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Evaluating Mergers and Acquisitions: A Belief Function Approach

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Abstract

Studies indicate that mergers and acquisitions are characterized by a high failure rate, often attributed to an inability on the part of the acquiring firm management to effectively evaluate potential acquisition candidates. This is not surprising given the considerable uncertainties surrounding both the relationships among factors in the evaluation process and also in the assessment of evidence. This paper develops a conceptual framework for acquisition and merger decisions using evidential reasoning approach under the belief function framework. It seeks to illustrate how expert knowledge of relevant factors can be mapped and how an evidential network can be used by decision makers to incorporate uncertainties in the evidence. Also highlighted is the fact that the nature and extent of evidence that needs to be collected depends on the rules assumed to govern relationships among input variables. Implications for theory and practice are discussed.

1. Introduction

Mergers and acquisitions (M&A) represent some of the most critical strategic decisions facing top management in corporations. They are often defining moments in a company's development with significant long-term performance implications. The impact of mergers and acquisitions on the U.S. business landscape is evident when one considers the fact that in 1999 the total dollar value of acquisitions exceeded $1 trillion. Paradoxically, however, the overall track record for corporate mergers and acquisitions has not been a resounding success. Available empirical evidence suggests that acquisitions are characterized by a high failure rate; with nearly 50% of all acquisitions being categorized as failures (e.g., Lajoux & Weston [1]; Ravenscraft & Scherer [2]). In addition, a meta-analytic synthesis of the empirical literature on the wealth effects in mergers and acquisitions (Datta, Narayanan & Pinches [3]) identified that acquisitions, on average, fail to create value for the shareholders of acquiring firms. Moreover, research by Datta and Puia [4] suggests that the same is also true in the case of cross-border (international) acquisitions undertaken by U.S. based firms.

While various reasons have been advanced to explain the high failure rate among acquisitions (e.g., major dislocations in given industry sectors or broader environmental shifts), much of it can be attributed to the failure on the part of firm management to effectively evaluate acquisition opportunities. This was borne out in a study of post-merger performance involving 150 large mergers between 1990-95 by Mercer Management Consulting and *Business Week.* They found that over 50% of the mergers eroded shareholder returns, with the primary cause being inadequate due diligence by the buyer. A failure to adequately evaluate the target firm often led to lack of a compelling strategy and overly optimistic expectations of possible synergies (Lajoux and Weston [1]).

A number of evaluation frameworks have been suggested in the literature (e.g., Achtmeyer & Daniell [5]; Rappaport [6]; Reilly [7]). However, current approaches have two key limitations. First, they are often driven by *rules of thumb* in the integration of evidence. As such, they do not account for the uncertainties in the evidence used in the evaluation and decision making process. However, in reality, considerable uncertainty exists both in terms of the relationships among the factors in the evaluation model, and in the assessment of factor levels. Second, current approaches do not account for interdependencies among various factors and items of evidence. It is, however, important that decision models incorporate such interdependencies for them to be meaningful and realistic.

The primary objective of this paper is to develop a conceptual framework for acquisition and merger decisions using evidential reasoning approach under the belief function framework. Belief functions are used to represent uncertainties in the variables involved in the decision process. Following Curley and Golden [8] and Harrison [9], we believe that uncertainties associated with evidence related to factors in a decision process are more realistically expressed in terms of belief functions than in terms of probabilities. In fact, uncertainties are so prevalent in all human decisions that the managing of such uncertainties has been the focal point of considerable research in AI (Artificial Intelligence) (see, e.g., Cohen [10]; Duda et al. [11]; Lauritzen & Spiegelhalter [12]; Pearl [13]; Shenoy & Shafer [14]). The outcome of such research has made possible not only the propagation of uncertainties in complex networks through local computations but has also led to the development of several computer systems that automate propagation of uncertainties in networks (e.g., Andersen et al. [15]; Saffiotti & Umkehrer [16];

Shafer, Shenoy & Srivastava [17]; Zarley et al. [18]). Such programs are helping researchers develop "real world" applications. For example, Srivastava, Dutta and Johns [19] have analyzed the use of "Auditor's Assistant" (Shafer et al [17]) for planning and evaluation of an audit in the healthcare industry. However, applications in the context of key strategic organizational decisions have been lacking.

2. Key Factors in the Evaluation of Mergers and Acquisitions

Firms engage in acquisitions for a variety of reasons. These include quicker market entry, avoiding costs and risks associated with new product development and the acquisition of critical resources and competencies (Datta et al [3]). However, from the perspective of an acquiring firm, the primary overall objective in an acquisition is the creation of economic value (Rappaport [20]). As such, given that acquisitions typically have a significant impact on the overall profitability and financial health of a firm, they deserve the same thoughtful and thorough evaluation and planning as a major new project or the building of a new plant. Yet, many otherwise well-run organizations fail to devote the necessary time and effort required in carefully analyzing acquisition opportunities.

Effective and accurate evaluation of acquisitions is critical to the valuation of potential targets. If an organization fails to diligently and thoroughly analyze a target firm, the consequences can be rather damaging. The firm might end up overpaying for the acquisition. Roll [21] attributes overpayment to managerial hubris and inadequate rational evaluation of targets in acquisitions.

The criteria for evaluating acquisitions entail creating a benchmark against which to evaluate candidates. This includes an assessment of the (1) attractiveness of the industry or industry segment, (2) competitive strengths of the target firm, (3) potential synergistic benefits, and (4) extent of organizational fit. While the first two define the attractiveness of the target as an independent or stand-alone entity, potential synergistic benefits and integration costs relate to the benefits and costs associated with combining the acquiring and acquired firms. These are discussed in greater detail in the following paragraphs.

2.1. Economic Desirability of a Target Firm as an Independent Entity

The extensive literature in industrial organization and strategic management (e.g., Barney [22]; Markides & Williamson [23]; Porter [24]; Scherer [25]) suggests that economic attractiveness of a firm in the market place is a function of (1) the structural attractiveness of the industry, and (2) the competitive strengths of the firm within the industry. Preacquisition screening aimed at increasing the acquiring firm's knowledge of potential takeover targets should incorporate analysis along both of these dimensions.

Industry attractiveness. Research in the area of strategic management indicates that the industry environment has a direct impact on the firm's efforts to achieve strategic competitiveness and earn above-average returns. According to research conducted by Robert Spitalnic and Acquisition Horizons (Lisle & Bartlam, [26]), some of the most commonly cited reasons for failed acquisitions have to do with market factors. These include market growth of the target firm industry being lower than expected, industry margins being less than expected, and competition being tougher than expected. In most cases, the preacquisition research and evaluation was either inadequate or inaccurate.

As such, in seeking an acquisition candidate (especially in the context of corporate diversification) the potential attractiveness of the industry structure is of utmost importance. Such attractiveness depends on the economic structure of the industry (Wernerfelt & Montgomery [27]; Porter [24]). Porter's "five forces" model suggests that, in addition to demand growth, industry attractiveness depends on the industry structure. These relate to the extent of threat of new entry, the bargaining power of buyers and suppliers, the extent of rivalry in the industry and, the threat of new substitutes. Thus, an important aspect of analyzing potential target firms would be the analysis of industry forces. Other things being equal, an acquiring firm would certainly desire that the target firm be in an attractive industry. In assessing the same, evidence needs to be gathered on the factors that determine the intensity of the competitive forces and demand growth, followed by an aggregation of the evidence along the different factors. However, problems encountered in the process are twofold: (1) the uncertainty underlying the assessment along the factors that contribute to the forces needs to be accounted for, and (2) the exact nature of the relationship between factors and the forces for the purpose of evidential reasoning is unknown. Simple weighting schemes can be used -- however, such schemes do not allow the users to incorporate such subjective judgments as "I believe that the factors tell me that the industry will be attractive with 80 percent confidence but I am not sure on the remaining 20 percent -- I probably need more information."

Thus, any meaningful process needs to account for the uncertainty not only in available evidence but also in the relationships that define the aggregation process.

Competitive strengths. Along with the attractiveness of its industry, the economic desirability of a target firm depends on its competitive position within the industry. Industry experts suggest the critical importance of conducting competitive intelligence analysis that probes the competitive position of the target firm vis-à-vis it rivals (Lisle & Bartlam, [26]). Competitive intelligence can be an extraordinarily useful tool in evaluating acquisition candidates and can help in uncovering the value of potential target firms along both tangible and intangible dimensions. Research (Hitt, Hoskisson & Ireland, [28]) indicates that acquisitions are likely to be successful when target firms have complementary resources and competencies.

The competitive strengths and capabilities are typically assessed by comparing the firm to its major competitors in the industry along key industry success factors (Barney [22]; Porter [29]). When using the value chain analysis mangers seek to study a firm's resources and capabilities in relationship to key activities in the value chain. However, rating a firm's capabilities along value chain-activities poses unique challenges -- it requires judgment and is associated with a high level of uncertainty. First, there is no obviously correct model or rule available to accomplish the process. Second, the data that can be accessed for such evaluation can, at times, be unreliable and/or incomplete. Moreover, there is no mechanism to account for the uncertainty in both the assessment and also in the relationship between the factors and the overall competitive position. In addition, there are likely to be situations where the items of evidence are conflicting. Such conflicting items of evidence obviously need to be properly aggregated – something that can be effectively accomplished using the belief-function framework.

2.2. Combination Benefits

The attractiveness of industry structure, along with the competitive position determines the desirability of a particular firm as a stand-alone entity. However, the economic justification behind an acquisition often rests on synergistic benefits or economies that the acquisition may provide.

Potential synergistic benefits. Simply stated, synergy represents an acquiring firm's potential capacity to use its capabilities to improve the performance of the target firms or vice versa. As argued by Porter [30], an acquisition must pass the "better off" test – i.e., there should be opportunities for value creation through the exploitation of synergies. Literature in industrial organization and strategic management suggest that synergies associated with acquisitions emanate from three types of economies: economies of scale, economies of scope, and pecuniary or market power based economies.

An important component of synergistic benefits in acquisitions (particularly in the acquisition of a firm in a related industry) are *scale economies* (Bielinski [31]; Datta [32]; Jensen & Ruback [33]; Salter & Weinhold [34]; Singh & Montogomery [35]). In addition, acquisitions, especially related acquisitions, can result in *economies of scope* (e.g., Singh & Montgomery [35]). Benefits of economies of scope are achieved when the cost of joint production of two goods by a multiproduct firm is less than the combined costs of production of these goods by two single product firms. Economies of scope may also arise from the reuse of an input, such as the sharing of know-how or other intangible assets. Intangible resources such as brands, corporate reputation, and technology offer economies of scope primarily due to the ability to transfer them from one business to another at low marginal cost. Finally, market power or *pecuniary economies* arise in acquisitions from the ability of the combined firms to dictate prices (Scherer [25]). The gains here are from a firm's ability to extract excess profits through the exercise of increased market power over buyers and suppliers (from an acquisition).

The process of evaluating synergistic benefits in a potential acquisition involves the identification of the areas or factors that will contribute to each of the three types of synergies followed by the determination of likely benefits along each factor. Again, such an assessment involves considerable uncertainty -- not just in the determination of factors along which synergistic benefits might accrue, but also in the assessment of the synergistic benefits itself (Bielinski [31]; Sirower [36]). Sirower [36] for example, details several acquisitions where synergies were grossly overestimated by acquiring firm management. One such transaction was the acquisition of NCR by AT&T in 1991 at a 125% premium. Unfortunately, failure at systems integration resulted in failure to generate needed synergies (to justify the steep premium), contributing to the acquisition’s downfall.

Organizational fit. In addition to the above, acquisition evaluation must include the consideration of significant (and, sometimes, hidden) costs associated with the post-acquisition implementation process. The assimilation and integration costs (arising out of organizational misfit), if overlooked, might even negate any available synergistic benefits (Datta [37]; Datta & Grant [38]; Jemison & Sitkin [39]; Slowinski [40]). Implementation impediments (and, hence, costs) are often associated with lack of *organizational fit* or compatibility. Existing incompatibility can be along organizational systems, prevalent organizational cultures, organizational structures, management styles etc. (Datta & Grant [38]; Datta [37]; Marks [41]). As researchers like Buono, Bowditch and Lewis [42], and Marks [41] have argued, such differences tend to destabilize the combined organization, contributing to enhanced probability of conflicts. Consequently, instead of anticipated benefits, the outcome is often a sharp decline in post-combination performance. The acquisition of MedPartners by PhyCor in 1997 provides an interesting illustration. While many analysts viewed it as a synergistic acquisition, some questioned whether the acquisition can be effectively implemented given the significant differences in their organizational cultures and operating styles (PhyCor had a slow and deliberative style of operating while MedPartners was much more aggressive). Market perception and investor concerns that integration challenges would destroy value resulted in PhyCor’s stock moving lower when the acquisition was announced.

Determining ways to measure the economic impact of organizational fit (or misfit) has been a vexing problem for organizational researchers. No generally accepted procedure exists (Marks [41]) and assessments are, at best, subjective and ad-hoc. Some formal tools have been developed - these include the Merging Cultures Evaluation Index (MCEI) that analyzes several dimensions of corporate culture. However, assessment of organizational misfit is fraught with uncertainty and inexactitude. Also, given the level of uncertainty, and, the sometimes unclear relationship to acquisition performance issues of organizational fit are often excluded from the purview of acquisition analysis. As discussed later, such uncertainties can be addressed using the belief-function framework -- the relationships between the factors that determine the net synergistic benefit will be expressed using subjective judgments about the uncertainties. We believe that this approach (in comparison to the traditional narrative approach) possesses greater objectivity and should, therefore, result in greater consistency.

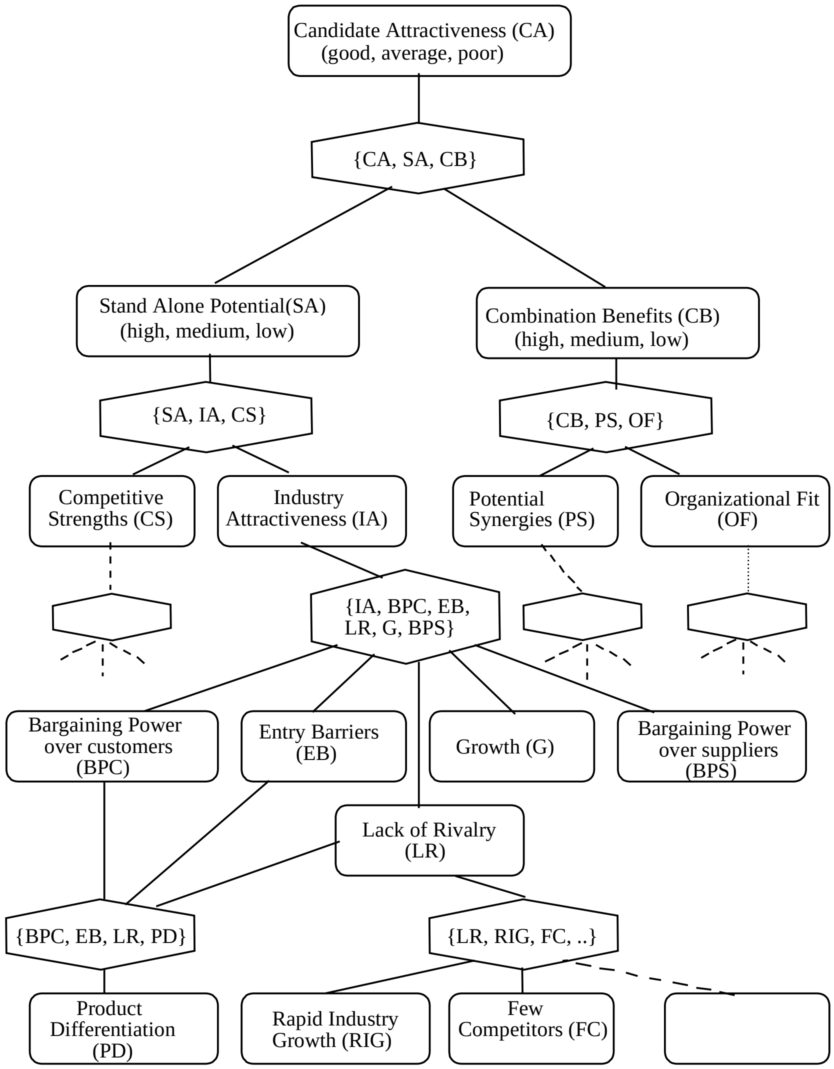
In sum, acquisition evaluation is characterized by the evaluation of a wide array of factors related to the target firm and the acquisition itself. These (and, the interrelationships between them) are partially depicted in the network presented in Figure 1. Moreover, it must be remembered that the decision process associated with the evaluation of acquisition candidates is characterized by (1) uncertainty in the evaluation of the candidates along the factors (the available evidence, especially on some of the key organizational and behavioral factors, is generally accompanied by a high level of uncertainty), (2) interrelationships amongst different factors and amongst various item of evidence, and (3) uncertainty pertaining to relationships between the factors. The best model is always the one that best fits the structure of evidence. At times it is a causal model and at other times it is not.

In this paper we seek to bridge the gap between work on the evaluation of mergers and acquisitions (primarily in the areas of strategic management and financial economics) and work in the area of decision making under uncertainty. A decision problem may involve multiple interrelated variables (or factors) with items of evidence that may bear on one or several variables. In other words, an M&A evaluation problem is likely to involve a network of variables, all with uncertainties associated with them.

Under the evidential reasoning approach, there are three main steps involved in the decision process. The first step is to identify all the relevant variables in the decision process. The second step is to identify the interrelationships among the variables and convert them into relational nodes under the belief-function framework. This step is quite complex because the interrelationships among variables are usually expressed in terms of 'if-then' rules. We need to convert each set of these 'if-then' rules to a relational node for propagation of beliefs in the network[1]. The third step is to build the appropriate network of variables based on the perceived interrelationships among the variables. This network is then used to combine information obtained on different variables in the network to make the final decision whether a particular company is a good candidate for acquisition or merger or not. In Appendix B we briefly discuss how logical relationships can be modeled in the belief function framework.

Figure 1: A Partial Evidential Network for Acquisition Decisions*

[1] An algorithm, based on Srivastava and Cogger [43], developed for this purpose is presented in Appendix A

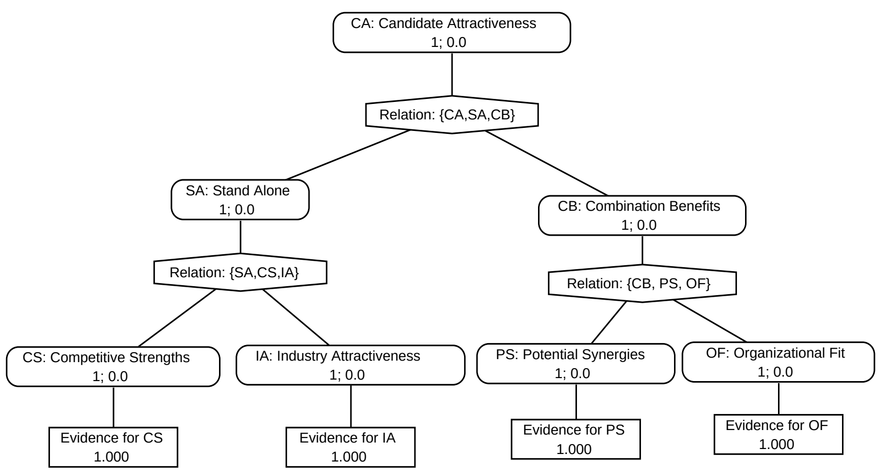


*The rectangular boxes with rounded corners represent variable nodes (factors) and the hexagonal boxes represent relational nodes.

3. Knowledge Representation in Mergers and Acquisitions Through An Evidential Network

In this section we map the expert knowledge of the relevant factors discussed in the previous section along with their interrelationships in terms of an evidential network. As seen in Figure 2, the main factor (node) 'Candidate attractiveness (CA)' depends on (i.e., is connected to) two other factors: 'Stand Alone Potential (SA),' and 'Combination Benefits (CB).' Let us assume that CA takes three possible values: good (g), average (a), or poor (p). Again, for simplicity, let's assume that the remaining variables in the network assume three possible values: high (h), medium (m), or, low (l). For example, "Stand Alone Potential (SA)" is a variable and it takes three values: high (h), medium (m), and low (l).

Figure 2: Evidential Network



The hexagonal boxes in Figure 2 represent relationships between variables. These relationships are usually expressed in terms of 'if-then' rules, which can be determined by interviewing experts in the field. These 'if-then' rules are then converted into relational nodes using the algorithm described in Appendix A for constructing the network. As an illustration of a relational node, let us consider a specific example involving the following variables: 'Candidate Attractiveness (CA)', 'Stand Alone Potential (SA)' and 'Combination Benefits (CB).' Lets also assume that the 'if-then' rules are obtained through an interview with an expert and they are given in Table 1.

Table 1: ‘If-Then’ rules relating the variables ‘Candidate Attractiveness (CA)’, ‘Stand Alone Potential (SA)’ and ‘Combination Benefits (CB).’ h, m, and l, respectively, represent high, medium, and low level of the variable.

‘If’ Condition		‘Then’ condition’s Confidence Level for Candidate Attractiveness (CA)			
SA: Stand Alone Potential	CB: Combination Benefits	Good (g)	Average (a)	Poor (p)	{g, a, p}*
h	h	1.0	0.0	0.0	0.0
h	m	0.7	0.3	0.0	0.0
h	l	0.5	0.5	0.0	0.0
m	h	0.5	0.5	0.0	0.0
m	m	0.2	0.8	0.0	0.0
m	l	0.0	0.6	0.4	0.0
l	h	0.0	0.4	0.6	0.0
l	m	0.0	0.2	0.8	0.0
l	l	0.0	0.0	1.0	0.0

*****{g,a,p} represent the set of all possible values of the variable CA with the values of the corresponding beliefs provided in the last column.

Each row in Table 1 represents a part of the ‘if-then’ rule relating to the two variables, SA and CB with CA. For example, the first row implies that if both ‘Stand Alone Potential (SA)’ and ‘Combination Benefits (CB)’ are high (h) then the variable ‘Candidate Attractiveness (CA)’ would be good (g). In other words, if both SA and CB are high then the acquisition candidate is a good candidate. However, from the second row, it appears that if SA is high (h) and CB is medium (m) then Candidate Attractiveness (CA) is 'good (g)' with 0.7 level of assurance (or belief) and 'average (a)' with 0.3 level assurance (or belief). Under this condition, there is no belief that 'CA' is 'poor (p)'. Using the algorithm described in Appendix A, we obtain the following belief function representation of the rules given in Table 1:

m({ghh, ghm, ghl, gmh, amm, aml, plh, plm, pll}) = 0.5,

m({ghh, ahm, ahl, amh, amm, pml, alh, plm, pll}) = 0.3,

m({ghh, ghm, ahl, amh, gmm, aml, alh, alm, pll}) = 0.1,

m({ghh, ghm, ahl, amh, gmm, pml, plh, alm, pll}) = 0.1,

where the symbol ***m*** stands for the ***m***-value. Each element in the argument of ***m*** represents a set of values for the three variables: ‘Candidate Attractiveness (CA)’, ‘Stand Alone (SA)’ and ‘Combination Benefits (CB).’ For example, an element, say ‘aml’ in the argument means that ‘CA’ is average (a), ‘SA’ is medium (m) and ‘CB’ is low (l).

As depicted in Figure 2, the variable Stand Alone (SA) depends on Competitive Strength (CS) and Industry Attractiveness (IA). The hexagonal box relating these variables represents the ‘if-then’ rule supposedly obtained from an expert. For the present discussion we assume the same rules as depicted in Table 1 for this relationship. From Figure 1, we see that the variable ‘Combination Benefits (CB)’ depends on the two variables: Potential Synergies (PS) and Organizational Fit (OF). Again, for simplicity, the relationship among these variables is assumed to be represented by ‘if-then’ rules as given in Table 1. As discussed in the previous section, these variables further depend on several other factors. For example, Industry Attractiveness (IA) depends on the following five factors: Bargaining power over customers (BPC), Entry barriers (EB), Lack of rivalry (LR), Growth (G), and Bargaining power over suppliers (BPS). All of these factors may further depend on another set of factors. For example, Lack of Rivalry (LR) would depend on “Rapid industry growth (RIG)”, “Few competitors (FC),” “High product differentiation (PD)”, “Low exit barriers (LEB)” and “High switching costs (HSC)”. On the other hand, “Entry barriers (EB)” would depend on “Economies of scale”, “Product differentiation (PD)”, “Capital requirements”, “Switching costs”, “Limited access to distribution channels” and “Government policy”. Figure 2 is only a partial evidential network for the decision process[2]. In the next section we show how the evidential network can help the decision maker to consider uncertainties in the evidence related to the factors relevant to the decision.

4. Sensitivity Analysis

Here we want to analyze the impact of the level of assurance for various possible values (‘high’, ‘medium’, and ‘low’) of the input variables: CS’, ‘IA’, ‘PS’, and ‘OF’, on the main variable ‘Candidate Attractiveness (CA)’.

[2] A complete network is beyond the scope of this paper. The objective of the present article is to show how one can develop an evidential network of relevant variables towards making merger and acquisition decisions.

4.1. Output Belief versus Input Assurance

The solid curve in Figure 3 depicts the overall belief, $Bel_{CA}(g)$, that the candidate attractiveness is good and the dotted curve depicts the overall belief, $Bel_{CA}(\{g,a\})$, that candidate attractiveness (CA) is either good or average. In this illustration, all input variables (CS, IA, PS, and OF) are assumed to be present at a 'high' level with various levels of assurance as shown on the x-axis. All the relational nodes (hexagonal boxes) in Figure 2 are assumed to be 'if-then' rules. The 'if-then' rules relating CA to SA and CB are given in Table 1 along with its belief function representation below the table in the previous section.

For simplicity (and, without the loss of generality), we assume the same structure of the 'if-then' rules relating SA to CS and IA, and CB to PS and OF as considered for relating CA to SA and CB as given in Table 1. As expected, we find the acquisition candidate to be a 'good' candidate with belief one, i.e., $Bel_{CA}(g) = 1$, when all the input variables (CS, IA, PS, and OF) are high with a level of assurance '1' on a scale of 0-1. In terms of belief masses or ***m***-values this assurance can be written as $\boldsymbol{m}_{CS}(h) = \boldsymbol{m}_{IA}(h) = \boldsymbol{m}_{S}(h) = \boldsymbol{m}_{IC}(h) = 1$. However, as the level of assurance for the input variables decreases below '1' the belief in CA that the firm will be a good candidate for acquisition decreases rapidly. For example, for 0.8 level of assurance that the input variables are 'high', the overall belief that the candidate attractiveness is good is only 0.692[3], i.e., $Bel_{CA}(g) = 0.692$, and the belief that CA is either 'good' or 'average' is 0.897, i.e., $Bel_{CA}(\{g, a\}) = 0.897$. In other words, these belief values imply that we have 0.897 level of belief on a scale of 0-1 that the candidate is either a 'good' or an 'average' acquisition candidate but we have only 0.692 level of belief that it is just a 'good' candidate.

Suppose the decision maker accepts the candidate firm for acquisition only when the overall belief that it is good is greater than a threshold value, say 0.8. The above value of 0.692 is low compared to the acceptable threshold value. The decision maker has two options in such a situation. First, he/she can decide not to acquire the firm and stop further collection of evidence relevant to the input variables. Alternatively, he/she can decide to collect further evidence to increase the level of support for a 'high' level of presence for all the input variables provided the cost of collecting evidence is reasonable[4].

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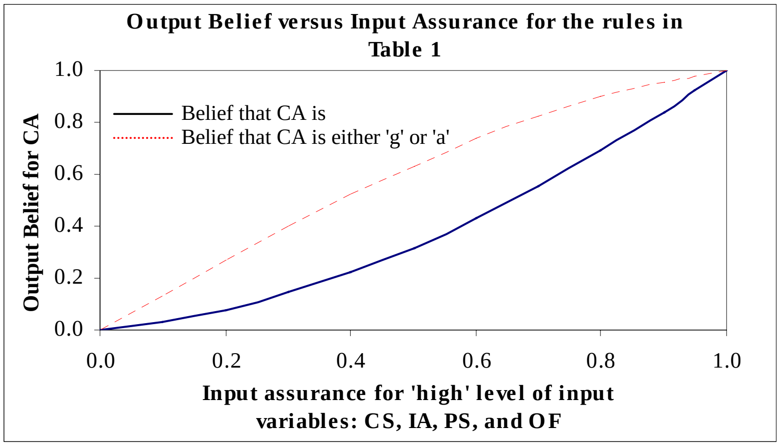
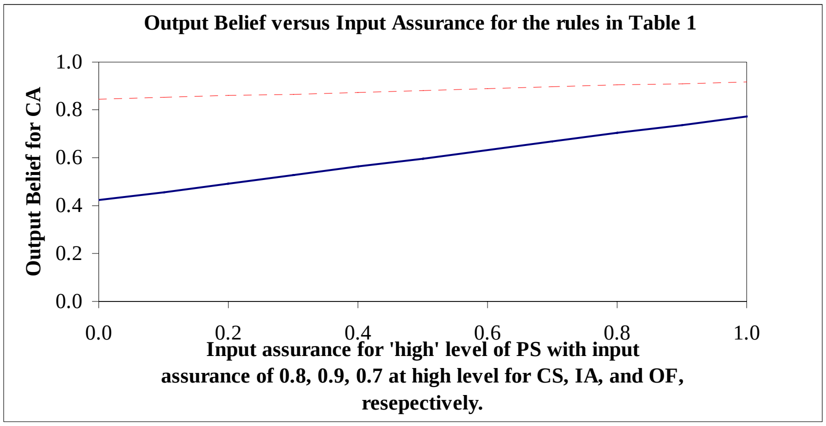
[3] Calculated using the program 'Auditor's Assistant' developed by Shafer, Shenoy, and Srivastava [17].
[4] We are only analyzing the impact of the strength of evidence on the overall belief and ignoring the impact of cost of collecting the evidence. This aspect can easily be incorporated by considering decision making under belief functions (e.g., see Smets [44])

Figure 3



The solid curve in Figure 4 depicts the overall belief that CA is ‘good’ and the dotted curve depicts the overall belief that CA is either ‘good’ or ‘average’. These beliefs are plotted as a function of input assurance for PS being ‘high’ with CS, IA, and OF being high at input assurance of 0.8, 0.9, and 0.7, respectively. The overall belief that CA is ‘good’ is relatively much higher than that depicted in Figure 3. This is due to the fact that three input variables, CS, IA, and OF, already have a high to medium level of support (0.8, 0.9, and 0.7, respectively), for them to be in “high” state. As the input assurance for Potential Synergies (PS) to be ‘high” increases from 0 to 1.0, the overall belief that Candidate Attractiveness (CA) is ‘good’ increases from 0.423 to 0.773, and the belief that CA is either ‘good’ or ‘average’ increases from 0.843 to 0.917. This is based on the assumption that the decision maker encounters a situation where he/she has a medium to high level of support for the notion that three variables, Competitive Strength (CS), Industry Attractiveness (IA), and Organization Fit (OF), are at a ‘high’ level. However, the decision maker may be more uncertain about the Potential Synergies (PS) associated with the merger. A zero level of support that PS is ‘high’ represents a situation where there is no evidence in support of PS being ‘high’. However, as the decision maker searches for more evidence and finds a greater level of support for Potential Synergies to be high, the overall belief that Candidate Attractiveness is ‘good’ also increases. Such an analysis would be useful in decision makers’ allocation of resources for evidence collection as they pertain to individual variables/factors. In the absence of such an analysis, it is difficult to focus efforts at gathering appropriate evidence towards making the final decision on whether the merger should be pursued or not.

Figure 4



4.2. Impact of the Nature of 'If-Then' Rules on the Output Belief

In order to analyze the impact of 'if-then' rules on the output belief that Candidate Attractiveness (CA) is 'good', we consider another set of 'if-then' rules as given in Table 2. This set involves rules that are fuzzier than those presented in Table 1. For example, in Table 1, when SA is 'high' and CB is 'medium' then CA is 'high' with 0.7 level of assurance and 'medium' with 0.3 level of assurance with no ignorance. However, with the information presented in Table 2, it can be observed that, under the above condition, CA is 'high', 'medium' and 'undecided' with 0.6, 0.3 and 0.1 levels of assurance, respectively. As another illustration of fuzziness of the 'if-then' rules in Table 2, consider the situation (row three in the table) where SA is 'high' and CB is 'low'. The 'then' part of the rule suggests that CA is 'good' with 0.4 level of confidence, 'average' with 0.4, and 0.2 level of assurance is still undecided. However, in the previous case (Table 1), if SA is 'high' and CB is 'low' then CA is 'good' or 'medium' with 0.5 level of assurance each with no ignorance.

Table 2: ‘If-Then’ rules relating the variables ‘Candidate Attractiveness (CA)’, ‘Stand Alone Potential (SA)’ and ‘Combination Benefits (CB).’ h, m, and l, respectively, represent high, medium, and low level of the variable.

‘If’ Condition		‘Then’ condition’s Confidence Level for Candidate Attractiveness (CA)			
SA: Stand Alone Potential	CB: Combination Benefits	Good (g)	Average (a)	Poor (p)	Undecided {g, a, p}*
h	h	1.0	0.0	0.0	0.0
h	h	0.6	0.3	0.0	0.1
h	l	0.4	0.4	0.0	0.2
m	h	0.3	0.4	0.0	0.3
m	m	0.2	0.5	0.0	0.3
m	l	0.0	0.4	0.3	0.3
l	h	0.0	0.2	0.4	0.4
l	m	0.0	0.1	0.7	0.2
l	l	0.0	0.0	1.0	0.0

Belief Function Representation using Appendix A:

m({ghh, ghm, ghl, amh, amm, aml, plh, plm, pll}) = 0.4,

m({ghh, ahm, ahl, gmh, gmm, amm, pmm, pml, glh, alh, plh, plm, pll }) = 0.3,

m({ghh, ghm, ghl, ahl, phl, gmh, amh, pmh, gmm, gml, aml, pml, alh, glm, alm, plm, pll}) = 0.2,

m({ghh, ghm, ahm, phm, ahl, gmh, amh, pmh, amm, gml, aml, pml, glh, alh, plh, alm,,pll}) = 0.1

*****{g,a,p} represent the set of all possible values of the variable CA with the values of the corresponding beliefs provided in the last column.

The solid curve in Figure 5 depicts the overall belief that CA is good for the set of ‘if-then’ rules given in Table 2 and the dotted curve depicts the overall belief that CA is either good or average for the case where all the input variables have evidence in support of the value ‘high’. Comparing Figure 3 to Figure 5, one observes that under the fuzzier ‘if-then’ rules given in Table 2, the two beliefs that CA is ‘good’ and ‘good’ or ‘average’ are much lower than the two beliefs under the ‘if-then’ rules given in Table 1. This has important implications for the collection of evidence. Under fuzzier rules, one would require stronger (or additional) evidence to achieve the desired level of overall belief for CA to be good. In other words, the nature and extent of evidence to be collected for a merger or acquisition decision depends on the type of ‘if-then’ rules assumed among the input variables.

Figure 5

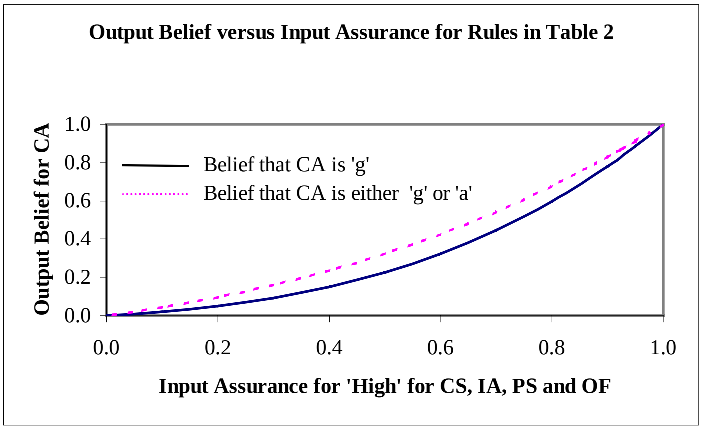


Figure 6

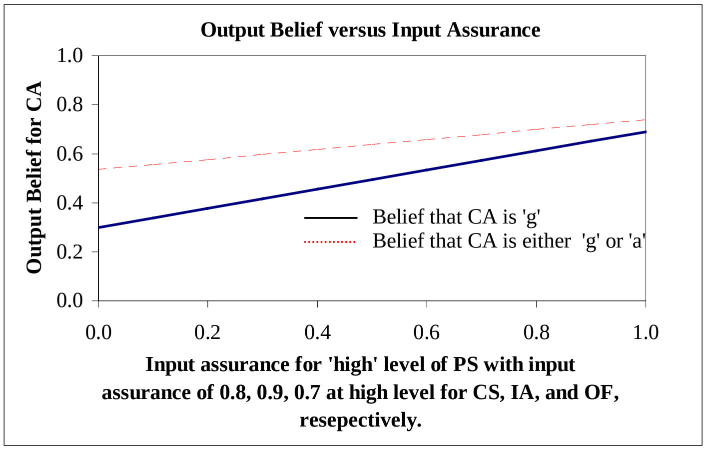


Figure 6 depicts the overall belief that CA is 'good' and CA is either 'good' or 'average' for the fuzzier rules given in Table 2. These beliefs are plotted as a function of input assurance for Potential Synergies being 'high' with input assurance of CS, IA, and OF at 'high' level being held constant at 0.8, 0.9, and 0.7 respectively. As the input assurance for Potential Synergies (PS) to be high increases from 0 to 1.0, the overall belief that Candidate Attractiveness (CA) is good also increases from 0.301 to 0.690, and the belief that CA is either 'good' or 'average' increases from 0.536 to 0.740. Comparing Figure 6 with Figure 4, it is evident that, for the same input conditions, the overall belief that Candidate Attractiveness is 'high' is much lower under the fuzzier rules than under the less fuzzy rules (Table 1). This, again, has important implications for the nature and extent of evidence that needs to be collected for decision making. In other words, when rules are fuzzy, additional and/or stronger supporting evidence needs to be collected to obtain the desired level of belief for the decision.

5. Conclusions

This paper has important theoretical and managerial implications for strategic decision making. From a theoretical perspective, the paper integrates concepts in the Dempster-Shafer theory of belief functions (Smets [45][44][46][47], Shafer [48], Yager et. al [49]) with the literature on mergers and acquisitions. The evidential reasoning approach under belief functions allows decision makers to incorporate the uncertainty judgment associated with various factors in a much more intuitive manner. The paper represents an initial effort at applying belief functions to represent uncertainties in mergers and acquisition decisions, one that results in a better understanding of how managerial decision making under uncertainty can be effectively and meaningfully addressed. In addition, we believe that the proposed framework and the approach suggested in the paper should provide the basis for future empirical studies on factors and relationships influencing acquisition performance.

Very importantly, the proposed framework should help managers in their decision making related to mergers and acquisitions. Fully developed, the suggested approach can help alleviate some of the critical shortcomings of currently used approaches – resulting in a more systematic and comprehensive evaluation of acquisition opportunities. Such evaluations are particularly critical when one considers the high failure rate among acquisitions and the considerable time and resources often expended in the pursuit of such transactions. In addition, sensitivity analysis on the relative importance of various input variables in the acquisition decision making model will help managers effectively and efficiently allocate scarce resources in the gathering of evidence (data/information) along various input factors.

In terms of its broad implications, the paper contributes to the development and building of decision making models under uncertainty in a variety of business situations. The present approach can be easily utilized in the context of other managerial decisions. For example, in recent years we have seen a surge in the number of strategic alliances between firms both in the international and domestic context. And, like mergers and acquisitions, strategic alliances are also plagued by high failure rates. Often the underlying cause is poor selection of partners. The approach developed and suggested in this paper can be easily extended (with appropriate modifications) to the evaluation of alliance partners.

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Appendix A

Algorithm for Belief Function Representation of 'If-Then' Rules

In order to propagate belief functions through a network of variables that are connected through 'if-then' rules, one must express these rules in terms of relational nodes. We will describe an algorithm to achieve this goal in this appendix and illustrate the process using a moderately complex example. Let us consider three variables A, B, and C with two possible outcomes for each variable, either the variable is true or false. Further consider the following 'if-then' rules involving A, B, and C.

Rule 1: If A is true (A = a), and B is true (B = b) then C is true (C = c) with 100% confidence.

Rule 2: If A is true (A = a) and B is false (B = ~b) then C is true (C = c) with 60% confidence and the state of C is not known whether it is true or false with 40% confidence.

Rule 3: If A is false (A = ~a) and B is true (B = b) then C is false (C = ~c) with 30% confidence, and the state of C is not known whether it is true or false with 70% confidence.

Rule 4: If A is false (A = ~ a) and B is false (B = ~ b) then C is false (C = ~c) with 100% confidence.

These rules can represented in the form of a table as given below in Table A1.

Table A1: 'If-Then' Rules Involving Three Variables A, B, and C, with Two outcomes for Each Variable

'If' Condition		'Then' Condition's confidence level		
A	B	c	~c	$\Theta c = \{c, \sim c\}$
a	b	1.0	0.0	0.0
a	~b	0.6	0.0	0.4
~a	b	0.0	0.3	0.7
~a	~b	0.0	1.0	0.0

The Algorithm

Step 1: Prepare a table similar to Table A1 with 'If" variables in the left columns and the 'Then' variable in the right side listed in a row with all its possible values and all the possible sets consisting of these values.

Step 2: Express the 'if-then' rule with its uncertainties for each condition of the rule in the body of the table. For example, the first row in Table A1 represents the rule that if A = a, and B = b, then C = c with 100% confidence. The second row represents the condition that if A = a, and B = ~b, then C = c, with a confidence of 0.6, and C could be either c or ~c with 0.4 level of confidence.

Step 3: Select the largest confidence number in each row. These values are written inside rectangular boxes in Table A2. These values define the focal elements of the ***m***-value to be determined next.

Step 4: Select the smallest value among the set of values from each row selected in Step 3. This value represents the m-value for the set of elements consisting of the elements corresponding to the values selected in step 3.

Step 5: Subtract the minimum number obtained in Step 4 from each selected numbers in Step 3.

Step 6: Repeat Steps 3 - 4 until all entries are zero.

Belief-Function Representation of 'If-Then' Rules

The following steps illustrate the use of the above algorithm to convert the 'if-then' rules described earlier in the appendix to a belief function representation[5].

[5] In general, there is no unique solution. There are several different belief-function representations for the same set of rules, i.e., we can obtain several different sets of m-values representing the same set of 'if-then' rules. We will investigate this issue further in a separate article. However, the overall combination of beliefs in a network of variables with 'if-then' rules where information flows from finer levels to a coarser level, which is the case in the present study, remains unaffected by the choice of the set of m-values representing the rules.

Iteration 1

Steps 1 – 3: The largest confidence number in each row is boxed in a rectangle. The lowest number among the boxed numbers is identified by an asterisk.

Table A2

'If' Condition		'Then' Condition's confidence level		
A	B	c	~c	Θc = {c,~c}
a	b	$\boxed{1.0}$	0.0	0.0
a	~b	$\boxed{0.6*}$	0.0	0.4
~a	b	0.0	0.3	$\boxed{0.7}$
~a	~b	0.0	$\boxed{1.0}$	0.0

Step 4: ***m***({abc, a~bc, ~abc, ~ab~c, ~a~b~c}) = 0.6.

Step 5: By subtracting the lowest number identified in Step 3 from each marked numbers, one obtains the following table:

'If' Condition		'Then' Condition's confidence level		
A	B	c	~c	Θc = {c,~c}
a	b	0.4	0.0	0.0
a	~b	0.0	0.0	0.4
~a	b	0.0	0.3	0.1
~a	~b	0.0	0.4	0.0

Iteration 2

Step 3: The largest number in each row is boxed in a rectangle. The lowest number among the boxed numbers is identified by an asterisk.

'If' Condition		'Then' Condition's confidence level		
A	B	c	c	Θc = {c, ~c}
a	b	$\boxed{0.4}$	0.0	0.0
a	~b	0.0	0.0	$\boxed{0.4}$
~a	b	0.0	$\boxed{0.3*}$	0.1
~a	~b	0.0	$\boxed{0.4}$	0.0

Step 4: ***m***({abc, a~bc, a~b~c, ~ab~c, ~a~b~c}) = 0.3.

Step 5: By subtracting the lowest number identified in Step 3 from each marked numbers, one obtains the following table:

'If' Condition		'Then' Condition's confidence level		
A	B	c	~c	Θc = {c,~c}
A	b	0.1	0.0	0.0
A	~b	0.0	0.0	0.1
~a	b	0.0	0.0	0.1
~a	~b	0.0	0.1	0.0

Iteration 3

Steps 3: This step yield the following table:

'If' Condition		'Then' Condition's confidence level		
A	B	c	~c	Θc = {c,~c}
a	b	$\boxed{0.1}$	0.0	0.0
a	~b	$\boxed{0.1}$	0.0	0.0
~a	b	0.0	0.0	$\boxed{0.1}$
~a	~b	0.0	$\boxed{0.1}$	0.0

Step 4: ***m***({abc, a~bc, ~abc, ~ab~c, ~a~b~c}) = 0.1.

The Relational Node

The above process yields the relational node with the following m-values for the 'If-Then' rule defined in this appendix:

m({abc, a~bc, ~abc, ~ab~c, ~a~b~c}) = 0.6,

m({abc, a~bc, a~b~c, ~ab~c, ~a~b~c}) = 0.3,

m({abc, a~bc, ~abc, ~ab~c, ~a~b~c}) = 0.1.

Appendix B

Modeling Logical Relationships and Fuzzy 'If-Then' Rules

As mentioned earlier, in order to combine or propagate uncertainties through a network, we need to convert the interrelationships among the variables of the network into relational nodes under belief functions. In general, some of these relationships are expressed in terms of logical relationships that are categorical such as 'and', 'or', and 'exclusive or', and some relationships are expressed through 'if-then' rules. As shown below, it is much easier to model the logical relationships in belief functions than to model 'if-then' rules, especially if the rules involve uncertainties. We demonstrate the use of the algorithm described in Appendix A for converting various interrelationships among the variables in a network.

'AND' Relationship

Assume that we have three variables: A, B and C. An 'and' relationship between C, and A and B implies that C is true if and only if A and B are true ($C = A \cap B$). Such a relationship will allow only the following possible set of values[6] for the variables: {abc, a~b~c, ~ab~c, ~a~b~c}. One can express the above relationship through the following conditional probabilities:

P(c|ab) = 1, P(~c|ab) = 0, P(c|a~b) = 0, P(~c|a~b) = 1,

P(c|~ab) = 0, P(~c|~ab) = 1, P(c|~a~b) = 0, P(~c|~a~b) = 1.

In order to develop the belief function representation of the above relationship, we complete Steps 1-3 of Appendix A and obtain the following table:

'If' Condition		'Then' Condition's confidence level		
A	B	c	~c	Θ_c = {c, ~c}
A	b	1.0	0.0	0.0
A	~b	0.0	1.0	0.0
~a	b	0.0	1.0	0.0
~a	~b	0.0	1.0	0.0

Using Steps 4-5 of the algorithm, we obtain the following belief function representation of the 'and' relationship:

[6] We represent the value of a variable, say A, by the lower case letter 'a' that it is true and the value that A is not true by '~a'.

m({abc, a~b~c, ~ab~c, ~a~b~c}) = 1.

'OR' Relationship

For an 'or' relationship between C, and A and B (i.e., $C = A \cup B$), we will have the following possible values: {abc, a~bc, ~abc, ~a~b~c}. This relationship implies that C is true when either both A and B are true or when any one of them is true but it is false when both A and B are false. In terms of probabilities, we can express the above relationship as:

P(c|ab) = 1, P(~c|ab) = 0, P(c|a~b) = 1, P(~c|a~b) = 0,

P(c|~ab) = 1, P(~c|~ab) = 0, P(c|~a~b) = 0, P(~c|~a~b) = 1.

Again by completing Steps 1-3 of Appendix A we obtain the following table:

'If' Condition		'Then' Condition's confidence level		
A	B	C	~c	Θ_c = {c,~c}
a	b	1.0	0.0	0.0
a	~b	1.0	0.0	0.0
~a	b	1.0	0.0	0.0
~a	~b	0.0	1.0	0.0

Using Steps 4-5 of the algorithm, we obtain the following belief-function representation of the 'or' relationship:

m({abc, a~bc, ~abc, ~a~b~c}) = 1.

Exclusive OR (EOR)

An 'exclusive or (eor)' relationship between C, and A and B implies that C is true only when either A is true or B is true but it is false when both A and B are either false or true. Such a relationship will allow only the following set of values: {ab~c, a~bc, ~abc, ~a~b~c}. The probability representation of this relationship is:

P(c|ab) = 0, P(~c|ab) = 1, P(c|a~b) = 1, P(~c|a~b) = 0,

P(c|~ab) = 1, P(~c|~ab) = 0, P(c|~a~b) = 0, P(~c|~a~b) = 1.

Again using the algorithm described in Appendix A, we obtain the following belief-function representation of the 'eor' relationship:

m({ab~c, a~bc, ~abc, ~a~b~c}) =1.

Fuzzy 'IF-Then' Relationship

Consider a simple 'if-then' rule: If A is true (A = a) then B is true (B = b) with 90% confidence but with 10% confidence B could be either true or false. Also, if A is false (A = ~a), then we do not know whether B is true or false. The following table is obtained after completing Steps 1-4 of Appendix A:

'If' Condition	'Then' Condition's confidence level		
A	b	~b	Θ_b = {b,~b}
a	$\boxed{0.9}$*	0.0	0.1
~a	0.0	0.0	$\boxed{1.0}$

And the corresponding m-value is given by:

m({ab, ~ab, ~a~b}) = 0.9.

Step 5 of the algorithm yields:

'If' Condition	'Then' Condition's confidence level		
A	b	~b	Θ_b = {b,~b}
A	0.0	0.0	0.1
~a	0.0	0.0	0.1

The second iteration of Steps 3-4 of Appendix A yields the following m-value:

m(ab, a~b, ~ab, ~a~b}) = 0.1.

Thus a belief-function representation of the above 'if-then' relationship is given by the following m-values:

m({ab, ~ab, ~a~b}) = 0.9,

m(ab, a~b, ~ab, ~a~b}) = 0.1.