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Belief Functions in Accounting Behavioral Research

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ABSTRACT

Behavioral accounting research deals with a complex set of phenomenon including the broad domain of human decision making under uncertainty. Two aspects of decision making of particular relevance to accounting and auditing research are two constructs that are inexorably interrelated: uncertainty and information (evidence). This paper introduces a theoretical perspective that enriches the knowledge-set that may be used in behavioral accounting research when confronting decision contexts that involve uncertainly.

The main body of the paper is an introduction to Belief Functions. The introduction includes a discussion of the fundamental constructs and then illustrates the use of belief functions in two audit settings: traditional financial statement audit planning and the evaluation of evidence in a cascaded-inference setting involving the evaluation of internal accounting control. The paper concludes with a brief exploration of some of the research issues and opportunities that are related to the potential use of belief functions in Behavioral Accounting Research.

I. INTRODUCTION

The refusal to choose is a form of choice; Disbelief is a form of belief. -Frank Barron

Human decision-making is a complex process, especially in situations where significant ambiguity exists. Ambiguity and uncertainty[1] are inherent characteristics of tasks in all disciplines whether it is accounting, auditing, law, or medicine. The primary objective of this paper is to consider some important aspects of belief formulation within the context of behavioral accounting research.

There are two major issues when it comes to dealing with belief formulation under ambiguity and uncertainty. The first issue deals with the framework that can be used to express or measure uncertainty and ambiguity present in the task setting. Shafer and Tversky (1985) and Shafer and Srivastava (1990) describe this process of choosing a framework as a process of choosing a "formal language" or "semantics" to express the uncertainties in analyzing a task. They argue that the context and the domain of the problem determine what language is appropriate.

[1] In general, uncertainty deals with situations where one is not sure about the outcome of an event. For example, consider an urn with 100 balls, 50 red and 50 black. The outcome whether the color would be red or black of a ball being picked from this urn is not sure. Given only two possible outcomes (red or balck) with equal likelihood (50 red and 50 black balls), one can make a judgment that each outcome has a 50-50 chance of occurring. Ambiguity, on the other hand, deals with those situations of uncertainty where one is not even able to make that judgment about the likely chance of each outcome to occur. For example, suppose you have a second urn with 100 balls of red and black color, but you are told that the proportion of red and black balls is not known. It may be in any proportion; from all being red to all being black. Einhorn and Hogarth (1986) call the situation of urn one a situation of complete knowledge and the situation of the second urn a situation of complete ambiguity or no knowledge.

The second issue deals with the calculus or “syntax” (Shafer and Tversky 1985) through which we combine information to make an overall judgment or decision. There are many frameworks and formal languages that can be used to represent uncertainties such as those based on probability theory, fuzzy logic, possibility theory (Zadeh 1978, 1979), belief functions (Shafer 1976, Smets 1990a, 1990b, 1998, Yager et. al 1994), and epistemic belief functions (Spohn 1990, 1998). These frameworks have different characteristics and thus seem to “work better” in certain problem domains than in other domains.

For example, Srivastava and Shafer (1992) and Akresh et al. (1988) argue that belief functions provide a more flexible and adaptable way to combine evidence from a variety of sources. One aspect of this flexibility is that the belief function framework reduces to the Bayesian framework under a special condition (Shafer and Srivastava 1990).

More importantly, belief functions provide a superior way of mapping uncertainty judgments in accounting and auditing (Harrison 1999), and incorporating ambiguity within the decision-making (Srivastava 1997a) process. For example, suppose the auditor has obtained and assessed inherent factors for a client such as its business environment, its economic condition, and the CEO’s honesty and integrity. Assume that all of these factors provide positive evidence to the auditor that the financial statements are fairly stated. Based on this, the auditor could attribute a low level of support, say 0.2 on a scale of 0-1, that the financial statements are fairly stated, a zero level of support that the financial statements are materially misstated, and a 0.8 level of uncommitted support.

Such a representation of uncertainty is difficult under the probability framework, as it is unclear as to how the uncommitted support (the ambiguity) should be treated.

Thus an advantage of using belief functions in behavioral research is that it provides a more natural and logical way to model ambiguity compared to the probability framework. To further illustrate this point, let us consider the urn example introduced in footnote 1.

In this example, a decision-maker is considering two urns – Urn One has an unknown distribution of red and black balls and Urn Two is known to have an equal number of red and black balls. Under the probability framework, one often assumes uniform priors in situations of ambiguity such as Urn One. Thus in either situation, using the probability framework, one would assign 0.5 that a drawn ball would be red or black. In an experimental context, this feature of the probability framework of not being able to distinguish between the two urns has led Einhorn and Hogarth (1986, p. S228) to observe a behavioral incongruity:

> ... either urn 2 has complementary probabilities that sum to more than one, or urn 1 has complementary probabilities that sum to less than one. As we will show, the nonadditivity of complementary probabilities is central to judgments under ambiguity.

Srivastava (1997a) has shown that *no such super- or sub-additivity is needed to explain the decision maker's behavior* if belief functions are used to treat ambiguity. This will be further elaborated in the paper later.

Representation of ignorance is another area where probability framework has problems, and leads to illogical implications, especially in an audit context. Consider an example where the auditor is trying to make a judgment about the probability of whether the total value of the inventory items stored at two locations is materially misstated. Suppose the auditor starts with no knowledge about the state of the inventory at the two locations, i.e., the auditor is completely ignorant about the state of the inventory. Under probability framework, if uniform priors are assumed, the auditor would assume a 50-50 chance of the inventory at the two locations to be fairly stated, ‘f’, and materially misstated, ‘~f’, i.e., $P(f_1) = 0.5$, $P(\sim f_1) = 0.5$, $P(f_2) = 0.5$, and $P(\sim f_2) = 0.5$, where subscript 1 and 2 stand for the two locations. If we further assume that the combined inventory is fairly stated only when the inventory at both the locations are fairly stated, then this situation yields a probability of 0.25 that the combined inventory is fairly stated and 0.75 that the combined inventory is materially misstated[2]. This seems to be illogical. Since we started with ignorance for the inventory at the two locations, we should end up with ignorance about the state of the inventory even after combining which is not the case. We seem to have more knowledge about the state of the combined inventory. In fact, the situation would be more confusing if we consider more locations. We will show later how a belief function treatment avoids this problem.

The main purposes of this article are to introduce belief functions (Shafer 1976, Smets 1990a, 1990b, 1998) and then to consider their application to decision-making in accounting and auditing. We will first introduce belief functions and discuss how belief functions help to overcome certain problems inherent in the use of probabilities. The

[2] The combined inventory is fairly stated only when inventory at the two locations are fairly stated, i.e., = $f_{12} = f_1 \cap f_2$. The combined inventory is materially misstated under the following conditions: inventory at only one location is materially misstated or inventory at both locations are materially misstated. This condition yields $P(f_{12}) = P(f_1 \cap f_2) = P(f_1)P(f_2) = 0.5x0.5 = 0.25$, and $P(\sim f_{12}) = P((\sim f_1 \cap f_2) \cup (f_1 \cap \sim f_2) \cup (\sim f_1 \cap \sim f_2)) = P(\sim f_1)P(f_2) + P(f_1)P(\sim f_2) + P(\sim f_1)P(\sim f_2) = 0.5x0.5 + 0.5x0.5 +0.5x0.5 = 0.75$. In the above argument, we have assumed that there are no off setting errors.

paper concludes with a discussion of several studies that have used belief functions in auditing (and accounting) research and of research opportunities.

II. INTRODUCTION TO BELIEF FUNCTIONS

The belief-function formalism is not new. It has its origin in the seventeenth century work of George Hooper and James Bernoulli (Shafer 1986, see also Gabbay and Smets 1998, Shafer 1976, Shafer and Srivastava 1990, Smets 1998, 1990a, 1990b, and Yager et. al 1994). It is based on the mathematical theory of probability similar to the Bayesian formalism. Also, the belief-function formalism reduces to the Bayesian formalism under a special condition, as discussed later. Here we will present the basics of belief functions (see also Srivastava 1993).

There are three basic functions that are important to understand the use of belief functions in a decision-making process: *basic belief mass functions* or m-values, *belief functions*, and *plausibility functions*. We discuss each of these functions below.

Basic Belief Mass Function (m-values)

The *basic belief mass*[3] function (or m-values) is similar to the probability function. Let us consider an example to illustrate the properties of this function. Consider a decision problem with *n* possible elements or states of nature forming a mutually exclusive and exhaustive set represented by $\{a_1, a_2, a_3, \ldots a_n\}$. We call this set a *frame*

[3] Shafer calls this function the *basic probability assignment* function.

and represent it by the symbol Θ. Under the probability framework, we assign probabilities to each state of nature and these probabilities must add to one[4].

Under the belief function framework, basic belief masses or m-values are assigned not only to each state of nature but also to all possible combinations of these states of nature. For example, m-values are assigned to all the single elements, to all the subsets consisting two elements, three elements, and so on, and to the entire frame Θ. Similar to probabilities, these m-values add to one[5]. We will consider Shafer's belief-function framework where the basic belief mass assigned to the empty set is zero by definition[6].

Let us consider an auditing example to help us understand the basic concepts of m-values. Suppose an auditor has performed certain ratio and trend analyses pertinent to the accounts receivable balance and has decided that the analyses provide a positive but low level of belief, say 0.2 on a scale of 0 - 1, that the account balance is fairly stated. If the state that the accounts receivables balance is fairly stated is represented by 'a' and the state that the account balance is materially misstated by '~a' then we have the following *basic belief masses*, i.e., m-values: m(a) = 0.2, m(~a) = 0, and m({a,~a}) = 0.8, and the sum is one.

In an audit context, one can interpret these m-values as the level of support directly obtained from the evidence for the argument of the m-value function. For

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[4] Probability mass assigned to each state of nature, a_i, is $P(a_i) \geq 0$ where i = 1, 2, … n and the sum of all these probabilities is equal to one, i.e., $\sum_{i=1}^{n} P(a_i) = 1$.

[5] The sum of all the basic belief masses also is equal to one, i.e., $\sum_{A \subseteq \Theta} m(A) = 1$, where A represents all the subsets of the frame.

example, in the above case, we have direct ratio and trend analysis evidence that the account is fairly stated with 0.2 level of belief, no belief that the account is materially misstated, and 0.8 belief still uncommitted. This 0.8 is assigned to the entire frame Θ = {a, ~a}. Note that in this example, the evidence is "positive" in that it supports the hypothesis that the accounts receivable is fairly stated.

A negative piece of evidence is represented by a finite (non-zero) belief mass assigned to '~a' that the account is materially misstated. Suppose that the ratio and trend analyses considered in the above example signal that the account balance may be materially misstated but with a low level of support, say 0.1, for ~a and no evidence that the account balance is fairly stated. This situation can be written as: m(a) = 0, m(~a) = 0.1, and m({a, ~a}) = 0.9.

Mixed audit evidence can be expressed by assigning some basic belief mass to 'a', some to '~a', and some to the entire frame Θ = {a, ~a}. An example of mixed evidence is where part of the support, say 0.2, is for 'a', 0.1 for '~a', and the remaining, 0.7 is assigned to {a, ~a}. In terms of m-values, one can express this evidence as: m(a) = 0.2, m(~a) = 0.1, and m({a, ~a}) = 0.7. This kind of evidence is not easy to model under the probability framework.

There are two ways one can assess these basic belief masses or m-values. The first approach is through the decision maker's (i.e. auditor's) subjective judgment as illustrated above. This judgment could be based on the experience or other knowledge.

The second approach is based on the *compatibility relationship* between two frames. Suppose the decision-maker is interested in making a decision in one frame but

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[6] Under Transferable Belief Functions of Smets (1990a, 1998), one can assign a non-zero mass to the empty set.

has no prior knowledge about the probability distribution of the possible outcomes in this space. However, assume the auditor has knowledge about the probability distribution on another frame that is compatible to the frame of interest. This compatible relationship may help the decision maker make inferences about the frame of interest from the knowledge of the probability distribution on the other frame. As an illustration let us consider the following situation.

The auditor asks the manager of a business unit whether his unit follows certain important control procedures and in response the management says, “Yes, we do follow the procedures.” The question is what would be the level of belief that the unit follows the control procedures? Before we answer this question let us consider that the auditor knows this manager and thinks that the manager is trustworthy about 80 percent of the time and remaining 20 percent of the time he is not trustworthy. The auditor has the following probability distribution on this frame, {t, ~t}, where t = ‘the manager is trustworthy” and ~t = ‘manager is not trustworthy”: P(t) = 0.8, and P(~t) = 0.2.

Assume this frame is compatible to the frame of interest {c, ~c} where c = ‘Unit follows the control procedures’ and ~c = ‘Unit does not follow the control procedures’. Assume further that the compatibility relationship is not necessarily one-to-one. If the manager is trustworthy when he says that the controls are being followed then the controls are being followed, i.e., ‘t’ is compatible to ‘c’. However, when the manager is not trustworthy, the controls may or may not be followed, i.e., ‘~t’ is compatible to the frame {c, ~c}. This compatibility relationship yields the following belief masses (see Figure 1): P(t) = 0.8 $\rightarrow$ m(c) = 0.8, and P(~t) = 0.2 $\rightarrow$ m({c,~c}) = 0.2 and we have no belief that controls are not being followed, i.e., m(~c) = 0.

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Belief Function

Belief on a set of elements, say, A of a frame Θ is defined as the total belief on A. This represents the sum of all the basic belief masses assigned to the elements contained in A plus the basic belief mass assigned to A[7]. Let us consider the example discussed earlier where the auditor has performed analytical procedures in testing the accounts receivable balance and based on the findings he concludes that the evidence provides a low level of support, say 0.2, that accounts receivable balance is fairly stated, no support for the account to be materially misstated, with 0.8 level of belief uncommitted. In terms of basic belief masses, one can write the auditor's judgment as: m(a) = 0.2, m(~a) = 0, and m({a, ~a) = 0.8.

Using the definition of belief functions as described above (see also footnote 8), the belief that the accounts receivable is fairly stated is 0.2, belief that the account is materially misstated is 0, i.e., Bel(a) = 0.2, and Bel(~a) = 0. Also, we have a belief of one in the entire frame, i.e., Bel({a, ~a}) = 1.0. This value is obtained as follows. According to the definition, the belief in a set A, say A = {a, ~a}, is the sum of the basic belief masses on all the subsets contained in A, and the belief mass on the entire set A. In our example, this definition implies that the belief on {a, ~a}, Bel({a,~a}), is the sum of the basic belief masses on 'a', '~a', and {a, ~a}, i.e., the sum of m(a) = 0.2, m(~a) = 0, and m({a, ~a}) = 0.8, which is one in the present example.

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[7] Mathematically this can be written as: $\mathrm{Bel}(A) = \sum_{B \subseteq A} m(B)$, where B is any subset of A.

Plausibility Function

The plausibility of an element or a set of elements, say A, of a frame, Θ, is defined to be the maximum possible belief that could be assigned to A if all future evidence were in support of A[8]. Consider the above example of analytical procedure with the basic belief masses: m(a) = 0.2, m(~a) = 0, and m({a, ~a}) = 0.8. This evidence provides a belief of 0.2 that the account is fairly presented. However, 0.8 level of belief which is assigned to the entire frame, {a, ~a}, is in principle uncommitted. In the best possible scenario, if new pieces of evidence provided only positive support then all of this uncommitted belief could be assigned to 'a'. This provides the following value for plausibility of 'a': Pl(a) = 0.2 + 0.8 = 1.0. Similarly, in the other extreme case, if new pieces of evidence provided support to only '~a' then the uncommitted belief of 0.8 could be assigned to '~a' yielding a 0.8 level of plausibility for '~a', i.e., Pl(~a) = 0.8.

Ambiguity Function

In belief functions, the ambiguity in a state A is defined (Srivastava 1997a, Wong and Wang 1993) as the difference between the plausibility of A and the belief in A, i.e.,

$$\text{Ambiguity in A} = \text{Pl(A)} - \text{Bel(A)}$$

In the above example, the belief that the account is fairly stated in 0.2 and its plausibility is 1.0. Therefore, the ambiguity in 'a' that the account is fairly presented is 0.8 (Ambiguity in 'a' = Pl(a) – Bel(a) = 1.0 – 0.2 = 0.8). Similarly, the ambiguity in ~a is 0.8.

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[8] Mathematically one can write plausibility as $\text{Pl(A)} = \sum_{A\cap B\neq\emptyset} m(B)$. Also, one can show that the plausibility function is related to the belief function through the following relationship: Pl(A) = 1- Bel(~A).

Consider now the example of two urns discussed earlier as considered by Enhorn and Hogarth (1986). Urn 1 (Ambiguous) is the urn with 100 balls of black and red color with no knowledge of the proportion of red and black balls. Urn 2 (Known) is the urn with complete knowledge that it contains 100 balls of 50 red and 50 black colors. The belief mass that a red ball is picked from Urn 1 is zero and similarly a belief mass that a black ball is picked is zero, i.e., m_1(red ball) = 0, and m_1(black ball) = 0 where the subscript represents the urn number. These values suggest that we have no direct evidence whether the ball picked will be of red or black color. The plausibility of picking a red ball would be one and similarly the plausibility of picking a black would be one. Thus, the ambiguity that the ball would be of red color is one (completely ambiguous) and so is the ambiguity that the ball would be of black color. However, in the case of Urn 2, the belief masses are: m_2(red ball) = 0.5, and m_2(black ball) = 0.5, and the corresponding beliefs and plausibilities are: Bel(red ball) = Bel(black ball) = 0.5, and = Pl(red ball) = Pl(black ball) = 0.5. In this case the ambiguity that a red ball is picked is zero and the ambiguity that a black ball is picked is also zero. Under the belief-function framework we see that the two urns are represented very differently.

III. ILLUSTRATIONS OF THE USE OF BELIEF FUNCTIONS IN AUDITING RESEARCH

In behavioral accounting and auditing research, research that considers uncertainly is rarely useful unless it can be applied to specific tasks. In this section we discuss two applications. The first application deals with the planning and evaluation of an audit as discussed by Srivastava and Shafer (1992) and Srivastava et. al (1996). The second application deals with belief revisions by auditors as studied by Krishnamoorthy, Mock & Washington (1999). Belief functions tend to have the following advantages over probability framework in these contexts

1. The audit risk model is a plausibility model (Srivastava and Shafer 1992) and a belief function interpretation of the components of the audit risk model makes more intuitive sense (see further discussions in the next section) [9].

2. There is some evidence that a judgment about the basic assessment of the strength of evidence using a belief function framework is more intuitive (Harrison 1999, and Golden an Curly 1995).

3. Representation of positive, negative, and mixed items of evidence is more convenient. For example, positive evidence in support of an audit objective 'o' can be expressed in terms of a non-zero m-value, say 0.2, for 'o' and zero value for '~o', i.e., m(o) = 0.2, m(~o) = 0, m({o, ~o}) = 0.8. A negative item of evidence would be represented by a non-zero m-value for '~o', say 0.1, a zero value for 'o', i.e., m(o) = 0, m(~o) = 0.1, and ({o, ~o}) = 0.9. A mixed item of evidence can be expressed by assigning a non-zero m-value for both 'o' and '~o', i.e., m(o) ≠ 0, and m(~o) ≠ 0. As an illustration, we can express a mixed item of evidence as m(o) = 0.3, m(~o) = 0.1, and m({o, ~o}) = 0.6.

4. Representation of different levels of assurance coming from the same item of evidence for two different audit objectives is expressed more conveniently. For example, in the case of a confirmation test, assume that the auditor finds that all the respondents have said that they owe money to the company, but some of them have made comments that their account balance is overstated. Based on this information the auditor's judgment is that a high level of support, say 0.8, is obtained for 'existence' objective (e) and a medium level of support, say 0.6, is obtained for 'valuation' objective (v). This can be expressed in terms of m-values as: m(e) = 0.8, m(~e) = 0, m{(e, ~e}) = 0.2, and m(v) = 0.6, m(~v) = 0, m({v, ~v}) = 0.4.

Audit Planning and Evaluation: Issues in the Combination of Evidence

The audit risk model of SAS 47 (AICPA 1983) has been proposed to be a framework that can be used for planning an audit. The model has been criticized on a

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[9] The audit risk model of SAS 47 is AR = IR×CR×DR, where AR is the risk that the auditor gives a clean opinion but the financial statements may contain material misstatements. IR, CR, and DR are respectively inherent risk, control risk and detection risk (see SAS 47 for definitions). SAS 47 suggests that if the auditor does not want to depend on the environmental factors then he should set IR to one and plan the audit based on the assessed control risk and planned detection risk. Under probability framework, IR = 1 implies that the auditor is assuming that there are material errors in the financial statement for sure, but that is not the auditor's thinking when he sets IR = 1. Under belief functions, IR = 1 implies that plausibility of

number of grounds, including Srivastava and Shafer (1992) who argued that the risks in the model really imply plausibility of material misstatements instead of probability of material misstatements. They argue that the plausibility interpretation of the risks in the audit risk model makes intuitive sense. Our first example of the use of belief functions in accounting and audit research reviews this idea.

The external auditor plans the audit to be able to reach an opinion considering the possibility of existence of material errors in the financial statements. If no material misstatements are found and the evidence supports the opinion that the financial statements are fairly stated with a high level of confidence, say 0.95, then the auditor issues a clean opinion. Given the earlier definition of plausibility, this implies that the plausibility of material misstatement is 0.05. This plausibility can be interpreted, as the maximum possible risk the auditor is willing to take that the financial statements might be materially misstated without giving a qualified opinion. Note that this interpretation does not imply that there is any evidence in support of material misstatements.

The audit process is a process of collecting, evaluating, and aggregating evidence. Srivastava and Shafer (1992) have shown analytically how one can combine positive items of evidence accumulated with respect to various accounts within the balance sheet. Srivastava et. al (1996, see also Srivastava 1995a, 1995b) have discussed the aggregation of evidence in a complex network of variables, where the variables are the balance sheet, the balance sheet accounts, and the audit objectives or the management assertions concerning the accounts. A network structure arises when one piece of evidence supports more than one audit objective or more than one account. For example, confirmations of

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material errors present in the financial statements is one, however, there is no evidence in support or against the financial statements being fairly stated.

accounts receivable pertain to the existence and valuation objectives of the account. We will demonstrate below through a simple example how two items of positive evidence can be combined in the belief-function framework.

Suppose that the auditor assesses a low level of belief, say 0.1 on a scale of 0–1, that the financial statements are fairly stated based on evidence concerning environmental factors, such as the economic conditions under which the client operates, the administrative style of the CEO, and the CEO's integrity. Assume there is no reason to believe that the financial statements are materially misstated given the economic conditions and a trustworthy CEO with integrity. These inherent factors (IF) yield the following m-values: $m_{IF}(f) = 0.1$, $m_{IF}(\sim f) = 0$, and $m_{IF}(\{f,\sim f\}) = 0.9$, where 'f = financial statements are fairly stated', and '~f = financial statements are not fairly stated'.

Suppose the auditor collects additional evidence by performing analytical procedures, such as comparing this year's financial statement balances with last year's balances and computing various ratios. Assume that the auditor again assesses a low, but positive, level of support, say 0.2 on a scale of 0–1, that the financial statements are fairly stated based on this evidence. Assume also that the analytical procedures provide no reasons to believe that there is material misstatement in the financial statements. This judgment about analytical procedures (AP) may be expressed in terms of the following basic belief masses or m-values as: $m_{AP}(f) = 0.2$, $m_{AP}(\sim f) = 0$, and $m_{AP}(\{f, \sim f\}) = 0.8$.

We have the following beliefs for 'f' and '~f' from the first piece of evidence, the inherent factors: $Bel_{IF}(f) = 0.1$, $Bel_{IF}(\sim f) = 0$. The corresponding plausibilities are: $Pl_{IF}(f) = 1.0$, and $Pl_{IF}(\sim f) = 0.9$. The plausibility of material misstatement based on the inherent factors is 0.9, although there is no belief that the financial statements are materially misstated. Srivastava and Shafer (1992) showed analytically that the plausibility of material misstatements in the financial statements based on inherent factors is equivalent to inherent risk (IR). Thus, in the current example IR = 0.9. Conceptually, this is equivalent to saying that the auditor has 0.1 level of support from environmental factors that the financial statements are fairly stated and 0.9 level of maximum possible support that the financial statements could be materially misstated,even though there is no evidence that they are materially misstated.

Similarly, the beliefs and plausibilities for 'f' and '~f' based on the analytical procedures are: $Bel_{AP}(f) = 0.2$, $Bel_{AP}(\sim f) = 0$, $Pl_{AP}(f) = 1.0$, and $Pl_{AP}(\sim f) = 0.8$. This means that the auditor has direct evidence from analytical procedures that the financial statements are fairly stated with 0.2 degree of belief, no evidence and thus zero belief for material misstatement, and 0.8 level of plausibility that material misstatement could exist (even though there is no direct evidence of misstatement). Similar to the prior case, a plausibility of 0.8 that the financial statements are materially misstated represents the risk associated to analytical procedures as considered in SAS 47.

One of the difficulties in actual audits is the combination of various items of evidence, such as those discussed above. Using belief functions, one would use Dempster's rule of combination[10]. In the above example, we have two items of evidence with the following m-values:

$$m_{IF}(f) = 0.1,\ m_{IF}(\sim f) = 0,\ m_{IF}(\{f, \sim f\}) = 0.9,$$

$$m_{AP}(f) = 0.2,\ m_{AP}(\sim f) = 0,\ m_{AP}(\{f, \sim f\}) = 0.8.$$

[10] Dempster's rule is similar to Bayes' rule. In fact, Dempster's rule reduces to Bayes' rule under the condition where the basic belief masses, i.e., m-values, are assigned to only singletons of the frame (Shafer 1976).

According to Dempster's rule, we cross multiply all the m-values and assign the resulting masses to the intersection of the arguments. In situations where the intersection is empty with a finite belief mass ("conflict situations"), we need to renormalize the remaining masses (Shafer and Srivastava 1990). In the above case, we do not have any conflict. Thus the following are the combined m-values as a result of cross multiplication of the two sets of m-values:

$$m_T(f) = m_{IF}(f)m_{AP}(f) + m_{IF}(f)m_{AP}(\{f,\sim f\}) + m_{IF}(\{f,\sim f\})m_{AP}(f)$$

$$= 0.1x0.2 + 0.1x0.8 + 0.9x0.2 = 0.28,$$

$$m_T(\sim f) = m_{IF}(\sim f)m_{AP}(\sim f) + m_{IF}(\sim f)m_{AP}(\{f,\sim f\}) + m_{IF}(\{f,\sim f\})m_{AP}(\sim f)$$

$$= 0 + 0x0.8 + 0.9x0 = 0,$$

$$m_T(\{f,\sim f\}) = m_{IF}(\{f,\sim f\})m_{AP}(\{f,\sim f\}) = 0.9x0.8 = 0.72.$$

The beliefs and plausibilities are: $Bel_T(f) = 0.28$, $Bel_T(\sim f) = 0$, $Pl_T(f) = 1.0$, and $Pl_T(\sim f) = 0.72$. Thus, based on the inherent factors and analytical procedures the auditor has 0.28 degree of belief that the financial statements are fairly presented. But there is still 0.72 degree of plausibility that material misstatements may be present. Usually, the auditor will need to collect more evidence to increase the belief that the financial statements are fairly stated to a threshold level, say 0.95, or reduce the plausibility of material misstatement to 0.05, i.e., reduce the overall audit risk to 0.05 in order to give a clean opinion[11]. Note that, although there is no belief that the financial statements are materially misstated, a 0.05 level of plausibility that the financial statements are materially misstated represents the maximum possible belief for material errors (a worst scenario case) if all additional items of evidence collected provide support for material

[11] See Shafer and Srivastava (1990) for an example of the use of Dempster's rule with conflicting pieces of evidence.

misstatements. This interpretation of risk is more intuitive than the interpretation provided under probability framework.

The analytical model proposed by Srivastava and Shafer (1992) for combining various items of evidence at different levels of the balance sheet is for planning purposes. One can also use their model for evaluation purposes, provided all the pieces of evidence gathered are positive in nature. This is not the case in general. In fact, in general, the auditor may encounter some positive evidence, some negative evidence, and some mixed items of evidence. In order to combine evidence from various sources at different level of the financial statement, Srivastava et. al (1996) used a software called "Auditor's Assistant" developed by Shafer et. al (1988). This software allows the user to draw the evidential network for audit planning and evaluation decisions. Srivastava et al (1996) applied the belief function approach for planning and evaluation of an audit of a healthcare unit. They showed how one could develop a network of variables with accounts and audit objectives and connect various items of evidence to pertinent variables (i.e., audit objectives of an account, the accounts on the balance sheet, and the balance sheet as whole).

The inputs to the network are the auditor's judgments about the planned (or achieved) level belief to be obtained (or obtained) from various items of evidence. The program would aggregate all the evidence and provide the overall belief at all the variables. If the overall belief obtained at the variable of interest, say the financial statement, is below the threshold level, say 0.95, the auditor would (1) collect more evidence to increase the overall belief to the threshold level and issue an unqualified opinion, (2) either issue a qualified opinion or an adverse opinion if the overall belief that the financial statements are materially misstated is more than the threshold value, 0.05 in the present example, or (3) issue a disclaimer if the auditor is not able to collect further evidence. If the overall belief that the financial statements are fairly stated is equal to or more than the threshold value then the auditor would not collect any further evidence and issue an unqualified opinion.

Evidential Assessment: Issues in Cascaded Inference Settings

In this section, we also discuss an audit task where multiple items of evidence need to be integrated to assess likelihood of error. The particular setting differs from the prior example in that the evidence is "cascaded", that is knowledge concerning one type of evidence (the quality of a client's system on internal controls) can be interpreted as affecting the strength of other audit evidence (an inventory price test). The audit task studied (see Krishnamoorthy et al 1999) involves the valuation of a retail store's inventory account. The context is one where auditors would normally review information on the internal control system for the acquisition and payment cycle and then ultimately collect and assess the results of tests of inventory details such as an inventory price test.

Given the background and environmental information, auditors would first assess the likelihood of material pricing error in the inventory account. This judgment would be the auditor's estimate of material error **prior** to assessing either internal control or price test information. Assume that next an evaluation of internal control system reliability would be obtained. For example, the control system reliability might range on a scale from 50% to 100% (completely reliable). A statistical price test could also be obtained. In statistical audit testing, the diagnosticity of such tests could be specified in terms of the risk of incorrect rejection or of incorrect acceptance. For example, the parameters used in planning the price test sample for the risk of incorrect rejection could be set at 5%, and the risk of incorrect acceptance (i.e. test of details risk) might vary from 2% to 20%. The test results could indicate a book value sufficiently close to the audited value so that either the book value could be accepted or a value that differed from the book value such that the book value would seem to be misstated.

The audit task would then be to reassess the likelihood of material pricing error in the inventory account. This decision represents the auditor's **posterior** belief and should reflect the auditor's evaluation of the evidence pertaining to internal control system reliability aggregated with the evidence pertaining to the price test.

One important issue in such a context is how an auditor should treat the uncertainty confronting him at each decision stage. The way uncertainty is treated has implications on how audit evidence ought to be utilized and thus potentially on audit efficiency and effectiveness.

The belief function model for this setting can be summarized as follows (see Krishnamoorthy et al 1999 for details). First, all possible states for the variables in the model must be defined, i.e., D (~D) = Pricing test reveals *no material error* (*material error*); D* (~D*) = Internal Control System is *reliable* (*not reliable*); and H_1 (H_2) = there is *no material error* (*material error*) in the inventory account. Next, consider frame Θ, a set of mutually exclusive and collectively exhaustive set of all possible combinations of the variables in the model. The evidential value (m-values) of the cues relating to the price tests and the reliability of the internal control system is propagated by vacuously extending the frames Θ_D and Θ_{D*} to Θ, and then marginalizing it to the frame of interest

Θ_H. Vacuous extension is the propagation of m-values from a smaller node (fewer variables) to a larger node (more variables). Marginalization refers to the propagation of m-values from a larger node to a smaller node (Srivastava 1995b, 340). Finally, prior beliefs are updated after a normalization procedure detailed in Shafer (1976). This provides the posterior beliefs on frame Θ_H.

To illustrate the model, assume an auditor's initial belief (prior) for "no material pricing error" is 0.75. But, without either internal control evidence or test of details, significant ambiguity exits as to whether the remaining probability mass should apply to material or no material error. This setting is easily modeled using belief functions as follows: m(D) = 0.75; m_1(~D) = 0 and m(D,~D) = .25.

Assume further that the auditor then learns that the internal control system reliability is 80% and that the test of details results with risk of incorrect acceptance at 2% imply that inventory is correctly valued. The following m-values[12] then become the inputs to the analysis: m_1(~D) = 0.02; m_1(D) = 0.98; $m_1(\Theta_D)$ = 0; m_2(~D*) = 0; m_2(D*) = 0.80; $m_2(\Theta_{D*})$ = 0.20; $m_3(H_1)$ = 0.75; $m_3(H_2)$ = 0; $m_3(\Theta_H)$ = 0.25.

In Krishnamoorthy et al (1999), the auditor's prior was elicited and they then mapped the following values into the belief function model: $m_3(H_1)$ = the elicited prior; $m_3(H_2)$ = 0; and $m_3(\Theta_H)$ = 1- elicited prior. The following posterior beliefs were then associated with the hypotheses of interest:

$m(H_1)$ = 0.945 (belief that there is no material pricing error);

$m(H_2)$ = 0.004 (belief that there is material pricing error); and

[12] Here it is assumed that a 2% risk of incorrect acceptance is equivalent to a belief of 2% to ~D and 80% to D. However, one could also use the Srivastava and Shafer (1994, see also Srivastava 1997b) approach of converting statistical evidence to belief function evidence.

$m(\Theta_H) = 0.051$ (belief in the set consisting of both H_1 and H_2).

The key behavioral result of this analysis is a calculation of a theoretical or hypothesized amount of belief revision given the evidence obtained. In most behavioral research settings, such a calculation is not provided and thus no theoretical benchmark exits with which to compare actual behavior. In the setting described above, the belief revision (benchmark) predicted by a model based on belief functions would be 0.195.

In Krishnamoorthy et al (1999) this formulation is contrasted with 3 other models that also capture the underlying subtleties of the task from different theoretical perspectives. Since the theories underlying the models are different, the measure of the strength of evidence varies across the models.

Several important differences exist between the belief function models and the other models, including descriptive models such as the Belief Revision model developed by Einhorn and Hogerth (1992). For example, there is an explicit parameter in the Einhorn and Hogarth model that allows for sensitivity to negative and positive evidence (α and β). Unlike heuristic models, theoretic models based on probability theory or on the belief function model do not predict differential sensitivity to positive and negative evidence, although it is feasible to include such a parameter in the analytical development of these models (e.g., Dutta and Srivastava 1992). This and other modeling differences are discussed as an avenue for future research in Krishnamoorthy et al. (1999).

Other Studies

There are a number of other studies the interested reader can consult to see further illustrations of the use of belief functions in behavioral auditing and accounting research.

Many of these are listed in our references and are discussed in a forthcoming book (Srivastava and Mock 2000). For example, Srivastava and Mock (1999) have recently used an evidential reasoning approach based on the belief-function framework for WebTrust assurance services (AICPA 1997a, 1997b). They have also applied the decision theoretical approach to estimate a minimum acceptable fee based on the costs associated with various risks of not meeting the objectives with a belief of 1.0.

Srivastava (1996) has used belief functions in value judgments and analyzed the impact of framing of evidence on decision-making. As discussed earlier, Srivastava (1997a) has also used belief functions to model ambiguity and demonstrated its benefits in decision making behavior.

IV. CONCLUSIONS

Ambiguity and uncertainty are common in behavioral auditing and accounting research. However, probability theory does not have a natural, logical way of dealing with ambiguity. One common way of dealing with ambiguity in probability is to assign uniform priors. This leads to some illogical implications, especially in an audit context. An important advantage of a formalism based on belief functions is that it does explicitly and logically deal with ambiguity. As shown, this approach can be applied to auditing and leads to more logical interpretations of audit risk and evidence.

Belief functions also have potential use and research implications for behavioral accounting research in general. As noted, uncertainty and ambiguity exist and should be formally dealt with in managerial, financial and accounting information systems contexts.

A number of research opportunities arise with respect to belief functions, especially in behavioral accounting research. For example, it would be interesting to see whether decision makers tend to think of uncertainty as depicted in the belief-function framework. Also, given that some research has shown that judgments about the level of belief in support of a given assertion is more natural when beliefs function are used, the basic measurement (calibration) issue of the level belief obtained from various types of audit evidence is an important area for future research. A related issue is whether decision makers are able to combine items of evidence according to Dempster's rule of combination. Lastly, there are a number if issues concerning the efficiency and effectiveness of the belief-function framework as compared to other approaches of dealing with uncertainty and ambiguity.

REFERENCES

American Institute of Certified Public Accountants. 1983. *Statement on Auditing Standards, No, 47: Audit Risk and Materiality in Conducting an Audit.* New York: AICPA.

American Institute of Certified Public Accountants. 1997a. *AICPA/CICA WebTrust Principles and Criteria for Business-to-Consumer Electronic Commerce*. http://www.aicpa.org/webtrust/princrit/htm.

American Institute of Certified Public Accountants. 1977b. Report of the Special Committee on Assurance Services. www.aicpa.org.

Akresh, A.D., J.K. Loebbecke, and W.R. Scott. 1988. Audit approaches and techniques. In *Research Opportunities in Auditing: The Second Decade,* edited by A. R. Abdel-khalik and Ira Solomon. Sarasota, FL: AAA, pp. 13-55.

Curley, S. P., and Golden, J.I. 1994. Using belief functions to represent degrees of belief. *Organization Behavior and Human Decision Processes*: 271 – 303.

Dutta, S. K., and R.P. Srivastava. 1992. Belief Revision in Auditing: A Theoretical Investigation. *Proceedings of the 21st Annual Western Decision Science Meeting,* Reno Nevada, March 24-27.

Einhorn, H. J., and Hogarth, R. M. 1992. Order Effects in Belief Updating: The Belief Adjustment Model. *Cognitive Psychology,* **24**: 1-55.

Einhorn, H. J., and Hogarth, R. M. 1986. Decision Making under Ambiguity. *The Journal of Business,* 59(4), Pt. 2 (October), S225-S250.

Gabbay, D. M., and P. Smets. 1998. *Handbook of Defeasible Reasoning and Uncertainty Management Systems.* Kluwer Academic Publishers.

Harrison, K. 1999. Evaluation and Aggregation of Audit Evidence under Uncertainty: An Empirical Study of Belief Functions. Ph.D. Dissertation, School of Business, University of Kansas.

Krishnamoorthy, G., T.J. Mock, and M. Washington. 1999. A Comparative Evaluation of Belief Revision Models in Auditing. *Auditing: A Journal of Practice and Theory,* Vol. 18, pp. 104-126.

Shafer, G. 1976. *A Mathematical Theory of Evidence*. Princeton University Press.

Shafer, G., P.P. Shenoy, and R. P. Srivastava. 1988. Auditor's Assistant: A knowledge engineering tool for audit decisions. *Proceedings of the 1988 Touche*

Ross/University of Kansas Symposium on Auditing Problems. Lawrence, KS: School of Business, University of Kansas, pp. 61-84.

Shafer, G., and R.P. Srivastava. 1990. The bayesian and belief-function formalisms: A general perspective for auditing. *Auditing: A Journal of Practice and Theory* 9 (Supplement), pp. 110-48.

Shafer, G., and A. Tversky. 1985. Languages and Designs for Probability Judgment. *Cognitive Science*, vol. 9, pp. 309-339.

Smets, P. 1998. The Transferable Belief Model For Quantified Belief Representation. *Quantified Representation for Uncertainty and Imprecision*, Vol. 1. Edited by P. Smets. Kluwer Academic Publishers

Smets, P. 1990a. The Combination of Evidence in the Transferable Belief Model. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 12, 5 (May).

Smets, P. 1990b. Constructing the Pignistic Probability Function in a Context of Uncertainty. *Uncertainty in Artificial Intelligence 5.* ed. by Henrion, M., Shachter, R.D., Kanal, L.N., and Lemmer, J.F. North-Holland: Elsevier Science Publishers B.V.

Spohn, W. 1998. Ordinal Conditional functions: A Dynamic Theory of Epistemic States. Causation. *Decision, Belief Changes, and Statistics*, II, edited by W. L. Harper and B. Skyrms. D. Reidel.

Spohn, W. 1990. A General Non-Probabilistic Theory of Inductive Reasoning, *Uncertainty in Artificial Intelligence.* Elsvier Science Publications: 149-158.

Srivastava, R. P. 1997a. Decision Making Under Ambiguity: A Belief-Function Perspective. *Archives of Control Sciences,* Vol. 6 (XLII): 5-27.

Srivastava, R. P. 1997b. Integrating Statistical and Non-Statistical Evidence Using Belief Functions. *Encyclopedia of Computer Science and Technology*, Vol. 37, edited by Allen Kent, James G. Williams, and Carolyn M. Hall, published by Marcel Dekker, Inc., New York (Supplement 22), pp. 157-174.

Srivastava, R. P. 1996. Value Judgments using Belief Functions. *Research in Accounting Ethics,* Vol. 2, pp. 109-130.

Srivastava, R. P. 1995a. A General Scheme for Aggregating Evidence in Auditing: Propagation of Beliefs in Networks. *Artificial Intelligence in Accounting and Auditing,* Vol. 3, Miklos A. Vasarhelyi, editor, Markus Wiener Publishers, Princeton: 55-99.

Srivastava, R. P. 1995b. The Belief-Function Approach to Aggregating Audit Evidence" *International Journal of Intelligent Systems* (March): 329-356.

Srivastava, R. P. 1993. Belief Functions and Audit Decisions. *Auditors Report,* Vol. 17, No. 1, (Fall): 8-12.

Srivastava, R. P., S. Dutta, and R. Johns. 1996. An Expert System Approach to Audit Planning and Evaluation in the Belief-Function Framework," *International Journal of Intelligent Systems in Accounting, Finance and Management*, Vol. 5, No. 3: 165-183.

Srivastava, R. P., and T. Mock. 2000. *Belief Functions in Business Decisions,* Physica-Verlag, Heidelberg, Springer-Verlag Company (forthcoming).

Srivastava, R. P., and T. Mock. 1999. Evidential Reasoning for WebTrust Assurance Services. *Journal of Management Information Systems* (forthcoming).

Srivastava, R. P., and G. Shafer. 1994. Integrating Statistical and Non-Statistical Audit Evidence Using Belief Functions: A Case of Variable Sampling. *International Journal of Intelligent Systems,* Vol. 9: 519-539.

Srivastava, R. P. and G. Shafer. 1992. Belief-Function Formulas for Audit Risk. *The Accounting Review,* Vol. 67, No. 2 (April): 249-283.

Wong, S.K.M., and Wang, Z.W. 1993. Qualitative Measures of Ambiguity. *Proceedings of The Ninth Conference on Uncertainty in Artificial Intelligence*. ed. by Hackerman, Davis and Mamdani, Abe. San Mateo, California: Morgan Kaufmann: 443-450.

Yager, R.R.; Kacprzyk, J.; and Fedrizzi, M. 1994. *Advances in the Dempster-Shafer Theory of Evidence*. New York, NY: John Wiley and Sons: 71-95.

Zadeh, L. A. 1978. Fuzzy sets as a Basis for a Theory of Probability. *Fuzzy Sets and Systems,* 1: 3-28.

Zadeh, L. A. 1979. A Theory of Approximate Reasoning. *Machine Intelligence,* edited by J. E. Ayes, D. Mitchie andn L. I. Mikulich. Chichester, UK: Ellis Horwood.

Figure 1

Compatibility Relationships Between Two Frames, {t, ~t} and {c, ~c}.