# Introduction to Belief Functions[1]

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## 1. Introduction

> The refusal to choose is a form of choice;
> Disbelief is a form of belief. -Frank Barron

This chapter introduces a theoretical perspective that may be used in business research and practice when confronting decision tasks that involve uncertainly. The main body of the chapter is an introduction to Belief Functions. The introduction includes a discussion of the fundamental constructs and then illustrates the use of belief functions in a business (audit) setting.

Human decision-making is a complex process, especially in situations where significant ambiguity exists. Ambiguity and uncertainty are inherent characteristics of tasks in all disciplines whether it is accounting, auditing, business, law, or medicine. The primary objective of this chapter is to introduce some important aspects of belief formulation within a business decision context.

There are two major issues when it comes to dealing with belief formulation under ambiguity and <u>uncertainty</u>. In general, uncertainty deals with situations where one is not sure about the outcome of an event. For example, consider an urn with 100 balls, 50 red and 50 black. If one is considering the event of drawing a ball from the urn, the outcome of whether the color would be red or black is uncertain (not sure).

The first issue deals with the framework that can be used to express or measure uncertainty and ambiguity present in such task settings. Shafer and Tversky (1985) and Shafer and Srivastava (1990) describe this process of choosing a framework as a process of choosing a "formal language" or "semantics" to express the uncertainties in analyzing a task. They argue that the context and the domain of

[1] Based on Srivastava and Mock 2000. Belief Functions in Accounting Behavioral Research. *Advances in Accounting Behavioral Research*, Vol. 3: 225-242.

the problem determine what language is appropriate. This chapter introduces one language and set of notation that is appropriate.

The second issue deals with the calculus or "syntax" (Shafer and Tversky 1985) through which information is combined to make a judgment or decision. There are many frameworks and formal languages that can be used to represent uncertainties such as those based on probability theory, fuzzy logic, possibility theory (Zadeh 1978, 1979), belief functions (Shafer 1976, Smets 1990a, 1990b, 1998, Yager et. al 1994), and epistemic belief functions (Spohn 1990, 1998). These frameworks have different characteristics and thus seem to "work better" in certain problem domains rather than in other domains.

As illustrated in this book, belief functions provide a flexible and adaptable way to combine evidence from a variety of sources. One aspect of this flexibility is that the belief function framework reduces to the Bayesian framework under a special condition (Shafer and Srivastava 1990).

Importantly, belief functions provide a superior way of mapping uncertainty judgments and for incorporating ambiguity within the decision-making process (Srivastava 1997a)[2]. For example, suppose an auditor has obtained and assessed audit risk factors (*inherent factors*) for a client such as its business environment, its economic condition, and the CEO's honesty and integrity. Assume that all of these factors provide positive evidence to the auditor that the financial statements are fairly stated. Based on this, the auditor could attribute a low level of support, say 0.2 on a scale of 0-1, that the financial statements are fairly stated, a zero level of support that the financial statements are materially misstated, and a 0.8 level of uncommitted support. Such a representation of uncertainty is difficult under the probability framework, as it is unclear as to how the uncommitted support (the ambiguity) should be treated.

Thus an advantage of using belief functions in business is that it provides a more natural and logical way to model ambiguity compared to the Classical probability framework. To further illustrate this point, let us consider the urn example introduced earlier. Consider a situation with two urns. Urn One has 100 balls, 50 red and 50 black. The outcome of whether the color of a ball drawn from Urn One is red or black is uncertain, but quantifiable. Given only two possible outcomes (red or black) with equal likelihood (50 red and 50 black balls), one can make a judgment that each outcome has a 50-50 chance of occurring.

Ambiguity deals with those situations of uncertainty where one is not able to quantify a judgment about the chance of each outcome occurring. For example,

[2] See chapter 2.4: "Auditors' Evaluation of Uncertain Audit Evidence" and also Harrison 1999.

suppose you have a second urn (Urn Two) with 100 balls of red or black color, but the proportion of red and black balls is not known. It may be in any proportion, from all being red to all being black. Einhorn and Hogarth (1986) call the situation of Urn One a situation of complete knowledge and the situation of Urn Two a situation of complete ambiguity.

Under the probability framework, one often assumes uniform priors in situations of ambiguity such as Urn Two. Thus in either situation, using the probability framework, one would assign 0.5 that a drawn ball would be red or black. This feature of the probability framework of not being able to distinguish between the two urns has led Einhorn and Hogarth (1986, p. S228) to a paradoxical conclusion based on observed behaviors of subjects:

> ... either urn 2 has complementary probabilities that sum to more than one, or urn 1 has complementary probabilities that sum to less than one. As we will show, the nonadditivity of complementary probabilities is central to judgments under ambiguity.

Srivastava (1997a) has shown that *no such super- or sub-additivity is needed to explain the decision maker's behavior* if belief functions are used to treat ambiguity.

Thus the main purposes of this chapter are to introduce belief functions and then to consider their application to decision-making in an auditing task. We first introduce belief functions and discuss how belief functions help to overcome certain problems inherent in the use of probabilities.

## 2. Introduction To Belief Functions

The belief-function formalism has its origin in the seventeenth century work of George Hooper and James Bernoulli (Shafer 1976, Gabbay and Smets 1998, Shafer and Srivastava 1990, Smets 1998, 1990a, 1990b, and Yager et. al 1994). It is based on the mathematical theory of probability and it reduces to the Bayesian formalism under a special condition, as discussed later. There are three basic functions that are important to understand the use of belief functions in a decision-making process: *basic belief mass functions* or m-values, *belief functions*, and *plausibility functions*.

**Basic Belief Mass Function (m-values)**

The *basic belief mass*[3] function (or m-values) is similar to the probability function. Consider an example to illustrate the properties of this function. Consider a decision problem with *n* possible elements or states of nature forming a mutually exclusive and exhaustive set represented by $\{a_1, a_2, a_3, \ldots a_n\}$. Call this set a *frame* and represent it by the symbol Θ. Under the probability framework, probabilities are assigned to each state of nature and these probabilities must add to one. More specifically, the probability mass assigned to each state of nature, $a_i$, is $P(a_i) \geq 0$ where i = 1, 2, … n and the sum of all these probabilities is equal to one, i.e., $\sum_{i=1}^{n} P(a_i) = 1$.

Under the belief function framework, basic belief masses or m-values are assigned not only to each state of nature but also to all possible combinations of these states of nature. For example, m-values are assigned to all the single elements, to all the subsets consisting two elements, three elements, and so on, and to the entire frame Θ. Similar to probabilities, these m-values add to one, i.e., $\sum_{A \subseteq \Theta} m(A) = 1$, where A represents all the subsets of the frame. We will consider Shafer's belief-function framework where the basic belief mass assigned to the empty set is zero by definition[4].

Consider an auditing example to help us understand the basic concepts of m-values. Suppose an auditor has performed certain ratio and trend analyses pertinent to the stated accounts receivable balance and has decided that the analyses provide a positive but low level of belief, say 0.2 on a scale of 0 - 1, that the account balance is fairly stated. If the state that the accounts receivables balance is fairly stated is represented by 'a' and the state that the account balance is materially misstated by '~a' then, based on the evidence obtained, we have the following *basic belief masses*, i.e., m-values: m(a) = 0.2, m(~a) = 0, and m({a, ~a}) = 0.8, and the sum is one.

In an audit context, one can interpret these m-values as the level of support directly obtained from the evidence for the argument of the m-value function. For the above example, we have direct ratio and trend analysis evidence that the account is fairly stated with 0.2 level of belief, no belief that the account is materially misstated, and 0.8 belief still uncommitted. This 0.8 is assigned to the

[3] Shafer calls this function the *basic probability assignment* function.
[4] Under Transferable Belief Functions of Smets (1990a, 1998), one can assign a non-zero mass to the empty set.

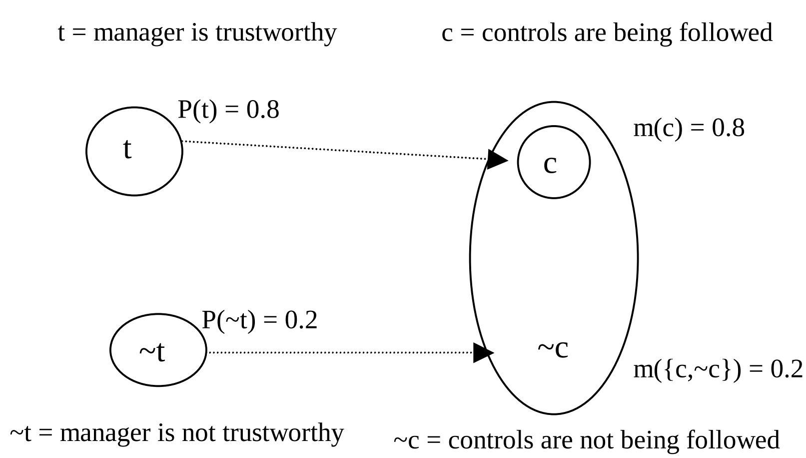
entire frame Θ = {a, ~a}. Note that in this example, the evidence is "positive" in that it supports the hypothesis that the accounts receivable is fairly stated.

A negative piece of evidence is represented by a finite (non-zero) belief mass assigned to '~a' that the account is materially misstated. Suppose that the ratio and trend analyses signal that the account balance may be materially misstated but with a low level of support, say 0.1, for ~a and no evidence that the account balance is fairly stated. This situation can be written as: m(a) = 0, m(~a) = 0.1, and m({a, ~a}) = 0.9.

Mixed audit evidence can be expressed by assigning some basic belief mass to 'a', some to '~a', and some to the entire frame Θ = {a, ~a}.  An example of mixed evidence is where part of the support, say 0.2, is for 'a', 0.1 for '~a', and the remaining, 0.7 is assigned to {a, ~a}. In terms of m-values, one can express this evidence as: m(a) = 0.2, m(~a) = 0.1, and m({a, ~a}) = 0.7. This kind of evidence is not easy to model under the probability framework.

There are two ways one can assess these basic belief masses or m-values. The first is through the decision maker's subjective judgment as illustrated above. This judgment could be based on experience or on other knowledge. The second approach is based on an assumed *compatibility relationship* between two frames (see Figure 1).

**Figure 1:** Assumed Compatibility Relationships Between Two Frames, {t, ~t} and {c, ~c} (Srivastava and Mock 2000. p 230).



**Belief Function**

Belief on a set of elements, say, A of a frame Θ is defined as the total belief on A. This represents the sum of all the basic belief masses assigned to the elements contained in A plus the basic belief mass assigned to A. Mathematically this can be written as: $\text{Bel}(A)=\sum_{B\subseteq A} m(B)$ , where B is any subset of A.

Consider the example discussed earlier where the auditor has performed analytical procedures in testing the accounts receivable balance. Based on the audit findings, assume that the auditor concludes that the evidence provides a low level of support, say 0.2, that accounts receivable balance is fairly stated, no support for the account to be materially misstated, with 0.8 level of belief uncommitted. In terms of basic belief masses, one can write the auditor's judgment as: m(a) = 0.2, m(~a) = 0, and m({a, ~a}) = 0.8.

Using the definition of belief functions as described above, the belief that the accounts receivable is fairly stated is 0.2, belief that the account is materially misstated is 0, i.e., Bel(a) = 0.2, and Bel(~a) = 0. Also, we have a belief of one in the entire frame, i.e., Bel({a, ~a}) = 1.0. This value is obtained as follows. According to the definition, the belief in a set A, say A = {a, ~a}, is the sum of the basic belief masses on all the subsets contained in A, and the belief mass on the entire set A. In our example, this definition implies that the belief on {a, ~a}, Bel({a,~a}), is the sum of the basic belief masses on 'a', '~a', and {a, ~a}, i.e., the sum of m(a) = 0.2, m(~a) = 0, and m({a, ~a}) = 0.8, which is one in the present example.

**Plausibility Function**

The plausibility of an element or a set of elements, say A, of a frame, Θ, is defined to be the maximum possible belief that could be assigned to A if all future evidence were in support of A. Mathematically one can write plausibility as Pl(A) = $\sum_{A\cap B\neq\varnothing} m(B)$. Also, one can show that the plausibility function is related to the belief function through the following relationship: Pl(A) = 1- Bel(~A).

Consider the above example of analytical procedures with the basic belief masses: m(a) = 0.2, m(~a) = 0, and m({a, ~a}) = 0.8. This evidence provides a belief of 0.2 that the account is fairly presented. However, 0.8 level of belief which is assigned to the entire frame, {a, ~a}, is in principle uncommitted. In the best possible scenario, if new pieces of evidence provided only positive support, then all of this uncommitted belief could be assigned to 'a'. This provides the following value for plausibility of 'a': Pl(a) = 0.2 + 0.8 = 1.0. Similarly, in the other extreme case, if new pieces of evidence provided support to only '~a' then the uncommitted belief of 0.8 could be assigned to '~a' yielding a 0.8 level of plausibility for '~a', i.e., Pl(~a) = 0.8.

**Ambiguity Function**

In belief functions, the ambiguity in a state A is defined as the difference between the plausibility of A and the belief in A, i.e.,

Ambiguity in A = Pl(A) – Bel(A)

In the above example, the belief that the account is fairly stated in 0.2 and its plausibility is 1.0. Therefore, the ambiguity in 'a' that the account is fairly presented is 0.8 (Ambiguity in 'a' = Pl(a) – Bel(a) = 1.0 – 0.2 = 0.8). Similarly, the ambiguity in ~a is 0.8.

Consider also the example of two urns discussed earlier. Urn Two (Ambiguous) is the urn with 100 balls of black and red color with no knowledge of the proportion of red and black balls. Urn One is the urn with complete knowledge that it contains 100 balls of 50 red and 50 black colors. The belief mass that a red ball is picked from Urn Two is zero and similarly a belief mass that a black ball is picked is zero, i.e., $m_2$(red ball) = 0, and $m_2$(black ball) = 0 where the subscript represents the urn number. These values suggest that we have no direct evidence whether the ball picked will be of red or black color. The plausibility of picking a red ball would be one and similarly the plausibility of picking a black would be one. Thus, the ambiguity that the ball would be of red color is one (completely ambiguous) and so is the ambiguity that the ball would be of black color.

However, in the case of Urn One, the belief masses are: $m_1$(red ball) = 0.5, and $m_1$(black ball) = 0.5, and the corresponding beliefs and plausibilities are: Bel(red ball) = Bel(black ball) = 0.5, and Pl(red ball) = Pl(black ball) = 0.5. In this case the ambiguity that a red ball is picked is zero and the ambiguity that a black ball is picked is also zero. Under the belief-function framework we see that the two urns are represented very differently.

## 3. Illustration of the Use of Belief Functions in Auditing

In business studies, research that considers uncertainly is rarely useful unless it can be applied to specific tasks. In this section we discuss two audit applications. The applications deal with the planning and evaluation of an audit as discussed by Srivastava and Shafer (1992) and Srivastava et. al (1996).

The audit risk model of SAS 47 (AICPA 1983) has been proposed to be a framework that can be used for planning an audit. The audit risk model introduced in SAS 47 is AR = IR×CR×DR, where AR is the risk that the auditor gives an opinion that the financial statements are fairly stated, but the financial statements may contain material misstatements. IR, CR, and DR are respectively inherent risk, control risk and detection risk. SAS 47 suggests that if the auditor does not want to depend on the environmental factors, then he should set IR to one and plan the audit based on the assessed control risk and planned detection risk. Under the probability framework, IR = 1 implies that the auditor is assuming that there are material errors in the financial statement for sure, but that may not be what the auditor's thinking when he sets IR = 1. Under belief functions, setting IR = 1 during initial audit planning implies that plausibility of material errors present in the financial statements is one, however, there is no evidence in support or against the financial statements being fairly stated.

The audit risk model has been criticized on a number of grounds, including Srivastava and Shafer (1992) who argue that the risks in the model really imply plausibility of material misstatements instead of probability of material misstatements. They argue that the plausibility interpretation of the risks in the audit risk model makes intuitive sense. Our first example of the use of belief functions in auditing reviews this idea.

The external auditor plans the audit to be able to reach an opinion considering the possibility of existence of material errors in the financial statements. If no material misstatements are found and the evidence supports the opinion that the financial statements are fairly stated with a high level of confidence, say 0.95, then the auditor issues a *clean opinion*[5]. Given the earlier definition of plausibility, this implies that the plausibility of material misstatement is 0.05. This plausibility can be interpreted, as the maximum possible risk the auditor is willing to take that the financial statements might be materially misstated without giving a *qualified opinion*[6]. Note that this interpretation does not imply that there is any evidence in support of material misstatements.

The audit process is a process of collecting, evaluating, and aggregating evidence. Srivastava and Shafer (1992) have shown analytically how one can combine positive items of evidence accumulated with respect to various accounts within the balance sheet. Srivastava et. al (1996, see also Srivastava 1995a, 1995b) have

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[5] The auditor can issue a number of possible opinions including a *clean opinion* meaning that the financial statements do not contain material misstatements.

[6] The auditor gives a *qualified opinion* when the evidence suggests that there is a material misstatement in one of the accounts of the financial statements but this misstatement is not so severe that it impacts the entire financial statements. In such a case the auditor would give an *adverse opinion*.

discussed the aggregation of evidence in a complex network of variables, where the variables are the balance sheet, the balance sheet accounts, and the audit objectives or the management assertions concerning the accounts. A network structure arises when one piece of evidence supports more than one audit objective or more than one account. For example, confirmations of accounts receivable provide evidence relevant to both the existence and valuation objectives of the account. We will demonstrate below through a simple example how two items of positive evidence can be combined in the belief-function framework.

Suppose that the auditor assesses a low level of belief, say 0.1 on a scale of 0–1, that the financial statements are fairly stated based on evidence concerning environmental factors, such as the economic conditions under which the client operates, the administrative style of the CEO, and the CEO's integrity. Assume there is no reason to believe that the financial statements are materially misstated given current economic conditions and a trustworthy CEO with integrity. Assume also that these inherent factors (IF) yield the following m-values: $m_{IF}(f) = 0.1$, $m_{IF}(\sim f) = 0$, and $m_{IF}(\{f,\sim f\}) = 0.9$, where 'f = financial statements are fairly stated', and '~f = financial statements are not fairly stated'.

Suppose the auditor collects additional evidence by performing analytical procedures, such as comparing this year's financial statement balances with last year's balances and computing various ratios. Assume that the auditor again assesses a low, but positive, level of support, say 0.2 on a scale of 0–1, that the financial statements are fairly stated based on this evidence. Assume also that the analytical procedures provide no reasons to believe that there is material misstatement in the financial statements. This judgment about analytical procedures (AP) may be expressed in terms of the following basic belief masses or m-values as: $m_{AP}(f) = 0.2$, $m_{AP}(\sim f) = 0$, and $m_{AP}(\{f, \sim f\}) = 0.8$.

We have the following beliefs for 'f' and '~f' from the first piece of evidence, the inherent factors: $Bel_{IF}(f) = 0.1$, $Bel_{IF}(\sim f) = 0$. The corresponding plausibilities are: $Pl_{IF}(f) = 1.0$, and $Pl_{IF}(\sim f) = 0.9$. The plausibility of material misstatement based on the inherent factors is 0.9, although there is no belief that the financial statements are materially misstated.

Srivastava and Shafer (1992) showed analytically that the plausibility of material misstatements in the financial statements based on inherent factors is equivalent to inherent risk (IR). Thus, in the current example IR = 0.9. Conceptually, this is equivalent to saying that the auditor has 0.1 level of support from environmental factors that the financial statements are fairly stated and 0.9 level of maximum possible support that the financial statements could be materially misstated, even though there is no evidence that they are materially misstated.

Similarly, the beliefs and plausibilities for ‘f’ and ‘~f’ based on the analytical procedures are: $Bel_{AP}(f) = 0.2$, $Bel_{AP}(\sim f) = 0$, $Pl_{AP}(f) = 1.0$, and $Pl_{AP}(\sim f) = 0.8$. This means that the auditor has direct evidence from analytical procedures that the financial statements are fairly stated with 0.2 degree of belief, no evidence and thus zero belief for material misstatement, and 0.8 level of plausibility that material misstatement could exist (even though there is no direct evidence of misstatement). Similar to the prior case, a plausibility of 0.8 that the financial statements are materially misstated represents the risk associated to analytical procedures as considered in SAS 47.

One of the difficulties in actual audits is the combination of various items of evidence, such as those discussed above. Using belief functions, one would use Dempster’s rule of combination. Dempster’s rule is similar to Bayes’ rule. In fact, Dempster’s rule reduces to Bayes’ rule under the condition where the basic belief masses, i.e., m-values, are assigned to only singletons of the frame (Shafer 1976).

In the above example, we have two items of evidence with the following m-values:

$$m_{IF}(f) = 0.1,\ m_{IF}(\sim f) = 0,\ m_{IF}(\{f,\sim f\}) = 0.9,$$

$$m_{AP}(f) = 0.2,\ m_{AP}(\sim f) = 0,\ m_{AP}(\{f, \sim f\}) = 0.8.$$

According to Dempster’s rule, we cross multiply all the m-values and assign the resulting masses to the intersection of the arguments. In situations where the intersection is empty with a finite belief mass (“conflict situations”), we need to renormalize the remaining masses (Shafer and Srivastava 1990). In the above case, we do not have any conflict. Thus the following are the combined m-values as a result of cross multiplication of the two sets of m-values:

$$m_T(f) = m_{IF}(f)m_{AP}(f) + m_{IF}(f)m_{AP}(\{f,\sim f\}) + m_{IF}(\{f,\sim f\})m_{AP}(f)$$

$$= 0.1x0.2 + 0.1x0.8 + 0.9x0.2 = 0.28,$$

$$m_T(\sim f) = m_{IF}(\sim f)m_{AP}(\sim f) + m_{IF}(\sim f)m_{AP}(\{f,\sim f\}) + m_{IF}(\{f,\sim f\})m_{AP}(\sim f)$$

$$= 0 + 0x0.8 + 0.9x0 = 0,$$

$$m_T(\{f,\sim f\}) = m_{IF}(\{f,\sim f\})m_{AP}(\{f,\sim f\}) = 0.9x0.8 = 0.72.$$

The beliefs and plausibilities are: $Bel_T(f) = 0.28$, $Bel_T(\sim f) = 0$, $Pl_T(f) = 1.0$, and $Pl_T(\sim f) = 0.72$. Thus, based on the inherent factors and analytical procedures the auditor has 0.28 degree of belief that the financial statements are fairly presented. But there is still 0.72 degree of plausibility that material misstatements may be present.

Usually, the auditor will need to collect more evidence to increase the belief that the financial statements are fairly stated to a threshold level, say 0.95, or reduce the plausibility of material misstatement to 0.05, i.e., reduce the overall audit risk to 0.05 in order to give a clean opinion[7].

The analytical model proposed by Srivastava and Shafer (1992) for combining various items of evidence at different levels of the balance sheet is for planning purposes. One can also use their model for evaluation purposes, provided all the pieces of evidence gathered are positive in nature. This is not the case in general. In fact, in general, the auditor may encounter some positive evidence, some negative evidence, and some mixed items of evidence. In order to combine evidence from various sources at different level of the financial statement, Srivastava et. al (1996) used computer software called "Auditor's Assistant" developed by Shafer et. al (1988). This software allows the user to draw the evidential network for audit planning and evaluation decisions and also calculates the beliefs for the specified network.

Srivastava et al (1996, see also, Wright et. al 1998) applied the belief function approach for planning and evaluation of an audit of a healthcare unit. They showed how one could develop a network of variables with accounts and audit objectives and connect various items of evidence to pertinent variables (i.e., audit objectives of an account, the accounts on the balance sheet, and the balance sheet as whole).

The inputs to such a network are the auditor's judgments about the planned (or achieved) level belief to be obtained (or obtained) from various items of evidence. The Auditor's Assistant program aggregates all the evidence and provides the overall beliefs. If the overall belief obtained at the variable of interest, say the financial statement, is below a specified threshold level, say 0.95, the auditor would (1) collect more evidence to increase the overall belief to the threshold level and issue an unqualified opinion, (2) either issue a qualified opinion or an adverse opinion if the overall belief that the financial statements are materially misstated is more than a threshold value, 0.05 in the present example, or (3) issue a disclaimer if the auditor is not able to collect further evidence. If the overall belief that the financial statements are fairly stated is equal to or more than the threshold value then the auditor would not collect any further evidence and would issue an unqualified opinion.

[7] See Shafer and Srivastava (1990) for an example of the use of Dempster's rule with conflicting pieces of evidence.

### Other Studies

There are a number of other studies the interested reader can consult to see further illustrations of the use of belief functions in behavioral auditing and accounting research. Many of these are listed in our references and are referenced in the following chapters. For example, Krishnamoorty, Mock and Washington (1999) utilized the belief-function framework in a cascaded inference context and Srivastava and Mock (1999) used an evidential reasoning approach based on the belief-function framework for WebTrust assurance services (AICPA 1997a, 1997b). They also applied a decision theoretical approach to estimate a minimum acceptable audit fee based on the costs associated with various risks.

Srivastava (1996) has used belief functions in value judgments and has analyzed the impact of framing of evidence on decision-making. As discussed earlier, Srivastava (1997a) has also used belief functions to model ambiguity and demonstrated its benefits in decision making behavior.

## 4. Conclusion

Ambiguity and uncertainty are common in business settings. However, probability theory does not have a natural, logical way of dealing with ambiguity. One common way of dealing with ambiguity in probability is to assign uniform priors. This leads to some illogical implications, as illustrated in an audit context. An important advantage of a formalism based on belief functions is that it does explicitly and logically deal with ambiguity. As shown, this approach can be applied to auditing and leads to more logical interpretations of audit risk and evidence. Belief functions tend to have the following advantages over the probability framework in these contexts

1. If the audit risk model (see SAS 47) is viewed as a plausibility model (Srivastava and Shafer 1992) with a belief function interpretation of its components, this interpretation makes more intuitive sense than alternative frameworks.

2. There is some evidence that a judgment about the basic assessment of the strength of evidence using a belief function framework is more intuitive to decision makers (Harrison 1999, and Curly and Golden 1994).

3. Representation of positive, negative, and mixed items of evidence is more convenient. For example, positive evidence in support of an audit objective ‘o’ can be expressed in terms of a non-zero m-value, say 0.2, for ‘o’ and zero

value for '~o', i.e., m(o) = 0.2, m(~o) = 0, m({o, ~o}) = 0.8. A negative item of evidence would be represented by a non-zero m-value for '~o', say 0.1, a zero value for 'o', i.e., m(o) = 0, m(~o) = 0.1, and ({o, ~o}) = 0.9. A mixed item of evidence can be expressed by assigning a non-zero m-value for both 'o' and '~o', i.e., m(o) ≠ 0, and m(~o) ≠ 0. As an illustration, we can express a mixed item of evidence as m(o) = 0.3, m(~o) = 0.1, and m({o, ~o}) = 0.6.

4. Representation of different levels of assurance coming from the same item of evidence for two different audit objectives is expressed more conveniently. For example, in the case of a confirmation test, assume that the auditor finds that all the respondents have said that they owe money to the company, but some of them have made comments that their account balance is overstated. Based on this information the auditor's judgment is that a high level of support, say 0.8, is obtained for 'existence' objective (e) and a medium level of support, say 0.6, is obtained for 'valuation' objective (v). This can be expressed in terms of m-values as: m(e) = 0.8, m(~e) = 0, m{(e, ~e}) = 0.2, and m(v) = 0.6, m(~v) = 0, m({v, ~v}) = 0.4.

Belief functions also have potential use and research implications for business problems in general as is illustrated in the following chapters of this book. As noted, uncertainty and ambiguity exist and should be formally dealt with in many business contexts.

A number of research opportunities arise with respect to belief functions, especially in behavioral research. For example, it would be interesting to see whether decision makers tend to think of uncertainty as depicted in the belief-function framework. Also, given that some research has shown that judgments about the level of belief in support of a given assertion is more natural when beliefs function are used, the basic measurement (calibration) issue of the level belief obtained from various types of audit evidence is an important area for future research. A related issue is whether decision makers are able to combine items of evidence according to Dempster's rule of combination. Lastly, there are a number of issues concerning the efficiency and effectiveness of the belief-function framework as compared to other approaches of dealing with uncertainty and ambiguity[8].

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[8] See, for example, Chapter 2.2: "The Descriptive Ability of Models of Audit Risk" by Monroe and Ng.

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