

Bayesian Fraud Risk Formula for Financial Statement Audits

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1. INTRODUCTION

Globally there have been several recent changes related to the accounting profession that have increased emphasis on the responsibility of the auditor to assess adequately the risk of fraudulent financial reporting. For example, in the USA, among those changes are the promulgation of SAS No. 99, *Consideration of Fraud in a Financial Statement Audit* (AICPA 2002), enactment of the Sarbanes–Oxley Act of 2002 (H.R. 3763, 107th Cong. 2d Sess. 2002), and, most recently, the release by the Public Company Accounting Oversight Board of Auditing Standard No. 5—*An Audit of Internal Control Over Financial Reporting Performed in Conjunction with An Audit of Financial Statements* (PCAOB, 2007). In Australia, those changes are reflected in ASA 240: *The Auditor's Responsibility to Consider Fraud in an Audit of a Financial Report*, published by the Australian Auditing Standard Board (AUASB, 2006). The International Federation of Accountants (IFAC) also has elaborated on the responsibilities of the auditor towards fraudulent financial reporting in ISA 240: *The Auditor's Responsibility to Consider Fraud in an Audit of Financial Statements* (IFAC, 2004). The increase in emphasis, however, has not been accompanied by methods for fraud risk assessment based on theoretically sound principles. Instead, only general guidance is provided and each audit firm has developed its own unique approach to such assessments (Shelton et al., 2001; Montgomery et al., 2002; Mock and Turner, 2005). While some approaches attempt to aggregate fraud risks from multiple sources, none appear to have a supportable theoretical basis either for assessment or for aggregation.

Assessing the risk of fraudulent financial reporting is an example of many real world problems where decisions are made based on multiple attributes related to both quantitative and

qualitative evidence. A circumstance where a reliable decision may not be made without fully taking into account all attributes in question is identified as multiple attribute decision analysis (MADA). In MADA, one often needs to deal with both numerical data and qualitative information with uncertainty. To support such decision analysis, an evidential reasoning (ER) approach has been developed (Pearl, 1990a, 1990b) and implemented in business and auditing settings including information systems security risk assessment (Sun et al., 2006).

The objective of this paper is to develop and illustrate the use of a formula for financial reporting fraud risk assessment. The research introduces an elegant ER approach of combining probabilities in a network of variables to derive analytical models under the Bayesian framework. This methodology has not been presented previously in the accounting, auditing or finance literature. The ER approach is used to derive a formula for fraud risk based on the “fraud triangle” factors: Incentive, Attitude and Opportunity (AICPA, 2002, ¶33, see also AUASB, 2006 and IFAC, 2004). The model allows a systematic, objective method of aggregating evidence from multiple sources regarding the likelihood of financial statement fraud.

Prior audit research has proposed various Bayesian or probability-based approaches to assist in arriving at the best decision. For example, the Bayesian framework using a decision tree approach (i.e., ladder diagram) has been used to develop an audit risk model under various assumptions (e.g., Leslie, 1984, Kinney, 1984, 1989, Sennetti, 1990). However, such an approach becomes intractable under complex situations, such as those involving multiple, interrelated items of evidence as in the case of financial statement fraud. Here there are three variables, Incentives, Attitude, and Opportunities, which can be considered to be interrelated, and each variable has multiple items of evidence. Such a complex situation cannot be handled by the decision tree approach for developing analytical models.

Our model and formula extend the work of Loebbecke et al. (1989), who propose a model assessing the likelihood of material fraud; the work of Dutta et al. (1998), who develop an audit risk model incorporating misstatement due to fraud; and the work of Turner et al. (2002), who incorporate a simple version of the fraud triangle into the overall evaluation of audit risk.¹ Loebbecke et al. (1989) propose a model consisting of some, but not all, components contained in our model. The model developed by Dutta et al. (1998) lacks several important dimensions needed to evaluate fraud risk within the current professional environment. First, their treatment is limited only to affirmative items of evidence. More importantly, their model does not consider the impact of fraud triangle conditions, does not incorporate any interrelationships among fraud triangle conditions, and does not consider the role of mitigating factors. The analysis by Turner et al. (2002) is limited in that they do not derive an analytical formula incorporating both the additional fraud risk and mitigating factors described in SAS No. 99 and do not consider forensic audit procedures or interrelationships among the fraud triangle factors.

To develop the formula and demonstrate its usefulness, the remainder of the paper is organized as follows. The next section provides an overview of the ER approach and details the steps necessary to derive an ER-based formula. The third section examines prior research on fraud risk assessment while the fourth section describes a fraud risk formula under the Bayesian framework. The final section provides a summary and conclusion. A detailed derivation of the fraud risk formula discussed in the body of the paper is included as Appendix A.

¹ Although the work of Dutta et al. (1998) and Turner et al. (2002) use Dempster-Shafer theory of belief functions (Shafer 1976) for managing uncertainties, their evidential diagrams are relevant to our discussion in the present paper which uses the more traditional Bayesian formulation of uncertainty.

2. EVIDENTIAL REASONING APPROACH UNDER THE BAYESIAN FRAMEWORK

Often, the variable of interest in a problem domain may not be directly observable. However, there may be other variables that could be related to the variable of interest and knowledge about these variables may provide information to infer about the variable of interest. A schematic diagram representing these variables and their interrelationship along with the items of evidence pertaining to various variables is known as an evidential diagram (see, e.g., Srivastava and Mock, 2000).

Usually, items of evidence in evidential diagrams contain uncertainties, i.e., one is never certain that the evidence supports the variable. Under the Bayesian framework, these uncertainties are modeled using probabilities. In general, in an evidential diagram one could make predictions about any one of the variables given the knowledge or partial knowledge about all the variables. This process of updating probabilities for each variable based on the knowledge about all the variables is known as the propagation of probabilities in a network of variables. Developments in the Bayesian literature on propagating probabilities in a network of variables through local computations (see, e.g., Shenoy and Shafer, 1990) now allows one to develop analytical models for complex problems such as the ones being discussed in this paper.² These models, known as Belief Networks, are being used in a wide variety of applications, such as safety assessment, information filtering, autonomous vehicle navigation, weapons scheduling, medical diagnosis, pattern recognition, and computer network diagnosis (Heckerman et al.,

² Recent development also has made it possible to develop computer software such as Netica (Norsys Software Corp. Available at <http://www.norsys.com/netica.html>) and Hugin (Hugin Expert A/S. Available at <http://www.hugin.com/>) to solve complex problems under Bayesian framework.

1995), operational risk (Cowell et al., 2006), marine decision-support systems (Eleye-Datubo et al., 2006), and financial auditor independence (Srivastava et al., 2008).

To develop analytical models using the ER approach under the Bayesian framework, the following four steps are followed:

- Step 1: Develop an evidential diagram (see Figure 1) for the problem. An evidential diagram is a schematic representation of the variables, their interrelationships, and the items of evidence for the problem of interest.
- Step 2: Determine the probability information for each variable in the evidential diagram and for their interrelationships. This information is represented as “probability potentials” as defined later.
- Step 3: Using the Shenoy and Shafer (1990) approach, combine the probability information determined in Step 2. Under the Shenoy and Shafer approach, one expresses all the probability potentials on the joint space of all the variables in the evidential diagram and performs a point-wise multiplication of all these potentials to obtain the overall potentials.
- Step 4: Finally, marginalize the combined probability potentials to the variable of interest. When normalized, this probability information yields the appropriate posterior probabilities which may be used to assess fraud risk.

3. BACKGROUND RESEARCH ON FRAUD

In 2002, the Auditing Standards Board released SAS No. 99, *Consideration of Fraud in a Financial Statement Audit* (AICPA, 2002), intended to expand required audit procedures

assessing the risk of material financial statement fraud.³ The SAS emphasizes considering a client's susceptibility to fraud, regardless of the auditor's past experience with the entity or prior beliefs about management's honesty and integrity. In 2004, the International Auditing and Assurance Standards Board issued ISA 240 (Revised) (IFAC, 2004, see also AUASB, 2006), that substantially aligns international auditing standards regarding fraud with the requirements of SAS No. 99.

Ramos (2003) provides an analysis of the requirements of SAS No. 99 and discusses the concept of the "fraud triangle," consisting of three conditions generally present when fraud occurs: incentives/pressures, opportunity, and attitude/rationalizations. Forensic experts, academics and others argue that evaluation of information about fraud is enhanced when auditors consider it in the context of these three conditions (Montgomery et al., 2002). For brevity, the remainder of this paper refers to incentives/pressures to commit fraud only as an *incentive* unless specifically discussing circumstances involving pressures, and attitudes/rationalizations to commit fraud are referred to as *attitude*.

SAS No. 99 emphasizes obtaining a broader range of information to serve as the foundation for an assessment that goes beyond considering the fraud risk factors provided in SAS No. 82 (AICPA, 1997). The various sources of information—audit team discussions, inquiries of management and others, consideration of fraud risk factors, results of planning analytical procedures, information from the client acceptance or continuance process and from reviews of interim financial statements—are compounded into the auditor's evaluation of fraud

³ Although the AICPA Auditing Standards Board no longer has authority to set audit standards for publicly traded companies, SAS No. 99 still remains in effect.

risk. However, neither SAS No. 99, ISA 240, nor ASA 240 provides much guidance as to how factors affecting the evaluation can be combined into a quantifiable measure.

SAS No. 99 notes “The auditor’s response to the assessment of the risks of material misstatement of the financial statements due to fraud is influenced by the nature and significance of the risks identified as being present and the entity’s programs and controls that address these identified risks” (¶47). Paragraph 48 of the SAS identifies three ways the auditor may respond to identified fraud risks:

- A response involving the nature, timing and extent of auditing procedures to be performed,
- A response addressing the risk of fraud due to management override of controls, and
- A response that has an overall effect on how the audit is conducted, such as the assignment of personnel and supervision, exhibiting greater concern over the selection of accounting principles, or incorporating an element of unpredictability in the year-to-year selection of audit procedures to be performed.

For our analysis, we define these fraud-specific actions to be *forensic procedures*. In addition, audit procedures intended to identify errors related to a financial statement assertion also may provide some level of belief regarding the absence or presence of fraud. We define non-fraud-specific procedures to be *other procedures*.

SAS No. 99 also leaves open the question as to at what level the fraud risk assessment should be focused. The SAS notes:

The auditor should evaluate whether identified risks of material misstatements due to fraud can be related to specific financial-statement account balances or classes of transactions and related assertions, or whether they relate more pervasively to the financial statements as a whole. Relating the risks of material misstatement due to fraud to the individual accounts, classes of

transactions, and assertions will assist the auditor in subsequently designing appropriate auditing procedures (AICPA, 2002, ¶38).

While some fraud risk may be pervasive to the financial statements taken as a whole, for the financial statements to contain material misstatements, one or more specific assertions related to specific account balances or classes of transactions must not be supportable. Accordingly, our analysis focuses on evaluating the risk of fraud for a specific assertion.

The second significant change increasing emphasis on the assessment of fraud risk was enactment of the Sarbanes-Oxley Act of 2002 (SOX). SOX codifies significant changes to corporate governance that mitigate incentives and opportunity, and encourages greater integrity on the part of management. For example, Section 301 of the Act strengthens the role of the audit committee of the board of directors, and requires establishing procedures for the receipt, retention, and treatment of complaints received by the issuer regarding accounting, internal controls, and auditing. Section 302 requires the CEO and CFO of each issuer to prepare a statement to accompany the audit report to certify the "appropriateness of the financial statements and disclosures contained in the periodic report, and that those financial statements and disclosures fairly present, in all material respects, the operations and financial condition of the issuer." Section 303 makes it unlawful for any officer or director of an issuer to take any action to fraudulently influence, coerce, manipulate, or mislead any auditor engaged in the performance of an audit for the purpose of rendering the financial statements materially misleading.

Other sections of the Act also are intended to reduce the risk of fraud. Section 402(a) prohibits loans to any director or executive officer, thereby potentially reducing both incentives and opportunities. Section 404 requires each annual report of an issuer to contain an internal control report that states the responsibility of management for establishing and maintaining an

adequate internal control structure and procedures for financial reporting and that contains an assessment, as of the end of the issuer's fiscal year, of the effectiveness of the internal control structure and procedures of the issuer for financial reporting.

Auditing Standard No. 5 (AS5)(PCAOB, 2007), which relates to auditing the Section 404 management report on internal controls, specifies that identifying fraud risk and assessing related internal controls are vital parts of the auditor's responsibilities when auditing public companies. AS5 indicates that as part of identifying and testing entity-level controls and selecting other controls to test, the auditor "should evaluate whether the company's controls sufficiently address identified risks of material misstatement due to fraud and controls intended to address the risk of management override of other controls" (§14). The standard notes that "If the auditor identifies deficiencies in controls designed to prevent or detect fraud during the audit of internal control over financial reporting, the auditor should take into account those deficiencies when developing his or her response to risks of material misstatement during the financial statement audit..." (§15). More specific guidance for the auditor's assessment of and response to the risk of fraud still is prescribed in SAS No. 99 as described above.

Subsequent to the issuance of SAS No. 99, no research has focused on how audit firms currently assess the risk of fraud. Two prior studies, however, examined assessment procedures specific audit firms implemented in response to SAS No. 82 (AICPA, 1997), a predecessor to SAS No. 99. Shelton et al. (2001) reviewed the audit manuals of the then-Big 5 firms and two second-tier firms shortly after issuance of SAS No. 82 and conducted conference-call interviews with one or two national office directors/partners in charge of auditing/assurance policy from each of the firms.

Shelton et al. found the practice aids used by firms to assess fraud risk during the client-acceptance/continuance stage frequently involved a check-off of fraud risk factors as being present or absent. Two firms, however, used ordinal measurement scales to assess the degree to which a factor was present. One Big 5 firm required the auditor to assess on an eight-point scale individual fraud risk factors pertaining to management-opportunities risk, pressures risk, integrity, and behavior. A second Big 5 firm required the auditor to assess some risk factors on five-point scales. That firm's expert system then was used to combine those results with a scoring system based on components of business risk (Z-score equivalent, industry, company, and management) and financial-reporting risk (incentive, integrity/ethics, control, recent audit results) to develop a quantitative measure of overall engagement risk. Finally, a third Big 5 firm required the auditor to provide narrative responses to questions about fraud risk factors.

Mock and Turner (2005) also examined the response to SAS No. 82 by three large international audit firms. Of the three audit firms examined, they found that two attempted to reach an assessment through some form of formal scoring system. One firm scored each fraud risk factor as being either one or two points depending on the nature of the factor. If the total points assigned exceeded a specific baseline cutoff, a response by the audit team was required. No response was required, but was optional, for points below the cutoff amount. The second audit firm also assigned points, but used a system of netting negative points for fraud risk factors identified against positive points for associated relevant internal controls. Net negative numbers required the auditor to develop a specific response for that specific area. The third firm required a narrative summary and left any possible response to the professional judgment of the audit team. There was no evidence that scoring methodologies used by any of the firms were based on

any formal probability-based assessment framework. The formula developed in this paper does provide such a framework.

4. FRAUD RISK FORMULA UNDER THE BAYESIAN FRAMEWORK

As mentioned previously, an evidential diagram is a graphical depiction of variables, interrelationships among variables, and items of evidence pertaining to various variables that provide support for the presence or absence of the pertinent variables. An evidential diagram allows one to assess the likelihood or belief related to one variable given what we know about the other variables (See, e.g., Srivastava and Mock, 2000, and Sun et al., 2006). For example, an evidential network related to fraud will facilitate the assessment of the likelihood of fraud given what is known about the accounting system and the entity being audited.

Evidential Diagram for Fraud Risk Assessment

In Figure 1, the three fraud risk factors, Incentive (I), Attitude (A), and Opportunity (O), are depicted as oval shaped boxes. These factors are connected to the variable ‘Fraud in Assertion (F)’ through an ‘AND’ relationship. The ‘AND’ relationship implies that fraud will occur if and only if all the factors, I, A, O, are present. The rectangular boxes represent items of evidence pertaining to the variables to which they are connected. The variables, I, A, O, and F are assumed to be binary variables, i.e., each has only two values; either the variable is present or absent.

Insert Figure 1 about here

We use upper case letters to represent the names and lower case letters to represent their values. For example, I represents incentive and ‘i’ represents the state that incentive is present, and ‘~i’ represents the state that incentive is not present. The circles in Figure 1 represent

interrelationships. The circle with ‘AND’ represents the relationship between the variable F with the three variables I, A, and O. The relationship R_1 represents the interrelationship between I and A. Similarly, R_2 and R_3 in Figure 1 represent interrelationships between A and O, and between I and O, respectively.

As seen in Figure 1, we consider three types of evidence at variable F. One type of evidence is based on the prior information (E_{PI}) about fraud in the industry of the given company being audited. The second type of evidence is based on Forensic Procedures (E_{FP}) that could be performed (see Table I for definitions of other symbols). The third type of evidence is ‘Other Procedures (E_{OP})’. This type of evidence is considered in Figure 1 to include any other evidence such as analytical procedures or other procedures performed in the traditional audit that might provide evidence pertaining to the presence of fraud.

Insert Table I about here

In general, we consider two sets of evidence for each variable, I, A, and O. One set pertains to risks that increase the likelihood of the presence of the variable and the other set of evidence pertains to controls put in place that decrease the likelihood of the presence of the related variable .

For example, there may be controls in place that will decrease any incentive to commit fraud. Similarly, there may be risk factors present related to ‘Incentive’ that will increase the likelihood of an incentive to commit fraud.

Fraud Risk Formula under the Bayesian Framework

As described in detail in Appendix A, after combining the information in terms of probability potentials related to various variables in Figure 1 using Shenoy and Shafer (1990)

approach, we obtain the following fraud risk assessment formula under the Bayesian framework in terms of prior odds and the likelihood ratios (see equation A19):

$$\begin{aligned} \text{FR} = \text{Fraud Risk} &= \text{P}(\text{Fraud} | E_{\text{TI}} E_{\text{CI}} E_{\text{TA}} E_{\text{CA}} E_{\text{TO}} E_{\text{CO}} E_{\text{OP}} E_{\text{FP}}) \\ &= \rho_1 \rho_2 \rho_3 \lambda_{\text{TI}} \lambda_{\text{CI}} \lambda_{\text{TA}} \lambda_{\text{CA}} \lambda_{\text{TO}} \lambda_{\text{CO}} \lambda_{\text{OP}} \lambda_{\text{FP}} \pi_{\text{I}} \pi_{\text{A}} \pi_{\text{O}} \pi_{\text{F}} / \text{D}. \end{aligned} \quad (1)$$

The symbols ρ_1 , ρ_2 , and ρ_3 , respectively, define the strength of interrelationships between I and A, A and O, and O and I. The value of ρ lies between 0.5 and 1.0; 0.5 meaning there is no interrelationship among the two variables. A value of 1.0 means that they are interrelated with the strongest relationship; if one variable is present then the other one is also present or if one is absent then the other one is also absent. The symbols λ_{TI} , λ_{TA} , and λ_{TO} , respectively, define the likelihood ratios whether threat factors related variables ‘I’, ‘A’, and ‘O’ are present. Similarly, the symbols λ_{CI} , λ_{CA} , and λ_{CO} , respectively, define the likelihood ratios whether controls or safeguard factors are present to reduce the effects of threat factors pertaining to variables ‘I’, ‘A’, and ‘O’. The symbols π_{I} , π_{A} , π_{O} , and π_{F} , respectively, define prior odds of variables ‘I’, ‘A’, ‘O’, and ‘F’. The above likelihood ratios and the prior odds are defined as:

$$\begin{aligned} \lambda_{\text{TI}} &= \text{P}(E_{\text{TI}} | i) / \text{P}(E_{\text{TI}} | \sim i), \quad \lambda_{\text{CI}} = \text{P}(E_{\text{CI}} | i) / \text{P}(E_{\text{CI}} | \sim i), \quad \lambda_{\text{TA}} = \text{P}(E_{\text{TA}} | a) / \text{P}(E_{\text{TA}} | \sim a), \\ \lambda_{\text{CA}} &= \text{P}(E_{\text{CA}} | a) / \text{P}(E_{\text{CA}} | \sim a), \quad \lambda_{\text{TO}} = \text{P}(E_{\text{TO}} | o) / \text{P}(E_{\text{TO}} | \sim o), \quad \lambda_{\text{CO}} = \text{P}(E_{\text{CO}} | o) / \text{P}(E_{\text{CO}} | \sim o), \\ \lambda_{\text{OP}} &= \text{P}(E_{\text{OP}} | f) / \text{P}(E_{\text{OP}} | \sim f), \quad \text{and } \lambda_{\text{FP}} = \text{P}(E_{\text{FP}} | f) / \text{P}(E_{\text{FP}} | \sim f), \end{aligned} \quad (2)$$

$$\pi_{\text{I}} = \text{P}(i) / \text{P}(\sim i), \quad \pi_{\text{A}} = \text{P}(a) / \text{P}(\sim a), \quad \pi_{\text{O}} = \text{P}(o) / \text{P}(\sim o), \quad \pi_{\text{F}} = \text{P}_{\text{PI}}(f) / \text{P}_{\text{PI}}(\sim f) \quad (3)$$

The denominator, D, in (1) is defined in terms of ρ s, λ s, and π s as follows:

$$\text{D} = \text{D1} + \text{D2} + \text{D3} + \text{D4} + \text{D5} + \text{D6} + \text{D7} + \text{D8}$$

$$\text{D1} = \rho_1 \rho_2 \rho_3 \lambda_{\text{TI}} \lambda_{\text{CI}} \lambda_{\text{TA}} \lambda_{\text{CA}} \lambda_{\text{TO}} \lambda_{\text{CO}} \lambda_{\text{OP}} \lambda_{\text{FP}} \pi_{\text{I}} \pi_{\text{A}} \pi_{\text{O}} \pi_{\text{F}}$$

$$\text{D2} = (1 - \rho_1) \rho_2 (1 - \rho_3) \lambda_{\text{TA}} \lambda_{\text{CA}} \lambda_{\text{TO}} \lambda_{\text{CO}} \pi_{\text{A}} \pi_{\text{O}}$$

$$\begin{aligned}
D3 &= (1-\rho_1)(1-\rho_2)\rho_3\lambda_{TI}\lambda_{CI}\lambda_{TO}\lambda_{CO}\pi_I\pi_O \\
D4 &= \rho_1(1-\rho_2)(1-\rho_3)\lambda_{TI}\lambda_{CI}\lambda_{TA}\lambda_{CA}\pi_I\pi_A \\
D5 &= \rho_1(1-\rho_2)(1-\rho_3)\lambda_{TO}\lambda_{CO}\pi_O \\
D6 &= (1-\rho_1)\rho_2(1-\rho_3)\lambda_{TI}\lambda_{CI}\pi_I \\
D7 &= (1-\rho_1)(1-\rho_2)\rho_3\lambda_{TA}\lambda_{CA}\pi_A \\
D8 &= \rho_1\rho_2\rho_3
\end{aligned} \tag{4}$$

Equation (1) provides the fraud risk formula under Bayesian theory. The numerator in Equation (1) determines the impact of all the evidence gathered on the variable F that fraud is present, including forensic procedures and other procedures. The denominator D consists of eight terms (D1, D2, ... D8) as a result of Bayes' rule of conditioning. There are eight possible states given that the presence or absence of fraud is determined by the presence or absence of variables I, A, and O.

The likelihood ratios defined in (2) determine the strength of the respective item of evidence (e.g., see Edwards, 1984, and Dutta and Srivastava, 1993, 1996) where $\lambda = 1$ implies that the evidence provides no information about the presence or absence of the corresponding variable. For example, if the forensic procedures (FP) and other procedures (OP) are not performed then we should set $\lambda_{FP} = 1$, and $\lambda_{OP} = 1$. A positive value greater than one ($1 < \lambda < \infty$) implies that the evidence supports the assertion or hypothesis and a value of less than one ($0 < \lambda \leq 1$) implies that the evidence negates the assertion. Theoretically, an infinitely large positive value of a likelihood ratio implies that the assertion is true with probability 1.0 and a value of zero, i.e., $\lambda = 0$, implies that the assertion is not true with probability 1.0.

Illustration of the Fraud Risk Formula

To illustrate the use of the fraud risk formula, we consider three situations. The first two are ‘extreme cases’ which help establish the logical correctness and thus ‘face validity’ of the formulations. The third situation resembles what might occur in audit practice and illustrates how the formula may be used to assess fraud risk and to assess the strength of evidence needed to achieve a desired low level of fraud risk, say 0.05.

Situation 1: No evidence is gathered about fraud and the three fraud factors, and there are no interrelationships among the fraud factors.

For situation 1 we assume we have gathered no audit evidence about fraud or about the three fraud factors (i.e., no evidence for variables, I, A, O, and F). No audit information on any of these factors implies that the likelihood ratios are equal to one, i.e., $\lambda_{TI} = \lambda_{CI} = \lambda_{TA} = \lambda_{CA} = \lambda_{TO} = \lambda_{CO} = \lambda_{OP} = \lambda_{FP} = 1$. If we also assume no information is available that allows the auditor to establish priors, then the prior odds for the fraud factors I, A, and O should also be one, i.e., $\pi_I = \pi_A = \pi_O = 1$. The last set of assumptions relate to the relationship parameters ρ_1 , ρ_2 , and ρ_3 . As we have argued in Appendix A (see equations A7-A9), when no relationships are assumed among the fraud factors (variables I, A, and O), the relationship parameters take the following values: $\rho_1 = 0.5$, $\rho_2 = 0.5$, $\rho_3 = 0.5$. Under these assumptions, the fraud risk formula reduces to:

$$\text{Fraud Risk} = \pi_F / (\pi_F + 7) \quad (5)$$

The above result is logical under the Bayesian formalism and given the fraud triangle framework depicted in Figure 1. It tells us that since fraud occurs only when all three fraud risk factors I, A, and O are present, there are seven possible combinations where fraud would not occur. That is, if any one, or two, or all the three factors are absent, then fraud will not occur. If

we assume we have no knowledge about the presence or absence of fraud (i.e., $\pi_F = 1$) then Fraud Risk is $1/8 = 0.125$. However, for situation 1 where we assume no evidence is gathered about fraud or the three fraud factors, it is logical that fraud risk depends only on the prior odds or the prior probability of fraud and on the structure of the framework assumed to specify fraud risk (Figure 1 and the fraud risk model).

Situation 2: There is no incentive to commit fraud and there are no interrelationships among fraud factors

This situation also tests the assumptions of the model. We have assumed in our framework and derivations that fraud will occur only when all three fraud factors are present. In situation 2 we assume that the fraud factor incentive (I) is absent, and also that there is no interrelationship among the three fraud factors. In this case, in principle, fraud cannot occur and fraud risk should be zero.

Indeed, this is what we obtain from Equation (1) when we substitute the condition that incentive is absent (i.e., the corresponding likelihood ratio is zero or $\lambda_{CI} = 0$) and there are no interrelationships (i.e., $\rho_1 = 0.5$, $\rho_2 = 0.5$, $\rho_3 = 0.5$, see Equations A7-A9). More generally, fraud risk will be zero if any one, or any two of the fraud factors are absent under the assumption that there are no interrelationships among these factors. Of course, fraud risk will be zero when all three factors are absent, whether or not there are interrelationships.

Situation 3: Two of the three interrelationships among the three fraud factors are the strongest possible

This situation is considered to demonstrate that if any two of the three interrelationships are assumed to be the strongest then the third interrelationship becomes irrelevant. Assume that the two strongest interrelationships are the relationship between incentives and attitude (i.e., $\rho_1 =$

1) and the relationship between opportunities and attitude (i.e., $\rho_2 = 1$). When these values are substituted in (1), one obtains the following expression for the fraud risk, irrespective of the strength of the interrelationship between incentives and opportunities:

$$\text{Fraud Risk} = \text{FR} = \frac{\lambda_{\text{TI}}\lambda_{\text{CI}}\lambda_{\text{TA}}\lambda_{\text{CA}}\lambda_{\text{TO}}\lambda_{\text{CO}}\lambda_{\text{OP}}\lambda_{\text{FP}}\pi_{\text{I}}\pi_{\text{A}}\pi_{\text{O}}\pi_{\text{F}}}{1 + \lambda_{\text{TI}}\lambda_{\text{CI}}\lambda_{\text{TA}}\lambda_{\text{CA}}\lambda_{\text{TO}}\lambda_{\text{CO}}\lambda_{\text{OP}}\lambda_{\text{FP}}\pi_{\text{I}}\pi_{\text{A}}\pi_{\text{O}}\pi_{\text{F}}} \quad (6)$$

The same result is obtained if we consider any two of the three interrelationships to be the strongest ($\rho_i = 1$). Thus, if two interrelationships are the strongest possible, then the third relationship becomes irrelevant.

The above assumption can be considered to be an extreme case where if very strong incentives are present then management will have a propensity to develop an attitude to commit fraud or vice versa. Similarly if very significant opportunities exist, management will have a propensity to develop an attitude to commit fraud or vice versa.

Figure 2 presents such a case, showing graphs of fraud risk as a function of the strength of evidence from forensic procedures. In the graph, a low level of overall support for the presence of all the three fraud factors = considering the impact of both the threat factors and control factors is assumed. Note that a low level of overall support can be expressed by setting the combined likelihood ratio $\lambda_{\text{TI}}\lambda_{\text{CI}}\lambda_{\text{TA}}\lambda_{\text{CA}}\lambda_{\text{TO}}\lambda_{\text{CO}}$ with a low value, say 5.⁴ We also assume that we have no prior information about the three fraud factors (i.e., prior odds of the three factors are

⁴ Recall that this combined likelihood ratio captures the (combined) strength of evidence where a value of 1 represents no evidence (complete uncertainty given the evidence) and infinity represents the strongest possible evidence (certainty given the evidence). Thus a combined likelihood ratio of five is at the ‘low’ end of the scale.

unity, $\pi_I = \pi_A = \pi_O = 1$). We have plotted three graphs, one for each of the following values of the prior odds of fraud: 0.01, 0.1, and 1.0 (i.e., $\pi_F = 0.01, 0.1, \text{ and } 1.0$). It is interesting that for a prior odds of fraud = 0.01 and when there is a low amount of evidence that incentives, attitude, and opportunities are present (i.e., $\lambda_{TI} \lambda_{CI} \lambda_{TA} \lambda_{CA} \lambda_{TO} \lambda_{CO} = 5$), the fraud risk is small (less than 0.05) even if no forensic procedures are performed (i.e., $\lambda_{FP} = 1$).

Insert Figure 2 about here

One can see from Figure 2 that, as the prior odds of fraud increases, fraud risk increases for the same level of support for the three factors. This finding suggests that if one considers the prior probability of fraud to be small (one fraud case in 100 firms or even much less), the Bayesian formula yields a very small posterior probability of fraud even when there is moderate level of evidence about the presence of incentives, attitude, and opportunities.

These situations only are a sample of the many that could be analyzed. However, they do demonstrate the value of having an analytical formulation such as that developed in this paper. Clearly, many other situations could be created, depending on the interests of the reader.

5. SUMMARY AND CONCLUSION

In this article we present an evidential reasoning approach to developing analytical models under the Bayesian framework. We argue that the approach used in prior literature of using a decision tree (ladder) diagram is not appropriate where there are several interrelated variables as in the case of fraud risk.

We illustrate the evidential reasoning approach by developing a Bayesian fraud risk model and presenting three scenarios. Our formula for assessing fraud risk facilitates the precise

assessment of the impact of the presence or absence of and interrelationships between the three important fraud risk factors, Incentives, Attitude, and Opportunities. In addition, it facilitates the assessment of the impact of risks and controls related to these three fraud risk factors as well as the impact of forensic audit procedures and relevant analytical and other procedures that provide evidence for the presence or absence of fraud. As the scenarios illustrate, the formula can be used for planning an audit as well as sequentially evaluating audit findings.

There are many directions for future research in this area. For example, while we focus in the present paper solely on assessing fraud risk, what analytical form would a full comprehensive audit risk model take if both fraud risk and risks due to error are integrated into the model? Other types of analytical research also would be valuable, such as different specifications of the relationships between the three basic fraud risk variables and the effect of controls and threats.

The formula sketched here also suggests needed empirical research to help assess the probability potentials, conditional probabilities and other probabilities included in the formula. For example, what is the extant frequency and strength of fraud risk factors in practice (e.g., Graham and Bedard, 2003 and Mock and Turner, 2005)? What is an appropriate assessment of the effectiveness of both specific and general mitigating factors such as those included in the Sarbanes–Oxley Act of 2002? To further apply such models in practice, empirical evidence also is needed on the strength of forensic procedures such as those suggested by SAS No. 99. Clearly, much theoretical and empirical research remains to be conducted in this most important area within the profession.

REFERENCES

- Auditing and Assurance Standards Board (AUASB). 2006. ASA 240 *The Auditor's Responsibility to Consider Fraud in an Audit of a Financial Report*. Melbourne: AUASB.
- American Institute of Certified Public Accountants (AICPA). 1997. SAS No. 82—*Consideration of Fraud in a Financial Statement Audit*. New York, NY: AICPA.
- American Institute of Certified Public Accountants (AICPA). 2002. SAS No. 99—*Consideration of Fraud in a Financial Statement Audit*. New York, NY: AICPA.
- Cowell, R.G., R. J. Verrall, and Y. K. Yoon, 'Modeling Operational Risk With Bayesian Networks', *The Journal of Risk and Insurance*, Vol. 74, No. 4, 2007.
- Dutta, S. K., and R. P. Srivastava, 'Aggregation of Evidence in Auditing: A Likelihood Perspective', *Auditing: A Journal of Practice and Theory*, Vol. 12, Supplement, 1993.
- Dutta, S. K., and R. P. Srivastava, 'A Bayesian Perspective on the Strength of Evidence in Auditing', *Indian Journal of Accounting*, Vol. XXVII, June 1996.
- Dutta, S.K., K. Harrison, and R.P. Srivastava, 'The Audit Risk Model Under the Risk of Fraud', In *Applications of Fuzzy Sets & The Theory of Evidence to Accounting II*, Vol. 7, edited by P. Siegel, K. Omer, A. Korvin, and A. Zebda. Jai Press Inc., 1998.
- Edwards, A. W. F., *Likelihood: An Account of the Statistical Concept of Likelihood and its Application to Scientific Inferences*, Cambridge University Press, Cambridge 1984.
- Eleye-Datubo, A. G., A. Wall, A. Saajedi, and J. Wang, 'Enabling a Powerful Marine and Offshore Decision-Support Solution Through Bayesian Network Technique', *Risk Analysis*, Vol. 26, No. 3, 2006.
- Graham, L. and J. Bedard, 'Fraud risk and audit planning'. *International Journal of Auditing*, Vol. 7 No. 1, March 2003.
- Heckerman, D., J.S. Breese, and K. Rommelse, 'Decision-theoretic troubleshooting. (Real-World Applications of Bayesian Networks)', *Communications of the ACM*, Vol. 38, No. 3, 1995.
- International Federation of Accountants (IFAC), International Standard on Auditing (ISA) 240 (Revised), *The Auditor's Responsibility to Consider Fraud in an Audit of Financial Statements*. New York, NY: International Auditing and Assurance Standards Board, 2004.
- Kinney, W. R., Jr., 'A Discussant Response to "An Analysis of the Audit Framework Focusing on Inherent Risk and the Role Statistical Sampling in Compliance Testing."' *Auditing Symposium VII*, University of Kansas, 1984.

- Kinney, W. R., Jr., 'Achieved Audit Risk and the Outcome Space', *Auditing: A Journal of Practice and Theory*, Supplement, 1989.
- Leslie, D. A., 'An Analysis of the Audit Framework Focusing on Inherent Risk and the Role of Statistical Sampling in Compliance Testing', *Auditing Symposium VII*, University of Kansas, 1984.
- Loebbecke, J., M. Eining, and J. Willingham, 'Auditors' Experience with Material Irregularities: Frequency, Nature and Detectability. *Auditing: A Journal of Practice and Theory*, Vol. 9 No. 1, Fall 1989.
- Mock, T. and J. Turner, 'Auditor Identification of Fraud Risk Factors and Their Impact on Audit Programs', *International Journal of Auditing*, Vol. 9, 2005.
- Montgomery, D., M. Beasley, S. Menelaides, and Z. Palmrose, 'Auditors' New Procedures for Detecting Fraud', *Journal of Accountancy*, May 2002.
- Pearl, J., 'Bayesian Decision Methods', In *Readings in Uncertain Reasoning*, edited by G. Shafer and J. Pearl, Morgan Kaufmann, 1990a.
- Pearl, J., 'On Evidential Reasoning in a Hierarchy of Hypotheses', In *Readings in Uncertain Reasoning*, edited by G. Shafer and J. Pearl, Morgan Kaufmann, 1990b.
- Public Company Accounting Oversight Board (PCAOB), Auditing Standard No. 5—*An Audit of Internal Control Over Financial Reporting Performed in Conjunction with An Audit of Financial Statements*, Washington, D.C., May 24, 2007.
- Ramos, M., 'Auditors' Responsibility for Fraud Detection', *Journal of Accountancy*, May 2003.
- Sennetti, J. T., 'Toward a More Consistent Model for Audit Risk', *Auditing: A Journal of Practice & Theory*, Vol. 9, No. 2, Spring 1990.
- Shafer, G., *A Mathematical Theory of Evidence*. Princeton, NJ: Princeton University Press 1976.
- Shenoy, P. P. and Shafer. G., 'Axioms for Probability and Belief-Function Computation', in Shachter, R. D., T. S. Levitt, J. F. Lemmer, and L. N. Kanal, eds., *Uncertainty in Artificial Intelligence*, Vol. 4, North-Holland, 1990.
- Shelton, S., R. Whittington, and D. Landsittle, 'Auditing Firms' Fraud Risk Assessment Practices', *Accounting Horizons* Vol. 15, No. 1, 2001.
- Srivastava, R. P. and T. J. Mock, 'Evidential Reasoning for WebTrust Assurance Services', *Journal of Management Information Systems*, Vol. 16, No. 3, Winter 2000.
- Srivastava, R.P., T.J. Mock, and J.L. Turner, 'Bayesian and Belief Functions Formulas for Auditor Independence Risk Assessment', *International Journal of Auditing*, forthcoming 2008.

Sun, L., R. P. Srivastava, and T. Mock, 'An Information Systems Security Risk Assessment Model under Dempster-Shafer Theory of Belief Functions', *Journal of Management Information Systems*, Vol. 22, No. 4, 2006.

Turner, J.L., T.J. Mock, and R.P. Srivastava, 'A Conceptual Framework and Case Studies on Audit Planning and Evaluation Given the Potential for Fraud', *Proceedings of the 2002 Deloitte & Touche University of Kansas Symposium on Auditing Problems*, 2002.

APPENDIX A

Derivation of Fraud Risk Assessment Formula

The purpose of this appendix is to describe the process of deriving the fraud risk assessment formula under Bayesian framework. To derive the fraud risk formula under the Bayesian framework, we first need to identify all the probability information, in terms of probability potentials for all the variables in the evidential diagram in Figure 1. In addition to prior probabilities of variables I, A, O, and F, we have conditional probabilities due to items of evidence related to risks and controls for each variable, I, A, and O. Also, we assume that we have probability potentials about variable F from two audit activities, “Other Procedures” and “Forensic Procedures,” and probability information related to three interrelationships, R_1 , R_2 and R_3 , among the variables, I, A, and O.

We use the Shenoy and Shafer (1990) approach to propagate all the probability potentials to variable F to determine the posterior probability of the existence of fraud given all the evidence. This approach is presented below through the following steps.

Step 1: Identify all the Probability Potentials in the Evidential Diagram

This section provides the probability potentials relevant to the fraud risk assessment framework given in Figure 1. These potentials are given below.

Probability Potentials at variable I:

Potential due to prior probabilities

$$\begin{array}{cc} \text{State Space} & \text{Probability Potential} \\ i & \left[P(i) \right] \\ \sim i & \left[P(\sim i) \right] \end{array} \quad (A1)$$

Potential due to Threat factors, Evidence E_{TI}

State Space	Probability Potential	
i	$\left[P(E_{TI} i) \right]$	(A2)
$\sim i$	$\left[P(E_{TI} \sim i) \right]$	

Potential due to Controls, Evidence E_{CI}

State Space	Probability Potential	
i	$\left[P(E_{CI} i) \right]$	(A3)
$\sim i$	$\left[P(E_{CI} \sim i) \right]$	

As one can see, these potentials are not normalized because $P(E_{TI}|i) + P(E_{TI}|\sim i) \neq 1$, and $P(E_{CI}|i) + P(E_{CI}|\sim i) \neq 1$.

The combined probability potentials at variable I are determined by point-wise multiplication, which is the process of multiplying each element of the potential by the corresponding element of the other potentials at the variable. This process yields the following potentials⁵ at node {I}.

⁵ Here we assume that evidence E_{TI} is conditionally independent of E_{CI} given ‘i’. This means that once we know that ‘i’ is true then the evidence E_{TI} pertaining to threat factors related to ‘I’ would not tell us any thing about evidence E_{CI} pertaining to controls related to incentives ‘I’. However, if in reality it is found that the two items of evidence, E_{TI} and E_{CI} are not independent then one can replace the terms $P(E_{TI}|i)P(E_{CI}|i)$ and $P(E_{TI}|\sim i)P(E_{CI}|\sim i)$ in (A4) by $P(E_{TI}E_{CI}|i)$ and $P(E_{TI}E_{CI}|\sim i)$, respectively, without any loss of generality. The only difference on the final formula would be to have one likelihood ratio determining the presence of the variable I based on the two items of evidence E_{TI} and E_{CI} jointly instead of having two separate likelihood ratios, one pertaining to threat factors and the other pertaining to control factors. Similar assumptions

$$\begin{array}{cc}
\text{State Space} & \text{Probability Potential} \\
i & \left[\begin{array}{c} P(i)P(E_{TI}|i)P(E_{CI}|i) \\ P(\sim i)P(E_{TI}|\sim i)P(E_{CI}|\sim i) \end{array} \right] \\
\sim i &
\end{array} \tag{A4}$$

If we normalize the above potentials, we obtain the posterior probabilities obtained by using Bayes' Rule. One advantage of using the Shenoy and Shafer (1990) approach for propagating probabilities to derive the fraud risk formula is that it does not require renormalization of the potentials at every stage of the combination.

Probability Potentials at variable A:

Similar to variable I, variable A has three sets of potentials—one set from the prior probabilities, a second from the risk factors, and a third set from the controls factors. The combined potentials can be written in the following form, similar to (A4):

$$\begin{array}{cc}
\text{State Space} & \text{Probability Potential} \\
a & \left[\begin{array}{c} P(a)P(E_{TA}|a)P(E_{CA}|a) \\ P(\sim a)P(E_{TA}|\sim a)P(E_{CA}|\sim a) \end{array} \right] \\
\sim a &
\end{array} \tag{A5}$$

Probability Potentials at Variable O:

Similar to variables I and A, variable O has three sets of potentials, one set from the prior probabilities, a second from the risk factors, and a third from the controls factors. The combined potentials can be written in the following form, similar to (A4) or (A5):

$$\begin{array}{cc}
\text{State Space} & \text{Probability Potential} \\
o & \left[\begin{array}{c} P(o)P(E_{TO}|o)P(E_{CO}|o) \\ P(\sim o)P(E_{TO}|\sim o)P(E_{CO}|\sim o) \end{array} \right] \\
\sim o &
\end{array} \tag{A6}$$

are made for E_{TA} and E_{CA} that they are conditionally independent given 'a', and E_{TO} and E_{CO} are conditionally independent given 'o'.

Probability Potentials related to the Three Interrelationships:

The probability potentials defining the relationships are conditional probabilities so if one variable is present, then the corresponding variable is present with a certain level of probability, ρ_1 , representing the strength of the interrelationship. The potentials associated with relationship R_1 , which represents the interrelationship between I and A, are given as:

$$\begin{array}{l} ia \\ i\sim a \\ \sim ia \\ \sim i\sim a \end{array} \left[\begin{array}{l} P(a|i) = \rho_1 \\ P(\sim a|i) = 1 - \rho_1 \\ P(a|\sim i) = 1 - \rho_1 \\ P(\sim a|\sim i) = \rho_1 \end{array} \right] \quad (A7)$$

Note that we no longer provide headers for the state space or probability potentials.

This relationship is described as follows. If there is an incentive to commit fraud, that is ‘i’ is true, then management will be more likely to compromise their ethical values to some extent—that is, if ‘i’ is true with probability 1 then ‘a’ will be true with probability ρ_1 where ρ_1 determines the strength of the relationship. We assume this relationship to be symmetric in the sense that if incentives are not present then there is less reason for management to compromise their ethical values. In other words, if ‘ $\sim i$ ’ is true then ‘ $\sim a$ ’ will be true at a level ρ_1 . The strongest relationship is represented by $\rho_1 = 1$, and no relationship is represented by $\rho_1 = 0.5$.

The potentials associated with relationship R_2 , representing the interrelationship between A and O, are given as:

$$\begin{array}{l} ao \\ a\sim o \\ \sim ao \\ \sim a\sim o \end{array} \left[\begin{array}{l} P(o|a) = \rho_2 \\ P(\sim o|a) = 1 - \rho_2 \\ P(o|\sim a) = 1 - \rho_2 \\ P(\sim o|\sim a) = \rho_2 \end{array} \right] \quad (A8)$$

where ρ_2 determines the strength of the relationship.

Similarly, the potentials associated with relationship R_3 , representing the interrelationship between O and I, are given as:

$$\begin{array}{l} oi \\ o\sim i \\ \sim oi \\ \sim o\sim i \end{array} \left[\begin{array}{l} P(i|o) = \rho_3 \\ P(\sim i|o) = 1 - \rho_3 \\ P(i|\sim o) = 1 - \rho_3 \\ P(\sim i|\sim o) = \rho_3 \end{array} \right] \quad (A9)$$

where ρ_3 determines the strength of R_3 relationship.

Probability Potentials for the ‘AND’ relationship:

The probability potentials for the ‘AND’ relationship are given by:

$$\begin{array}{l} iaof \\ \sim iaof \\ i\sim aof \\ ia\sim of \\ \sim i\sim aof \\ i\sim a\sim of \\ \sim ia\sim of \\ \sim i\sim a\sim of \end{array} \left[\begin{array}{l} P(f|iao) = 1.0 \\ P(\sim f|\sim iao) = 1.0 \\ P(\sim f|i\sim ao) = 1.0 \\ P(\sim f|ia\sim o) = 1.0 \\ P(\sim f|\sim i\sim ao) = 1.0 \\ P(\sim f|i\sim a\sim o) = 1.0 \\ P(\sim f|\sim ia\sim o) = 1.0 \\ P(\sim f|\sim i\sim a\sim o) = 1.0 \end{array} \right] \quad (A10)$$

The above potentials define the relationship between variable F and the variables I, A, and O. This implies that fraud is present if and only if an incentive is present (i), a compromised attitude is present (a), and an opportunity is present (o). There is no fraud, that is ‘ $\sim f$ ’ is true, when any one, any two, or all three variables are not present.

Probability Potentials at Variable F

As seen from Figure 1, there are three types of potentials directly related to variable F. The first potential is due to prior information or prior probabilities while the second is based on other procedures (OP), consisting of analytical procedures, third-party confirmations, and other normally-performed audit procedures that potentially could detect fraud. The third potential is

due to any forensic procedures (FP) specifically performed to detect whether fraud is present or absent for an assertion. Similar to variables I, A or O, the combined potentials at F due to prior probabilities, other procedures, and forensic procedures can be written as:

$$\begin{matrix} f \\ \sim f \end{matrix} \left[\begin{matrix} P_{PI}(f)P(E_{OP}|f)P(E_{FP}|f) \\ P_{PI}(\sim f)P(E_{OP}|\sim f)P(E_{FP}|\sim f) \end{matrix} \right] \quad (A11)$$

where $P_{PI}(f)$ and $P_{PI}(\sim f)$, represent the prior probabilities of ‘f’ and ‘~f’ based on the prior information about ‘F’. Similarly, $P(E_{OP}|f)$; $P(E_{OP}|\sim f)$ are the likelihoods or conditional probabilities related to the evidence E_{OP} , and $P(E_{FP}|f)$, and $P(E_{FP}|\sim f)$ are the likelihoods or conditional probabilities related to evidence E_{FP} .

Step 2: Combined Potentials at Variable F

Having identified all the probabilities potentials related to the framework in Figure 1, the potentials are combined using the Shenoy and Shafer (1990) approach. To combine various potentials defined on different state spaces, they need to be vacuously extended⁶ to a common state space of the joint space of the binary variables I, A, O, and F. In general, there will be 16

⁶ Vacuous extension can be illustrated through an example. Suppose we have the following probability potentials on the state space $\{i, \sim i\}$ of the variable I: $\phi(i) = P(i)$, and $\phi(\sim i) = P(\sim i)$. We can extend this information onto the joint state space of the binary variables I and A, $\{ia, i\sim a, \sim ia, \sim i\sim a\}$, as follows: $\phi(ia) = \phi(i) = P(i)$, $\phi(i\sim a) = \phi(i) = P(i)$, $\phi(\sim ia) = \phi(\sim i) = P(\sim i)$, $\phi(\sim i\sim a) = \phi(\sim i) = P(\sim i)$. This probability potential is not normalized because the sum of all the potentials, $\phi(ia)$, $\phi(i\sim a)$, $\phi(\sim ia)$, and $\phi(\sim i\sim a)$ on the space $\{ia, i\sim a, \sim ia, \sim i\sim a\}$ is not equal to 1.0. The normalized potentials would be $\phi(ia) = 0.5P(i)$, $\phi(i\sim a) = 0.5P(i)$, $\phi(\sim ia) = 0.5P(\sim i)$, and $\phi(\sim i\sim a) = 0.5P(\sim i)$.

possible states in the joint space. However, the ‘AND’ relationship restricts the state space of interest to the following set $\{iaof, \sim iaof, i\sim aof, ia\sim of, \sim i\sim aof, i\sim a\sim of, \sim ia\sim of, \sim i\sim a\sim of\}$.

The combination is accomplished through a point-wise multiplication as described earlier.

To demonstrate, the vacuous extension of one potential at ‘I’ follows and the rest are very similar:

Potential at variable I

$$\begin{array}{l}
 iaof \\
 \sim iaof \\
 i\sim aof \\
 ia\sim of \\
 \sim i\sim aof \\
 i\sim a\sim of \\
 \sim ia\sim of \\
 \sim i\sim a\sim of
 \end{array}
 \left[\begin{array}{l}
 P(i)P(E_{TI}|i)P(E_{CI}|i) \\
 P(\sim i)P(E_{TI}|\sim i)P(E_{CI}|\sim i) \\
 P(i)P(E_{TI}|i)P(E_{CI}|i) \\
 P(i)P(E_{TI}|i)P(E_{CI}|i) \\
 P(\sim i)P(E_{TI}|\sim i)P(E_{CI}|\sim i) \\
 P(i)P(E_{TI}|i)P(E_{CI}|i) \\
 P(\sim i)P(E_{TI}|\sim i)P(E_{CI}|\sim i) \\
 P(\sim i)P(E_{TI}|\sim i)P(E_{CI}|\sim i)
 \end{array} \right] \quad (A12)$$

Similar results can be obtained for the potentials at variables A and O.

The combined potentials of the vacuously extended potentials for the three relationships using point-wise multiplication are given as:

$$\begin{array}{l}
 iaof \\
 \sim iaof \\
 i\sim aof \\
 ia\sim of \\
 \sim i\sim aof \\
 i\sim a\sim of \\
 \sim ia\sim of \\
 \sim i\sim a\sim of
 \end{array}
 \left[\begin{array}{l}
 \rho_1\rho_2\rho_3 \\
 (1-\rho_1)\rho_2(1-\rho_3) \\
 (1-\rho_1)(1-\rho_2)\rho_3 \\
 \rho_1(1-\rho_2)(1-\rho_3) \\
 \rho_1(1-\rho_2)(1-\rho_3) \\
 (1-\rho_1)\rho_2(1-\rho_3) \\
 (1-\rho_1)(1-\rho_2)\rho_3 \\
 \rho_1\rho_2\rho_3
 \end{array} \right] \quad (A13)$$

The following potentials are obtained by vacuously extending the potentials defined at variable F (see Equation 8):

$$\begin{array}{l}
\text{iaof} \\
\sim\text{iao}\sim\text{f} \\
\text{i}\sim\text{ao}\sim\text{f} \\
\text{ia}\sim\text{o}\sim\text{f} \\
\sim\text{i}\sim\text{ao}\sim\text{f} \\
\text{i}\sim\text{a}\sim\text{o}\sim\text{f} \\
\sim\text{ia}\sim\text{o}\sim\text{f} \\
\sim\text{i}\sim\text{a}\sim\text{o}\sim\text{f}
\end{array}
\left[\begin{array}{l}
P_{PI}(f)P(E_{OP}|f)P(E_{FP}|f) \\
P_{PI}(\sim f)P(E_{OP}|\sim f)P(E_{FP}|\sim f) \\
P_{PI}(\sim f)P(E_{OP}|\sim f)P(E_{FP}|\sim f)
\end{array} \right] \quad (\text{A14})$$

To determine the overall potentials on the state space $\{\text{iaof}, \sim\text{iao}\sim\text{f}, \text{i}\sim\text{ao}\sim\text{f}, \text{ia}\sim\text{o}\sim\text{f}, \sim\text{i}\sim\text{ao}\sim\text{f}, \text{i}\sim\text{a}\sim\text{o}\sim\text{f}, \sim\text{ia}\sim\text{o}\sim\text{f}, \sim\text{i}\sim\text{a}\sim\text{o}\sim\text{f}\}$, we point-wise multiply the eight sets of potentials (three sets from the three variables, I, A, and O, three sets from the three relationships, one set from the ‘AND’ relationship, and one set from the variable F). In other words, we point-wise multiply the potentials in (A12), (A13), and (A14) and the extended potentials at A and O. This multiplication yields the following potentials:

$$\begin{array}{l}
\text{iaof} \\
\sim\text{iao}\sim\text{f} \\
\text{i}\sim\text{ao}\sim\text{f} \\
\text{ia}\sim\text{o}\sim\text{f} \\
\sim\text{i}\sim\text{ao}\sim\text{f} \\
\text{i}\sim\text{a}\sim\text{o}\sim\text{f} \\
\sim\text{ia}\sim\text{o}\sim\text{f} \\
\sim\text{i}\sim\text{a}\sim\text{o}\sim\text{f}
\end{array}
\left[\begin{array}{l}
T1.P_{PI}(f)P(E_{OP}|f)P(E_{FP}|f) \\
T2.P_{PI}(\sim f)P(E_{OP}|\sim f)P(E_{FP}|\sim f) \\
T3.P_{PI}(\sim f)P(E_{OP}|\sim f)P(E_{FP}|\sim f) \\
T4.P_{PI}(\sim f)P(E_{OP}|\sim f)P(E_{FP}|\sim f) \\
T5.P_{PI}(\sim f)P(E_{OP}|\sim f)P(E_{FP}|\sim f) \\
T6.P_{PI}(\sim f)P(E_{OP}|\sim f)P(E_{FP}|\sim f) \\
T7.P_{PI}(\sim f)P(E_{OP}|\sim f)P(E_{FP}|\sim f) \\
T8.P_{PI}(\sim f)P(E_{OP}|\sim f)P(E_{FP}|\sim f)
\end{array} \right] \quad (\text{A15})$$

where the T’s are defined below:

$$\begin{aligned}
T1 &= \rho_1\rho_2\rho_3P(i)P(a)P(o)P(E_{TI}|i)P(E_{CI}|i)P(E_{TA}|a)P(E_{CA}|a)P(E_{TO}|o)P(E_{SO}|o), \\
T2 &= (1-\rho_1)\rho_2(1-\rho_3)P(\sim i)P(a)P(o)P(E_{TI}|\sim i)P(E_{CI}|\sim i)P(E_{TA}|a)P(E_{CA}|a)P(E_{TO}|o)P(E_{CO}|o), \\
T3 &= (1-\rho_1)(1-\rho_2)\rho_3P(i)P(\sim a)P(o)P(E_{TI}|i)P(E_{CI}|i)P(E_{TA}|\sim a)P(E_{CA}|\sim a)P(E_{TO}|o)P(E_{CO}|o), \\
T4 &= \rho_1(1-\rho_2)(1-\rho_3)P(i)P(a)P(\sim o)P(E_{TI}|i)P(E_{CI}|i)P(E_{TA}|a)P(E_{CA}|a)P(E_{TO}|\sim o)P(E_{CO}|\sim o), \\
T5 &= \rho_1(1-\rho_2)(1-\rho_3)P(\sim i)P(\sim a)P(o)P(E_{TI}|\sim i)P(E_{CI}|\sim i)P(E_{TA}|\sim a)P(E_{CA}|\sim a)P(E_{TO}|o)P(E_{CO}|o),
\end{aligned} \quad (\text{A16})$$

$$T6 = (1-\rho_1)\rho_2(1-\rho_3)P(i)P(\sim a)P(\sim o)P(E_{TI}|i)P(E_{CI}|i)P(E_{TA}|\sim a)P(E_{CA}|\sim a)P(E_{TO}|\sim o)P(E_{CO}|\sim o),$$

$$T7 = (1-\rho_1)(1-\rho_2)\rho_3P(\sim i)P(a)P(\sim o)P(E_{TI}|\sim i)P(E_{CI}|\sim i)P(E_{TA}|a)P(E_{CA}|a)P(E_{TO}|\sim o)P(E_{CO}|\sim o),$$

$$T8 = \rho_1\rho_2\rho_3P(\sim i)P(\sim a)P(\sim o)P(E_{TI}|\sim i)P(E_{CI}|\sim i)P(E_{TA}|\sim a)P(E_{CA}|\sim a)P(E_{TO}|\sim o)P(E_{CO}|\sim o).$$

To determine the combined potentials at variable F, we marginalize⁷ the above potential in (15a). The marginalization process yields the following potentials at F.

$$\begin{matrix} f \\ \sim f \end{matrix} \left[\begin{matrix} T1.P_{PI}(f)P(E_{OP}|f)P(E_{FP}|f) \\ (T2+T3+T4+T5+T6+T7+T8)P_{PI}(\sim f)P(E_{OP}|\sim f)P(E_{FP}|\sim f) \end{matrix} \right] \quad (A17)$$

Fraud Risk Formula under the Bayesian Framework

Equation (A17) provides the overall potentials at node {F} after combining all the items of evidence including the priors. When normalized, the potentials in (A17) yield the posterior probability that fraud is present given all the evidence about F, I, A, and O. To simplify the final result, we divide both the numerator and denominator of the normalized potentials by

$$P(\sim i)P(\sim a)P(\sim o)P_{PI}(\sim f)P(E_{TI}|\sim i)P(E_{CI}|\sim i)P(E_{TA}|\sim a)P(E_{CA}|\sim a)P(E_{TO}|\sim o)P(E_{CO}|\sim o)P(E_{OP}|\sim f)P(E_{FP}|\sim f)$$

and define the following likelihood ratios:

$$\lambda_{TI} = P(E_{TI}|i)/P(E_{TI}|\sim i), \quad \lambda_{CI} = P(E_{CI}|i)/P(E_{CI}|\sim i), \quad \lambda_{TA} = P(E_{TA}|a)/P(E_{TA}|\sim a),$$

$$\lambda_{CA} = P(E_{CA}|a)/P(E_{CA}|\sim a), \quad \lambda_{TO} = P(E_{TO}|o)/P(E_{TO}|\sim o), \quad \lambda_{CO} = P(E_{CO}|o)/P(E_{CO}|\sim o),$$

⁷ Marginalization of potentials is similar to marginalization of probabilities where variables not needed are eliminated by summing the probabilities over those variables. For example, if we marginalize the un-normalized potentials in footnote 4 from the joint space of {ia, i~a, ~ia, ~i~a} to the space {i, ~i}, we get $\phi(i) = \phi(ia) + \phi(i\sim a) = P(i) + P(i) = 2P(i)$, and $\phi(\sim i) = \phi(\sim ia) + \phi(\sim i\sim a) = P(\sim i) + P(\sim i) = 2P(\sim i)$. Once these potentials are normalized, then the probability distribution on the space {i, ~i} can be obtained.

$$\lambda_{OP} = P(E_{OP}|f)/P(E_{OP}|\sim f), \text{ and } \lambda_{FP} = P(E_{FP}|f)/P(E_{FP}|\sim f). \quad (\text{A18})$$

We thus obtain the following posterior probability of fraud being present:

$$\begin{aligned} \text{FR} = \text{Fraud Risk} &= P(\text{Fraud}|E_{TI}E_{CI}E_{TA}E_{CA}E_{TO}E_{CO}E_{OP}E_{FP}) \\ &= \rho_1\rho_2\rho_3\lambda_{TI}\lambda_{CI}\lambda_{TA}\lambda_{CA}\lambda_{TO}\lambda_{CO}\lambda_{OP}\lambda_{FP}\pi_I\pi_A\pi_O\pi_F/D \end{aligned} \quad (\text{A19})$$

Where D's are defined as:

$$\begin{aligned} D &= D1 + D2 + D3 + D4 + D5 + D6 + D7 + D8 \\ D1 &= \rho_1\rho_2\rho_3\lambda_{TI}\lambda_{CI}\lambda_{TA}\lambda_{CA}\lambda_{TO}\lambda_{CO}\lambda_{OP}\lambda_{FP}\pi_I\pi_A\pi_O\pi_F \\ D2 &= (1-\rho_1)\rho_2(1-\rho_3)\lambda_{TA}\lambda_{CA}\lambda_{TO}\lambda_{CO}\pi_A\pi_O \\ D3 &= (1-\rho_1)(1-\rho_2)\rho_3\lambda_{TI}\lambda_{CI}\lambda_{TO}\lambda_{CO}\pi_I\pi_O \\ D4 &= \rho_1(1-\rho_2)(1-\rho_3)\lambda_{TI}\lambda_{CI}\lambda_{TA}\lambda_{CA}\pi_I\pi_A \\ D5 &= \rho_1(1-\rho_2)(1-\rho_3)\lambda_{TO}\lambda_{CO}\pi_O \\ D6 &= (1-\rho_1)\rho_2(1-\rho_3)\lambda_{TI}\lambda_{CI}\pi_I \\ D7 &= (1-\rho_1)(1-\rho_2)\rho_3\lambda_{TA}\lambda_{CA}\pi_A \\ D8 &= \rho_1\rho_2\rho_3 \end{aligned} \quad (\text{A20})$$

and the prior odds, π 's, are defined as:

$$\pi_I = P(i)/P(\sim i), \quad \pi_A = P(a)/P(\sim a), \quad \pi_O = P(o)/P(\sim o), \quad \pi_F = P_{PI}(f)/P_{PI}(\sim f) \quad (\text{A21})$$

The above symbols are defined in Table I.

Table I
List of Symbols and Their Descriptions

Symbol	Description
$A \{a, \sim a\}$	A represents the variable 'Attitude'. a and $\sim a$, respectively, represent that A is true, and not true. In other words, ' a ' represents that management's attitude rationalizes the commitment of fraud, and ' $\sim a$ ' is opposite to a .
$F \{f, \sim f\}$	F represents the variable 'Fraud'. F and $\sim f$, respectively, represent that F is true, and not true. In other words, ' f ' represents that fraud is present and ' $\sim f$ ' that fraud is not present.
$I \{i, \sim i\}$	I represents the variable 'Incentive'. i and $\sim i$, respectively, represent that I is true and not true. In other words, ' i ' represents that there is an incentive, and ' $\sim i$ ' represents that there is no incentive.
$O \{o, \sim o\}$	O represents the variable 'Opportunity'. o and $\sim o$, respectively, represent that O is true, and not true. In other words, ' o ' represents that there is an opportunity, and ' $\sim o$ ' represents that there is no opportunity.
E_{TI}	Evidence relevant to Risks related to 'Incentives'
E_{CI}	Evidence relevant to Controls related to 'Incentives'
E_{TA}	Evidence relevant to Risks related to 'Attitude'
E_{CA}	Evidence relevant to Controls related to 'Attitude'
E_{TO}	Evidence relevant to Risks related to 'Opportunities'
E_{CO}	Evidence relevant to Controls related to 'Opportunities'
E_{PI}	Evidence based on prior information (PI) relevant to fraud, F, whether it is present or not
E_{OP}	Evidence other than forensic procedures related to 'F'
E_{FP}	Evidence based on Forensic procedures related to 'F'
R_1	Represents relational node between 'Incentive' and 'Attitude' with ρ_1 being its strength.
R_2	Represents relational node between 'Attitude' and 'Opportunity' with ρ_2 being its strength.
R_3	Represents relational node between 'Opportunity' and 'Incentive' with ρ_3 being its strength.
$P(a), P(i), P(o)$	Prior probability that ' a ', ' i ', and ' o ' is true, respectively.
$P(\sim a), P(\sim i), P(\sim o)$	Prior probability that ' $\sim a$ ', ' $\sim i$ ', and ' $\sim o$ ' is true, respectively.
$P_{PI}(f), P_{PI}(\sim f)$	Prior Probabilities of fraud being present, ' f ', and fraud being not present, ' $\sim f$ '.
$P(E_{TI} i), P(E_{TI} \sim i)$	Conditional Probabilities that evidence E_{TI} exists given that ' i ' is true and ' $\sim i$ ' is true.
$P(E_{CI} i), P(E_{CI} \sim i)$	Conditional Probabilities that evidence E_{CI} exists given that ' i ' is true and ' $\sim i$ ' is true.
$P(E_{TA} a), P(E_{TA} \sim a)$	Conditional Probabilities that evidence E_{TA} exists given that ' a ' is true and ' $\sim a$ ' is true.
$P(E_{CA} a), P(E_{CA} \sim a)$	Conditional Probabilities that evidence E_{CA} exists given that ' a ' is true and ' $\sim a$ ' is true.
$P(E_{TO} o), P(E_{TO} \sim o)$	Conditional Probabilities that evidence E_{TO} exists given that ' o ' is true and ' $\sim o$ ' is true.
$P(E_{CO} o), P(E_{CO} \sim o)$	Conditional Probabilities that evidence E_{CO} exists given that ' o ' is true and ' $\sim o$ ' is true.
$P(E_{OP} f), P(E_{OP} \sim f)$	Conditional Probabilities that evidence E_{OP} exists given that ' f ' is true and ' $\sim f$ ' is true.
$P(E_{FP} f), P(E_{FP} \sim f)$	Conditional Probabilities that evidence E_{FP} exists given that ' f ' is true and ' $\sim f$ ' is true.
λ_{TI}	The Likelihood Ratio representing the strength of evidence E_{TI} , $\lambda_{TI} = P(E_{TI} i)/P(E_{TI} \sim i)$.
λ_{CI}	The Likelihood Ratio representing the strength of evidence E_{CI} , $\lambda_{CI} = P(E_{CI} i)/P(E_{CI} \sim i)$.
λ_{TA}	The Likelihood Ratio representing the strength of evidence E_{TA} , $\lambda_{TA} = P(E_{TA} a)/P(E_{TA} \sim a)$.
λ_{CA}	The Likelihood Ratio representing the strength of evidence E_{CA} , $\lambda_{CA} = P(E_{CA} a)/P(E_{CA} \sim a)$.
λ_{TO}	The Likelihood Ratio representing the strength of evidence E_{TO} , $\lambda_{TO} = P(E_{TO} o)/P(E_{TO} \sim o)$.
λ_{CO}	The Likelihood Ratio representing the strength of evidence E_{CO} , $\lambda_{CO} = P(E_{CO} o)/P(E_{CO} \sim o)$.
λ_{OP}	The Likelihood Ratio representing the strength of evidence E_{OP} , $\lambda_{OP} = P(E_{OP} f)/P(E_{OP} \sim f)$.
λ_{FP}	The Likelihood Ratio representing the strength of evidence E_{FP} , $\lambda_{FP} = P(E_{FP} f)/P(E_{FP} \sim f)$.
π_I, π_A, π_O	π_I, π_A and π_O , respectively, represent the prior odds of variables 'I', 'A', and 'O'.
π_F	π_F represents the prior odds of variables 'F'.

Figure 1: Evidential Diagram for Fraud Risk

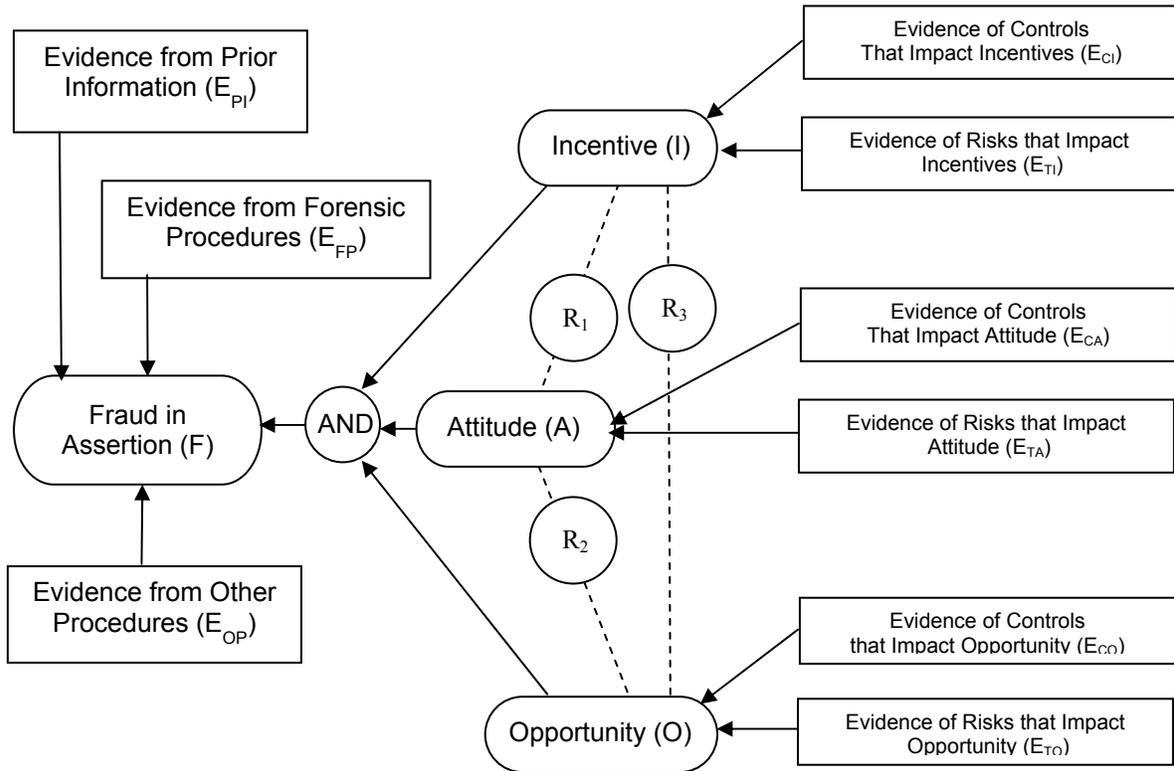
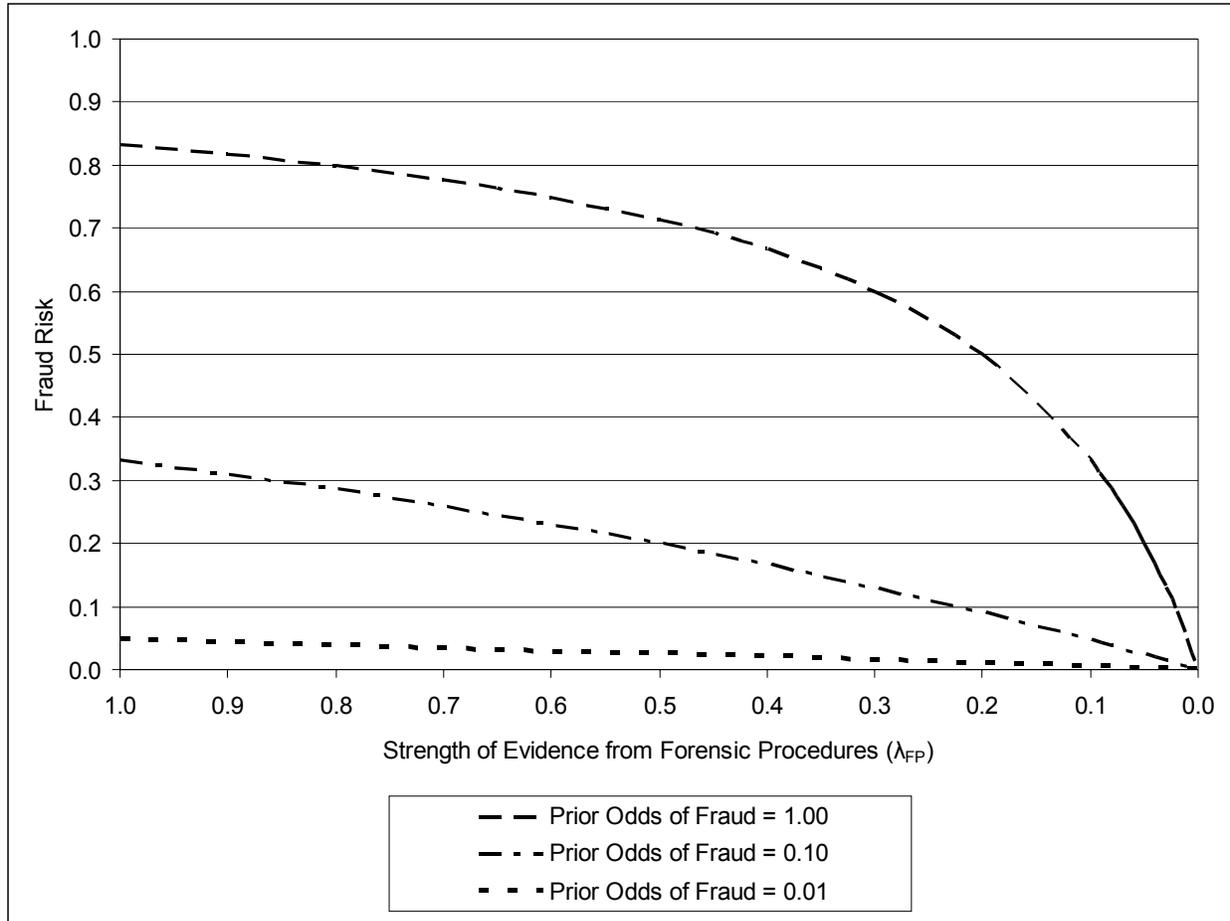


Figure 2: Fraud Risk as a function of the strength of evidence from the forensic procedures for the presence of fraud when there is a low level of overall support that all the three fraud factors are present.*



* For this graph we assume a perfect interrelationship between incentives and attitude, and between opportunities and attitude.