THEORETICAL INVESTIGATION OF BELIEF REVISIONS IN AUDITING

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Abstract. In this paper we examine the empirical findings of belief revision under two alternatives: the Bayesian framework and the Dempster-Shafer theory of belief functions. Bayesian theory is very stringent in its requirement that the probability of mutually exclusive and collectively exhaustive events sum to one. It requires that the belief be increased in light of supporting evidence, and be decreased in light of conflicting evidence. These results are consistent with the empirical findings. However, the Bayesian analysis entails the largest increase (decrease) in belief for medium priors in light of supporting (conflicting) evidence. This is contrary to empirical findings. The largest increase was observed for smallest priors when presented with supporting evidence. The largest decrease was observed for highest priors when presented with conflicting evidence. Further, Bayesian theory fails to explain the 'recency' and the 'dilution' effects.

In the belief-function formalism, the belief in a proposition increases in light of supporting (positive) evidence. Further, the largest increase is for the lowest priors. This is consistent with the empirical findings. The belief in a proposition decreases in light of conflicting (negative) evidence and the largest decrease is for medium priors. This is not totally consistent with the empirical findings. However, with discounting we can attain the largest decrease for the highest prior, consistent with the empirical findings. By incorporating discounting in belief functions, we are able to model the 'recency' and the 'dilution' effects.

Key Words: Belief Functions, Bayesian, Audit Evidence, Aggregation, Belief Revision.

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1. INTRODUCTION

The purpose of this paper is to investigate theoretically the belief revision process in auditing using two approaches, Bayes' theorem and the Dempster-Shafer theory of *belief-function*[1], and to compare and contrast the findings with the empirical results of Ashton and Ashton (1988) and Tubbs et al. (1990). Both the approaches considered here are based on the mathematical theory of probability. Bayesian formalismis based on additive probabilities, whereas the *belief-function* formalism is based on non-additive *beliefs* (see Shafer 1976; and Shafer and Srivastava 1990 for details).

Recent studies on auditors' behavior in aggregating evidence have furnished some very interesting results (Ashton and Ashton 1988; Tubbs et al. 1990). First, in the case of positive evidence, auditors revise their belief the most for the lowest prior (anchor). Second, in the case of conflicting evidence, auditors revise their belief the most for the highest prior. Third, in the case of mixed evidence, auditors put more credence on the recent piece of evidence. Fourth, for a given set of evidence, an auditor's belief revision varies with how the items of evidence are processed, i.e., there is a significant difference between the belief revisions when the items of evidence are presented simultaneously versus sequentially.

The Bayesian approach fails to model some of the empirical findings. In contrast, the Dempster-Shafer theory of *belief* functions models most of the findings. The Bayesian theory suggests that the belief be increased in light of positive evidence, and be decreased in light of conflicting evidence. This is consistent with the empirical findings. However, the Bayesian analysis entails the largest increase (decrease) in belief for medium priors in light of supporting (conflicting) evidence. This is contrary to the empirical findings: The largest increase (decrease) was observed for the smallest (highest) priors when presented with supporting (conflicting) evidence. Also, in the Bayesian analysis, the extent of belief revision is independent of the order in which the evidence is presented. Thus, the Bayesian formalism is unable to model the recency effect. The mode of presentation of evidence, sequential versus simultaneous, does not affect the extent of belief revision in Bayesian formalism. Thus, the dilution effect cannot be explained by Bayesian formalism.

The *belief*-function formalism, on the other hand, is able to model the behavior of auditors. In *belief* functions, the *belief* in a proposition increases in light of supporting (positive) evidence. Further, the largest increase is for the lowest priors. This is consistent with the empirical findings. The *belief* decreases when a conflicting item of evidence is presented. The decrease in *belief* is the most for the medium prior. This is inconsistent with the empirical findings. However, with discounted *belief* functions, the largest decrease for the highest priors can be attained.

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[1] The word "belief" in italics means the level of support in belief-function formalism, otherwise it represents the generic meaning as used by Ashton and Ashton (1988).

In this paper, the concept of discounted *belief* functions is applied to model an auditor's behavior in aggregating evidence. Belief functions have previously been employed in an audit setting to aggregate audit evidence (Srivastava 1995; Srivastava, Dutta and Johns 1996). However, there has not been any empirical justification provided that the auditor's revision of beliefs are in accordance to the prescription of *belief* functions. Though, auditors belief revision process has been studied, no attempt has been made to analyze these findings on the basis of normative theories of belief revision. Here we are attempting to model the belief revision proces using *belief* functions.

As mentioned earlier, our objective here is to make an attempt to model the empirical findings regarding the auditor's behavior in aggregating evidence by using Bayes' theorem and the *belief*-function framework. We will assume here that the reader has basic familiarity with the Bayesian and *belief*-function formalisms (for details see Shafer 1976; Shafer and Srivastava 1990).

The rest of the paper is divided into five sections. Section II presents belief revisions with positive and negative items of evidence using both the Bayesian and the belief -function formalisms. In Section III, belief revisions with mixed evidence are discussed using discounted belief functions. A discussion on simultaneous versus sequential aggregation of evidence is presented in Section IV. Summary and conclusions are presented in Section V.

2. EFFECT OF PRIOR BELIEF ON BELIEF REVISION

2.1 Belief Revision with Consistent Positive Items of Evidence

Ashton and Ashton (1988) found that belief increases with additional positive evidence. Furthermore, the study showed that the maximum increase was for the lowest priors. That is, when presented with positive evidence, the subjects who had lower anchors (smaller priors) revised their beliefs the most. Their study also showed that there was no order effect with consistent positive evidence, i.e., changing the order of presentation of the same set of evidence had no effect on the extent of revision. In this section we will analyze these findings from both, the Bayesian as well as the belief function perspectives.

<u>*Bayesian Perspective*</u>

In Bayesian formalism, the prior belief, $\mathbf{P}(A)$, in an event A is updated when an evidence E is obtained. This updated belief is called the posterior belief and is represented by $\mathbf{P}(A|E)$. Bayes' theorem is used to determine the posterior belief:

$$\mathbf{P}(A|E) = \mathbf{P}(E|A)\mathbf{P}(A)/[\mathbf{P}(E|A)\mathbf{P}(A) + \mathbf{P}(E|\sim A)\mathbf{P}(\sim A)] \qquad (1)$$

The likelihood, $\mathbf{P}(E|A)$, is the probability of an observation E, given that the event A has occurred. The likelihoods, $\mathbf{P}(E|A)$ and $\mathbf{P}(E|\sim A)$, depend on the audit procedure and should be known to the auditor. The likelihoods are independent of the population characteristics, whereas the prior belief or anchor $\mathbf{P}(A)$ depends on the population and is an estimate of the auditor.

An item of evidence is positive if our belief in A increases after we have observed E, i.e., when $\mathbf{P}(A|E) > \mathbf{P}(A)$ (Toba, 1975; Kissinger, 1977). This was shown to be equivalent to $P(E|A) > P(E|\sim A)$ (Dutta and Srivastava 1993). And, an item of evidence is negative if our belief in A decreases after we have observed E, i.e., when **P**(A|E) < **P**(A), or equivalently P(E|A) < P(E|~A). If we observe E and if E implies A then we are sure that A is true, that is, if E implies A then **P**(A|E) = 1 and **P**(~A|E) = 0. This type of evidence is known as categorical positive evidence (Dutta and Srivastava 1996). Similarly, if E is a categorical negative evidence, that is, if E implies ~A and if we observe E then A is false, i.e., **P**(A|E) = 0 and **P**(~A|E) = 1. Thus, it is obvious that for categorical positive evidence, the lower the prior the more the increase. And similarly, for categorical negative evidence, the higher the prior the more the decrease.

TABLE 1
Bayesian Probability Revision

Prior	Evidence E1		Posterior	Evidence E2		Posterior	Increase
(Anchor)	P(E1|A)	P(E1|~A)	P(A|E1)	P(E2|A)	P(E2|~A)	P(A|E1&E2)	Probability
0.2	0.6	0.10	0.600	0.800	0.10	0.923	0.723
0.5	0.6	0.10	0.857	0.800	0.10	0.980	0.480
0.8	0.6	0.10	0.960	0.800	0.10	0.995	0.195
0.2	0.8	0.10	0.667	0.600	0.10	0.923	0.723
0.5	0.8	0.10	0.889	0.600	0.10	0.980	0.480
0.8	0.8	0.10	0.970	0.600	0.10	0.995	0.195
0.2	0.4	0.30	0.250	0.600	0.40	0.333	0.133
0.5	0.4	0.30	0.571	0.600	0.40	0.667	0.167
0.8	0.4	0.30	0.842	0.600	0.40	0.889	0.089
0.2	0.6	0.40	0.273	0.400	0.30	0.333	0.133
0.5	0.6	0.40	0.600	0.400	0.30	0.667	0.167
0.8	0.6	0.40	0.857	0.400	0.30	0.889	0.089

Empirical studies have focused on the amount of increase (decrease) in belief for different priors given the same item of evidence. The results of combining the same items of evidence with different priors are shown in Table 1. These items of evidence are all positive because the posterior probabilities for A in the two cases are greater than the prior probabilities for A. We have used three different priors, 0.2, 0.5 and 0.8, representing weak, moderate and strong priors, respectively, in our calculation. It is observed that the largest increase in belief is for the lowest prior (see rows 1 and 4 in Table 1). Also, the increase in the posterior belief is larger for the stronger evidence as seen from comparing rows 1 and 7 of Table 1. These results are consistent with the findings of Ashton and Ashton (1988). Further, the ordering of the evidence has no effect on the extent of belief revision as evident from comparing rows 1-3 with row 4-6 in Table 1. This finding is also consistent with the empirical results.

However, it is interesting to find that the increase in the revised belief peaks at an intermediate value of the prior under Bayesian formalism as seen from Figure 1 and also from rows 8 and 11 of Table 1. The peak occurs at a prior[2]:

$$\mathbf{P}(A)_{max} = [\{\mathbf{P}(E|A).\mathbf{P}(E|\sim A)\}^{1/2} - \mathbf{P}(E|\sim A)]/[\mathbf{P}(E|A) - \mathbf{P}(E|\sim A)] \qquad (2)$$

As evident from Equation (2), $\mathbf{P}(A)_{max}$ value depends on the strength of evidence, [**P**(E|A) - **P**(E|~A)]. For example, for a strong positive evidence, i.e., for a large value of [**P**(E|A) - **P**(E|~A)], the increase in the prior belief is the highest at a low value of **P**(A) as seen from Case 3 in Figure 1. Whereas, for a weak evidence, i.e., for a small value of [**P**(E|A) - **P**(E|~A)], the peak occurs at a medium value of **P**(A) as seen from Case 1 in Figure 1. Such a phenomenon has not been observed empirically.

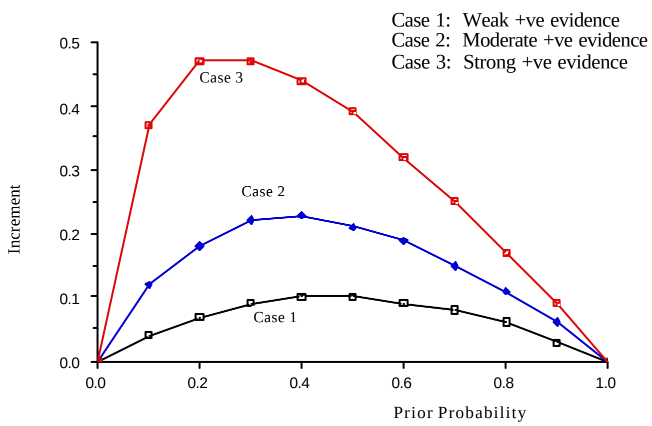

Figure 1 The Difference Between Posterior and Prior Probabilities, P(A|E) - P(A), as a Function of Prior Probability, P(A). Case 1: P(E|A) = .6 and P(E|~A) = .4. Case 2: P(E|A) = .5 and P(E|~A) = .2. Case 3: P(E|A) = .8 and P(E|~A) = .1.

Belief Function Perspective

In *belief-function* formalism two items of evidence are combined using Dempster's rule (Shafer 1976). Here, a general case of combining corroborating evidence is illustrated.

Without loss of generality, let m_o be the prior *belief*[3] in proposition A, i.e., $\mathbf{Bel}_0[A] = m_0$, and assume that no prior is assigned to ~A, i.e., $\mathbf{Bel}_o[\sim A] = 0$. Also, let m_1 be

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[2] The increase in the posterior can be written as:

$$\mathbf{P}(A|E) - \mathbf{P}(A) = \mathbf{P}(E|A)\mathbf{P}(A)/[\mathbf{P}(E|A)\mathbf{P}(A) + \mathbf{P}(E|\sim A)\mathbf{P}(\sim A)] - \mathbf{P}(A).$$

The value of **P**(A) for which the above increase in the prior is maximum is obtained by differentiating the increase with respect to **P**(A) and equating the resulting expression to zero.

the *belief* obtained from the new piece of evidence, i.e., $\mathbf{Bel}_1[A] = m_1$, and no *belief* for negation of proposition A, i.e., $\mathbf{Bel}_1[\sim A] = 0$. The combined *belief* in A using Dempster's rule is $\mathbf{Bel}[A] = m_0 + m_1 - m_0 m_1$. Thus, the increase in *belief* due to the new item of evidence is given by:

$\mathbf{Bel}[A] - \mathbf{Bel}_0[A] = (m_0 + m_1 - m_0 m_1) - m_0 = m_1(1 - m_0)$

From the above expression it can be seen that the increase in the revised *belief* is higher for lower prior *belief*s. This is consistent with the empirical findings of Ashton and Ashton (1988). Table 2 and Figure 2 both clearly show this effect.

TABLE 2
Belief Revision with Positive Evidence

Prior Belief in A (Anchor) Belo[A]	Support for A from 1st Evidence Bel1[A]	Revised Belief in A	Support for A from 2nd Evidence Bel2[A]	Revised Belief in A	Support for A from 3rd Evidence Bel3[A]	Revised Belief in A	Support for A from 4th Evidence Bel4[A]	Revised Belief in A	Total increase in Belief in A
0.2	0.3	0.44	0.4	0.66	0.6	0.87	0.7	0.96	0.760
0.5	0.3	0.65	0.4	0.79	0.6	0.92	0.7	0.97	0.475
0.8	0.3	0.86	0.4	0.92	0.6	0.97	0.7	0.99	0.190
0.2	0.7	0.76	0.6	0.90	0.4	0.94	0.3	0.96	0.760
0.5	0.7	0.85	0.6	0.94	0.4	0.96	0.3	0.97	0.475
0.8	0.7	0.94	0.6	0.98	0.4	0.99	0.3	0.99	0.190
0.2	0.7	0.76	0.3	0.83	0.4	0.90	0.6	0.96	0.760
0.5	0.7	0.85	0.3	0.90	0.4	0.94	0.6	0.97	0.475
0.8	0.7	0.94	0.3	0.96	0.4	0.97	0.6	0.99	0.190

Table 2 presents the computations for belief revision employing *belief-function* formalism with three priors, 0.2, 0.5 and 0.8 representing low, medium, and high anchors, respectively. Figure 2 represents a plot of rows 1-3 of Table 2. As one can see from comparing rows 1, 2, and 3, as well as, rows 7, 8 and 9 of Table 2 that the greatest revision is observed for the lowest prior. Also, the order in which different items of evidence are aggregated does not affect the revised *belief* as seen by comparing rows 1-3 with rows 4-6, and rows 7-9 with rows 10-12 in Table 2. These results are in full agreement with Ashton and Ashton's findings. However, the results using the Bayesian formalism were not in full agreement with the empirical findings.

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[3] For the sake of exposition the degree of support is represented as an **m**-function. Here all the examples consist of only two possible states, say A and ~A, thus the **m**-value for any proposition is also equal to the **Bel**-function for the proposition. For example, **Bel**(A) = **m**(A) and **Bel**(~A) = **m**(~A). By definition **Bel**(∅) = 0 where ∅ is an empty set, and **Bel**(A,~A) = 1.

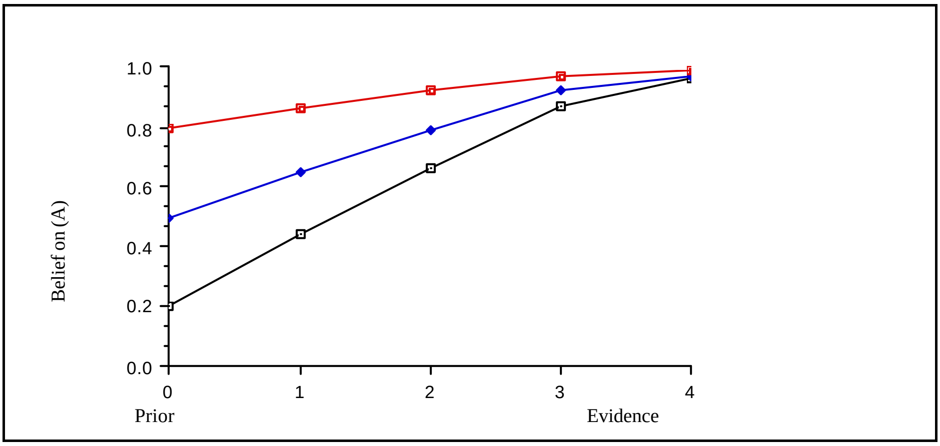

Figure 2 Belief Revision with Consistent Positive Evidence using *Belief-Function* Formalism for Three Priors, $\mathbf{Bel}_0(A)$ = 0.2, 0.5 and 0.8. The **m**-values for the Four Items Evidence are: $\mathbf{m}_1(A) = 0.3$,and $\mathbf{m}_1(\sim A) = 0$; $\mathbf{m}_2(A) = 0.4$, and $\mathbf{m}_2(\sim A)=0$; $\mathbf{m}_3(A)= 0.6$, and $\mathbf{m}_3(\sim A) = 0$; and $\mathbf{m}_4(A) = 0.7$ and $\mathbf{m}_4(\sim A)=0$.

3.2 Belief Revision with Consistent Negative Items of Evidence

The empirical findings indicate that the subjects decreased their belief when they were furnished with a negative (conflicting) item of evidence. Furthermore, in light of negative evidence the largest anchors were affected the most, i.e., the decrease in belief was the highest for the highest prior belief. The order of presentation of the evidence again had no effect on the extent of revision as observed in the case of consistent positive evidence. Again, we will analyze these findings from both the Bayesian as well as the *belief-function* perspectives.

Bayesian Perspective

In Bayesian formalism, an item of evidence is negative if $\mathbf{P}(A|E) < \mathbf{P}(A)$. The decrease in belief, $\mathbf{P}(A|E) - \mathbf{P}(A)$, is plotted in Figure 3 against the prior belief $\mathbf{P}(A)$. It shows that the decrease in belief peaks at an intermediate value of the prior belief which is similar to the result obtained earlier for a positive item of evidence under Bayesian formalism.

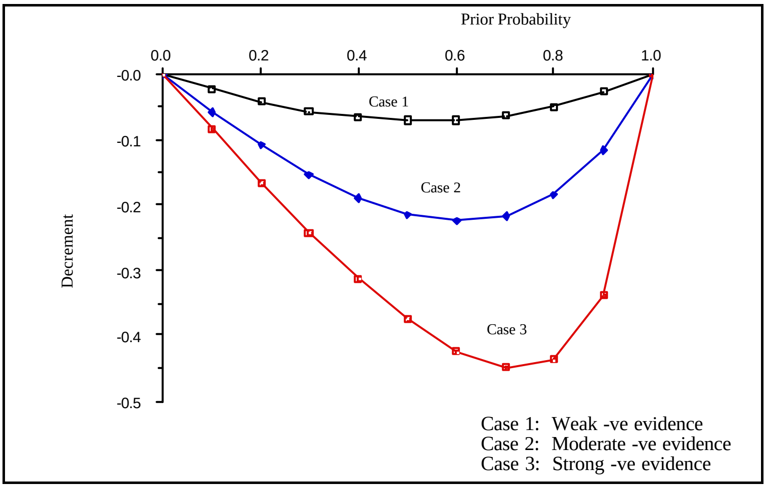

Figure 3. Belief Revision with Negative Evidence using Bayesian Formalism. Difference Between Posterior and Prior Probabilities, **P**(A|E) - **P**(A), as a Function of Prior Probability, **P**(A). Case 1: **P**(E|A) = .3 and **P**(E|~A) = .4. Case 2: **P**(E|A) = .2 and P(E|~A) = .5. Case 3: **P**(E|A) = .1 and **P**(E|~A) = .6.

It is clear from Figure 3 that the decrease in belief is the highest for the medium prior belief using Bayesian formalism, contrary to the empirical findings (Ashton & Ashton 1988). However, it can be easily shown that the order of presentation of the evidence under Bayesian formalism will not have any effect on the extent of revision. This result is in agreement with the empirical findings of Ashton & Ashton.

Belief Function Perspective

In general, the largest decrease in the revised belief is not for the highest prior but for a certain intermediate value of the prior *belief*. This value depends on the strength of evidence. For a weak evidence, i.e., for a small value of $\mathbf{m}_1$[~A], the largest decrease in the revised belief occurs around a prior *belief* of 50%. Whereas, for a strong negative evidence, i.e., for a large value of $\mathbf{m}_1$[~A], the decrease in *belief* peaks at a large value of the prior *belief*. In fact, one can obtain an analytical expression[4] for which the decrease is the largest at a prior *belief*:

$$\mathbf{Bel}_0[A] = [\{1 - (1 - \mathbf{m}_1(\sim A))^{0.5}\}/\mathbf{m}_1(\sim A)]. \qquad (3)$$

[4] The decrease in the revised *belief* in A can be written as:

$$\mathbf{Bel}_0[A] - \mathbf{Bel}[A] = m_0 - m_0(1-m_1)/(1 - m_0 m_1).$$

The value of the prior belief, m_0, at which the decrease is maximum is obtained by differentiating the decrease with respect to m_0 and equating the resulting expression to zero.

Thus, it is seen that like Bayesian formalism, *belief-function* formalism does not explain the empirical findings regarding belief revision under consistent negative items of evidence.

TABLE 3

Belief Revision with Negative Evidence using Discounting.

Prior Belief in A	Evidence 1: Support for ~A	Level of Conflict*	Discounted Prior in A	Revised Belief in A	Revised Belief in ~A	Decrease in Belief
0.2	0.1	0.02	0.1976	0.18	0.08	0.02
0.5	0.1	0.05	0.485	0.46	0.05	0.04
0.8	0.1	0.08	0.7616	0.74	0.03	0.06
0.2	0.3	0.06	0.1928	0.14	0.26	0.06
0.5	0.3	0.15	0.455	0.37	0.19	0.13
0.8	0.3	0.24	0.6848	0.60	0.12	0.20
0.2	0.6	0.12	0.1856	0.08	0.55	0.12
0.5	0.6	0.3	0.41	0.22	0.47	0.28
0.8	0.6	0.48	0.5696	0.35	0.39	0.45

*The level of conflict is defined to be the probability mass assigned to the impossible outcome.

** A discount rate of 60% of the level of conflict is used in the table.

However, if we discount the prior *belief* as shown in Table 3, the decrease in the revised *belief* becomes the largest for the highest prior[5] as observed empirically. Thus, the Dempster-Shafer theory of *belief function* with discounting can be used to explain the empirical findings. The discounting of prior *belief* makes more intuitive sense, especially in situations with large positive priors and many negative items of evidence.

3. BELIEF REVISION WITH MIXED EVIDENCE

In this section we consider aggregation of mixed evidence. Mixed evidence implies that one item of evidence supports a hypothesis, while the other rejects the hypothesis. It is interesting to note that the order of presentation of the evidence in a mixed evidence situation affects the belief revision process. In fact, it has been observed that more weight or credence is attached by the auditor to the latest

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[5] The decrease in the revised *belief* in A can be written as:

$$\mathbf{Bel}_0[A] - \mathbf{Bel}[A] = m_0 - (1-k.m_o m_1)m_0(1-m_1)/(1 - (1-k.m_o m_1)m_0 m_1).$$

The above expression is differentiated with respect to m_0. The resulting expression is positive for all values of m_1, provided k >0.5. Thus, $\mathbf{Bel}_0[A] - \mathbf{Bel}[A]$, is a monotonically increasing function with respect to m_0. The maximum decrease is thus attained for the highest value of m_0, provided the discount rate is not too small.

information (e.g., see Ashton & Ashton 1988; Tubbs et. al. 1990). This effect has been called the 'recency' effect in the accounting literature.

Bayesian theory cannot explain the recency effect, neither can *belief-function* formalism when applied without discounting. However, when previous items of evidence are *discounted* in light of new evidence, *belief-function* formalism does explain the recency effect.

In Table 4, we present the results of combining mixed items of evidence using *belief-function* formalism with discounting. However, we would like to discuss the essential steps involved in calculating the final *beliefs* in Table 4 in detail and provide the logical reasoning behind it. First, we assume here that the discounting rate is proportional to the probability mass assigned to the impossible state. Let us consider row one of Table 4(a) for our illustration. The prior probability mass (**m**-values) assigned to A, ~A and {A,~A} for this case are: $\mathbf{m}_0$(A) = 0.2, $\mathbf{m}_0$(~A) = 0, and $\mathbf{m}_0$({A,~A}) = 0.8. The **m**-values obtained from the first evidence in row one and column two of Table 4(a) are: $\mathbf{m}_1$(A) = 0.4, $\mathbf{m}_1$(~A) = 0, and $\mathbf{m}_1$({A,~A}) = 0.6. These two items of evidence are not conflicting, i.e., when we multiply the two sets of **m**-value we do not get any non-zero product for the state (A&~A = ∅). Since we are assuming the discounting rate to be proportional to the level of conflict, the discounting rate is zero as there is no conflict.

TABLE 4

Belief Revision with Mixed Evidence: Recency Effect by Discounting

a) Sequencing of Evidence (+,+,-,-)

Prior Belo[A]	Evidence Bel1[A]	Revised Belief	Evidence Bel2[A]	Revised Belief	Evidence Bel3[~A]	Revised Belief	Evidence Bel4[~A]	Revised Belief	Increase in Belief
0.20	0.40	0.52	0.60	0.81	0.50	0.44	0.30	0.32	0.12
0.50	0.40	0.70	0.60	0.88	0.50	0.48	0.30	0.35	-0.15
0.80	0.40	0.88	0.60	0.95	0.50	0.52	0.30	0.38	-0.42

b) Sequencing of Evidence (-,-,+,+)

Prior Belo[A]	Evidence Bel1[~A]	Revised Belief	Evidence Bel2[~A]	Revised Belief	Evidence Bel3[A]	Revised Belief	Evidence Bel4[A]	Revised Belief	Increase in Beleif
0.20	0.50	0.05	0.30	0.04	0.40	0.23	0.60	0.59	0.39
0.50	0.50	0.19	0.30	0.14	0.40	0.33	0.60	0.65	0.15
0.80	0.50	0.47	0.30	0.35	0.40	0.52	0.60	0.77	-0.03

N.B.- A discount rate of 0.6 was used in the computations.

Using Dempster's rule (Shafer 1976) to combine these two items of evidence, one obtains the following **m**-values: **m**(A) = 0.52, **m**(~A) = 0, and **m**({A,~A}) = 0.48 and the corresponding *beliefs* are: **Bel**[A] = 0.52, and **Bel**[~A] = 0, **Bel**[{A,~A}] = 1. Column 3 of Table 4(a) represents the *belief* in A computed at this stage. The second item of evidence is also not conflicting to the previous two. The **m** values for the second evidence are given as: $\mathbf{m}_2$(A) = 0.6, $\mathbf{m}_2$(~A) = 0, and $\mathbf{m}_2$({A,~A}) = 0.4. This evidence when combined with the first two items then yields the following new set of **m**-values: $\mathbf{m}'(A) = 0.81$, $\mathbf{m}'(\sim A) = 0$, and $\mathbf{m}'(\{A,\sim A\}) = 0.19$. The **m**-values for the third evidence are: $\mathbf{m}_3(A) = 0$, $\mathbf{m}_3(\sim A) = 0.5$, and $\mathbf{m}_3(\{A,\sim A\}) = 0.5$. This evidence is conflicting to all the previous items of evidence. The level of conflict in this case is 0.405 (the level of conflict = probability mass associated with the impossible state ($A\&\sim A = \emptyset$) which is equal to $\mathbf{m}'(A) x \mathbf{m}_3(\sim A) = 0.81x\ 0.5 = 0.405$).

In this situation, before the the third item is combined with the previous two, the previous *beliefs* will be discounted. Just to illustrate the discounting process we assume here that the dis count rate, δ, is equal to 60% of the level of conflict, i.e., in this case, $\delta = 0.6x0.405 = 0.243$. The reason for making δ dependent on the level of conflict is intuitive, since the decision maker will be more prone to reevaluating the previous items of evidence if he encounters a new piece of evidence that is conflicting. The higher the conflict the bigger the concern. The discounted *beliefs*, $\mathbf{m}_\delta'$, are obtained by multiplying each **m**-values with $(1 - \delta)$ and adding δ to the **m**-value for the frame $\{A,\sim A\}$:

$$\mathbf{m}_\delta'(A) = (1 - 0.243)x0.81 = 0.613,\ \mathbf{m}'(\sim A) = 0, \text{ and}$$

$$\mathbf{m}'(\{A,\sim A\}) = (1 - 0.243)x0.19 + 0.243 = 0.387$$

These discounted *beliefs* are now combined with $\mathbf{m}_3$'s resulting into a new set of **m**-values:

$$\mathbf{m}''(A) = 0.442,\ \mathbf{m}''(\sim A) = 0.279, \text{ and } \mathbf{m}''(\{A,\sim A\}) = 0.279.$$

The corresponding *belief* in A is equal to 0.442 as shown in row one and column seven of Table 4(a).

The fourth item of evidence is a weak negative item for which the **m** values are given by:

$$\mathbf{m}_4(A) = 0,\ \mathbf{m}_4(\sim A) = 0.3, \text{ and } \mathbf{m}_4(\{A,\sim A\}) = 0.7$$

This evidence is in conflict with the aggregate evidence evaluated so far. We will discount this aggregate evidence and then combine that with the fourth item of evidence. The level of conflict, in this case, is 0.1326 (= $\mathbf{m}''(A) x \mathbf{m}_4(\sim A) = 0.442x0.3 = 0.1326$). Thus, the dis count rate is 0.07956 ($\delta = 0.6x0.1326$). The discounted *beliefs* are given in terms of **m**-values as: $\mathbf{m}'''(A) = 0.4068$, $\mathbf{m}'''(\sim A) = 0.2568$, and $\mathbf{m}'''(\{A,\sim A\}) = 0.3364$.

Combining the above **m** functions with $\mathbf{m}_4$'s one obtains the final **m** values as:

$$\mathbf{m}_f(A) = 0.3243,\ \mathbf{m}_f(\sim A) = 0.4074, \text{ and } \mathbf{m}_f(\{A,\sim A\}) = 0.2683.$$

The overall *belief* in A is 0.3243 which is given in row one and column 9 of Table 4(a). Similarly, all the other numbers in Tables 4(a) and 4(b) can be calculated. The results of Table 4 are plotted in Figure 4.

The results in Table 4 and Figure 4 indicate that, in a mixed evidence situation, the extent of belief revision is dependent upon the order in which the items of evidence are presented, provided the previous evidence is discounted. Thus, it appears that the Dempster-Shafer theory of *belief function* with discounting can be used to model the recency effect.

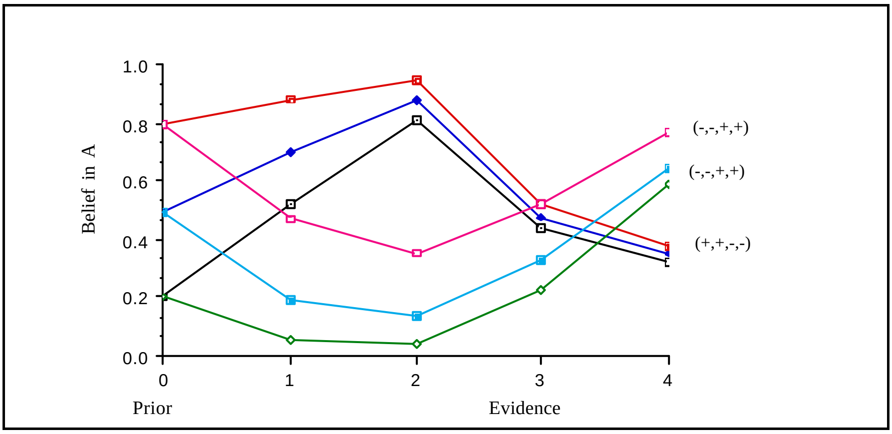

Figure 4 Belief Revision under Mixed Items of Evidence using *Belief-Function* Formalism with Discounting. Data used in the Graph are Given in Table 8.

4. SIMULTANEOUS VERSUS SEQUENTIAL AGGREGATION OF EVIDENCE

In this section, we examine the 'dilution' effect. The empirical studies revealed that the belief revisions were diluted for simultaneous processing. The simultaneous evaluation of evidence resulted in a less extreme belief change than the sequential processing of the same set of evidence. The effect was more pronounced for consistently negative evidence (Ashton & Ashton 1988).

Bayesian theory cannot explain the dilution effect, neither can *belief functions* without discounting. However, when items of evidence are discounted, in light of conflict, the *belief-function* formalism does explain the dilution effect.

In the previous section, we discussed discounting when mixed items of evidence were presented sequentially. The previous items of evidence were discounted in light of new evidence. Discounting was done in favor of the latest information.

In case of simultaneous processing, a uniform discounting of all items of evidence seems more reasonable. Unlike sequential processing, there are no previous or later items of evidence, since all items of evidence are presented simultaneously. Thus, when there is conflict between different items of evidence, all items of evidence are discounted uniformly.

The discounting methods could be different in both cases. While in the sequential processing the discount rate is biased by the last piece of evidence, there is no bias in simultaneous processing. One would expect an averaging, or uniform discounting, when conflicting items of evidence are presented simultaneously. Thus, there is no order effect and the revision of belief is less pronounced compared to sequential evaluation.

In Table 5 (a), we present the results of simultaneously combining negative items of evidence using *belief function* with discounting. Since all items of evidence are simultaneously presented, a uniform discounting is applied to all the items. Here we will illustrate with an example the mechanism of uniform discounting. Let us consider row 1 of Table 5(a) for an illustration. The prior probability mass (**m**-value) assigned to A, ~A and (A,~A) for this case are: $\mathbf{m}_o$(A) = 0.2; $\mathbf{m}_o$(~A) = 0; and $\mathbf{m}_o$(A,~A) = 0.8. The **m**-values obtained from evidence 1 and 2 are, $\mathbf{m}_1$(A) = 0, $\mathbf{m}_1$(~A) = 0.3, $\mathbf{m}_1$(A,~A) = 0.7, $\mathbf{m}_2$(A) = 0, $\mathbf{m}_2$(~A) = 0.25 and $\mathbf{m}_2$(A,~A) = 0.75, respectively. We first compute the level of conflict of all the items of evidence. In this case the level of conflict is 0.095[6]. We assume that the discounting rate, δ, is 60% of the level of conflict. Thus, δ = 0.6X0.095 = 0.057. The discounted **m**-values are obtained by multiplying each **m**-value by a factor (1-δ) and adding a δ degree of belief to the **m**-value for the entire frame. We thus obtain the dis counted **m**-values of 0.189, 0.283 and 0.236, for the prior, evidence 1 and evidence 2, respectively. The overall revised belief in A is obtained by combining the discounted **m**-values (columns 6-8 of Table 5a) using Dempster's rule of combination. The resultant belief in A, **m**(A) = **Bel**(A)=0.113.

TABLE 5

Simultaneous Aggregation of Evidence with Uniform Discounting

a) Consistent Negative Evidence

Prior Bel0(A)	Evidence m1(~A)	Evidence m2(~A)	Conflict	Discount Rate	Discounted Prior	Discounted Evidence1	Discounted Evidence 2	Revised Belief	Decrease in Belief
0.20	0.30	0.25	0.095	0.057	0.189	0.283	0.236	0.113	0.087
0.50	0.30	0.25	0.238	0.143	0.429	0.257	0.214	0.305	0.195
0.80	0.30	0.25	0.380	0.228	0.618	0.232	0.193	0.500	0.300
0.20	0.50	0.20	0.120	0.072	0.186	0.464	0.186	0.090	0.110
0.50	0.50	0.20	0.300	0.180	0.410	0.410	0.164	0.255	0.245
0.80	0.50	0.20	0.480	0.288	0.570	0.356	0.142	0.422	0.378

6 In general, the level of conflict = $\Sigma\{\mathbf{m}_o(B)\mathbf{m}_1(C)\mathbf{m}_2(D)|B\cap C\cap D = \emptyset\}$ where B, C, and D represent subsets of the elements of the frame {A,~A}. The nonzero terms of the above expression yields:

Level of conflict = $\mathbf{m}_o(A)\mathbf{m}_1(\sim A)\mathbf{m}_2(\sim A)+\mathbf{m}_o(A)\mathbf{m}_1(\sim A)\mathbf{m}_2(\{A,\sim A\})$ + $\mathbf{m}_o(A)\mathbf{m}_1(\{A,\sim A\})\mathbf{m}_2(\sim A)$

= 0.2 x0.3x0.25 + 0.2x0.3x0.75 + 0.2 x 0.7 x 0.25 = 0.095

b) Mixed Evidence

Prior Belief	Evidence on (A)*	Evidence on (~A)**	Conflict	Discount Rate	Discounted Prior	Discounted Evidence	Discounted Evidence	Revised Belief	Change in Belief
0.20	0.76	0.65	0.525	0.315	0.137	0.521	0.445	0.440	0.240
0.50	0.76	0.65	0.572	0.343	0.328	0.499	0.427	0.531	0.031
0.80	0.76	0.65	0.619	0.371	0.503	0.478	0.409	0.628	-0.172

N.B.- A discount rate of 0.6 of the level of conflict was used.

* The number is obtained by combining two positive items of evidence in Table 4(a) (columns 2 & 4)

** The number is obtained by combining two negative items of evidence in Table 4(a) (columns 6 & 8).

Simultaneous processing of mixed evidence results in less extreme belief changes. This can be verified by comparing the results of Table 5(b)with those of Tables 4(a) and (b). In Table 4(a) and (b), we showed that the extent of revision was dependent on the order in which the items of evidence were presented. For example, the anchor of 0.5 was revised to 0.35 when the items of evidence were presented in +,+,-,- sequence (row 2, col. 9, Table 4a), and it was revised to 0.65 when the order of presentation was -,-,+,+ (row 2, col. 9, Table 4b). However, when the items of evidence were simultaneously presented the belief was revised to an intermediate value of 0.53 (row 2, col. 9, Table 5b).

In case of consistently positive evidence, both sequential as well as simultaneous processing attain the same degree of belief revision. There is no conflict when all items of evidence are confirming (positive). Since there is no conflict, the discount rate is zero. Hence, the items of evidence are aggregated without discounting in either case. Thus, both the processes attain the same degree of belief revision. This is consistent with the empirical findings (Footnote 9, Ashton & Ashton, 1988).

To sum up, a uniform discounting of all items of evidence is appropriate when all the items are presented simultaneously. There is no difference in sequential versus simultaneous processing for confirming (positive) items of evidence. Simultaneous processing leads to dilution for mixed and negative items of evidence.

5. SUMMARY AND CONCLUSION

In this paper we examined the empirical findings under the framework of probability theory. We employed two alternatives: the Bayesian framework and the Dempster-Shafer theory of belief functions. The scope of the paper is limited to explaining the empirical results using the two approaches. This paper provides no normative prescriptions for belief revision.

The Bayesian theory of probability is unable to explain most of the empirical findings. Bayesian theory is very stringent in its requirement that the probability of mutually exclusive and collectively exhaustive events sum to one. Bayesian theory requires that the belief be increased in light of supporting evidence, and be decreased in light of conflicting evidence. This is consistent with the empirical findings. However, the Bayesian analysis entails the largest increase (decrease) in belief for medium priors in light of supporting (conflicting) evidence. This is contrary to empirical findings. The largest increase was observed for smallest priors when presented with supporting evidence. The largest decrease was observed for highest priors when presented with conflicting evidence. Further, Bayesian theory fails to explain the 'recency' and the 'dilution' effects.

Belief-function formalism, on the other hand, can model the behavior of the auditors. In belief functions, the belief in a proposition increases in light of supporting (positive) evidence. Further, the largest increase is for the lowest priors. This is consistent with the empirical findings. The belief in a proposition decreases in light of conflicting (negative) evidence and the largest decrease is for medium priors. This is not totally consistent with the empirical findings. However, with discounting we can attain the largest decrease for the highest prior, consistent with the empirical findings. By incorporating discounting in belief functions, we have explained the 'recency' and the 'dilution' effects.

With the advent of expert systems in auditing such studies have become imperative. In the construction of an expert system, the issue of how auditors combine evidence is enigmatic. Expert systems are built to emulate experts, thus the need to understand the expert's behavior is imperative. Belief function formalism seems to provide a theory which can model an expert's decision process and hence is highly viable.

The theory of evidence aggregation will ease the process of audit planning and audit risk assessments (Srivastava and Shafer 1992; Dutta, Harrison and Srivastava 1997). Once we know how auditors aggregate evidence, we can formulate an efficient way of evidence search. This would lead to more efficient audits and will subsequently reduce the cost of audits. We feel this is a very crucial and fundamental area in auditing and much research effort should be expended in understanding the evidence aggregation process.

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