L. E. 1985a. Audit risk - Part I. *The CPA Journal* (August): 12-21.

———. 1985b. Audit risk - Part II. *The CPA Journal* (September): 34–40.

———. 1985c. Audit risk - Part III. *The CPA Journal* (October): 36–43.

———. 1985d. Audit risk - Part IV. *The CPA Journal* (November): 38–45.

———. 1985e. Audit risk - Part V. *The CPA Journal* (December): 26–33.

Jeffrey, R. C. 1983. *The Logic of Decisions* 2d ed. New York, NY: McGraw Hill.

Kinney, W. R. Jr. 1984. Discussant's response to an analysis of the audit framework focusing on inherent risk and the role of statistical sampling in compliance testing. *Proceedings of the 1984 Touche Ross/University of Kansas Symposium on Auditing Problems* (May): 126–132.

———. 1989. Achieved audit risk and the audit outcome space. *Auditing: A Journal of Practice & Theory* (Supplement): 67–97.

Lauritzen, S. L., and D. J. Speigelhalter. 1988. Local computations with probabilities on graphical structures and their application to expert systems. *Journal of Royal Statistical Society* (Series B, Vol. 50): 157–224.

Leslie, D. A. 1984. An analysis of the audit framework focusing on inherent risk and the role of statistical sampling in compliance testing. *Proceedings of the 1984 Touche Ross/University of Kansas Symposium on Auditing Problems* (May): 89–125.

———, S. J. Aldersley, D. J. Cockburn, and C. J. Reiter. 1986. Discussant's response to an assertion-based approach to auditing. *Proceedings of the 1986 Touche Ross/University of Kansas Symposium on Auditing Problems* (May): 31–67.

Pearl, J. 1986. Fusion, propagation and structuring in belief networks. *Artificial Intelligence* 29: 241–288.

———. 1988. *Probabilistic Reasoning in Intelligent Systems: Networks of Plausible Inference.* San Mateo, CA: Morgan Kaufmann Publishers, Inc.

Savage, L. J. 1961. The foundations of statistics reconsidered. *Proceedings of the Fourth Berkeley Symposium on Mathematical Statistics and Probability,* I.

Sennetti, J. T. 1990. Toward a more consistent model for audit risk. *Auditing: A Journal of Practice & Theory* (Spring): 103–112.

Shafer, G., and P. P. Shenoy. 1990. Probability propagation. *Annals of Mathematics and Artificial Intelligence* 2: 327–352.

———, and R. P. Srivastava. 1990. The Bayesian and belief-functions formalisms I: A general perspective for auditing. *Auditing: A Journal of Practice & Theory* (Supplement): 110–148.

Shenoy, P. P., and G. Shafer. 1988. An axiomatic framework of Bayesian and belief-function propagation. *Proceedings of the Fourth Workshop on Uncertainty in Artificial Intelligence.* St. Paul, MN: 307–314.

———, and ———. 1990. Axioms for probability and belief-function propagation. In *Uncertainty in Artificial Intelligence* 4, eds. R. D. Shachter, T. S. Levitt, J. F. Lemmer, and L. N. Kanal. North-Holland: 169–198.

Srivastava, R. P., and G. Shafer. 1992. Belief-function formulas for audit risk. *The Accounting Review* (April): 1.

———, P. P. Shenoy, and G. Shafer. 1990. Belief propagation in networks for auditing. Audit Judgment Symposium. University of Southern California (February 19–20).

———. 1991. A general scheme for aggregating evidence in auditing: propagation of beliefs in networks. Working paper, School of Business, University of Kansas.

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DISCUSSION OF
Aggregation of Evidence in Auditing: A Likelihood Perspective

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The paper by Dutta and Srivastava (1992) (hereinafter "DS") discusses a general framework for aggregating evidence from a network of variables using the likelihood perspective. The paper relates to a relevant and important area of research because combining and aggregating evidence directly bears on the efficiency and effectiveness of audits.

The primary focus of my discussion relates to the validation of the aggregation scheme proposed by DS. Following the issues raised in model validation, an alternative hierarchical structure is proposed and discussed.

MODEL VALIDATION

There are several ways in which network-based models such as the one proposed in DS can be validated. One validation technique is the use of sensitivity analysis. For evidential networks, sensitivity analysis can be performed by computing the output (posterior odds) of the model after systematically varying input numbers (likelihood ratios, conditional probabilities) and/or model structure (dependencies, independencies, and relational operators).

Sensitivity analysis is important and useful for at least two reasons. First, it is a mathematical technique for validating analytical models, particularly when alternative methods are less feasible. Second, evidential networks in auditing may be improperly specified. For example, there may be errors and/or differences specific to the auditor, the firm, or the client in specifying the network. Further, we should not expect inter/intra auditor or inter/intra firm consistency in specifying the likelihood ratios, which are inputs to the model. Even if we do have consistency, there is the issue of "accuracy" in specifying the network and the likelihood ratios. This issue is perhaps unresolvable, given the lack of well-defined criteria in auditing. My point is simply that there is potential for significant variations and errors in specifying the evidential network in auditing. We must understand the implications of such variations on the aggregation scheme proposed in the paper, especially because DS describe the evidence aggregation process as involving three distinct steps, and seek to deal with all three steps of the process.

For the sake of clarity and computational tractability, I performed sensitivity analysis primarily on the model presented in figure A-1 (DS), although the implications from the analysis are relevant to the DS aggregation scheme in general. The first part of the analysis examined the impact on the output of the model (posterior odds) of varying the model structure.[1] Results of the analysis revealed significant differences in posterior odds even when seemingly minor

[1]Specifically, the model represented by equation A-10 (DS, Appendix) (independent variable case) was compared with an extended version of the model represented by equation A-18 (DS, Appendix) (dependent variable case). A copy of the derivation and details of the sensitivity analysis are available from the author.

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