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THE AUDIT RISK MODEL UNDER THE RISK OF FRAUD

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ABSTRACT

In this article, we derive an audit risk formula for a simple situation. This formula closely resembles the SAS 47 model when we assume that no material misstatement due to fraud exists. A simple case illustrates how the risk of material misstatement due to management fraud impacts audit risk and how performing special audit procedures to detect such irregularities can decrease overall audit risk. While SAS 53 requires auditors to assess the risk of errors and irregularities and plan the audit to provide reasonable assurance of detecting errors and irregularities, the audit risk model of SAS 47 does not directly address the risk of management fraud. This article develops an audit risk model using the belief-function framework that considers the risks faced by auditors due to random errors, defalcations (employee fraud) and management fraud. We consider two cases. In the first, we consider only affirmative items of evidence and derive an analytical formula for the audit risk. In the second, we consider mixed items of evidence (both affirmative and negative), which models the situation more frequently faced by auditors. We demonstrate that a serious underestimation of audit risk can occur if the audit risk model is used without specifically considering the risks associated with management fraud.

Key Words: Belief-functions, Audit risk model, Evidence, Material irregularities, Fraud.

The Audit Risk Model Under the Risk of Fraud

I. INTRODUCTION

In recent years, the societal concerns regarding the responsibility of the auditor is on the rise. Congress, juries, and regulatory bodies want to increase the auditor's responsibility in detecting management fraud in a financial audit. Jenkins Committee report on the responsibilities of the auditor in detecting fraud is an epitome of the societal expectation of the auditor's role. In response, the accounting profession through the Statement on Auditing Standards No. 53, *The Auditor's Responsibility to Detect and Report Errors and Irregularities,* (AICPA 1988) explicitly requires the auditor (1) to assess the risk that errors and irregularities may cause the financial statements to contain a material misstatement, and (2) to design the audit to provide reasonable assurance of detecting such errors and irregularities. Additionally, a new auditing standard is forthcoming[i] by mid-1997 which will further increase the responsibility of the auditors in detecting fraud. However, there exists limited guidance for numerically evaluating audit risk in such situations.

The purpose of this paper is to develop audit risk formulae that will explicitly consider the risk of material misstatements due to management fraud along with the risk of material misstatements due to random errors. This is achieved by considering a properly specified structure of audit evidence, as it relates to errors and material irregularities. We use the belief function framework to represent the auditor's judgment about the risk and the strength of evidence. Srivastava and Shafer (1992) have argued that the belief-function framework is a better representation for depicting uncertainties in audit evidence than probability theory (see also Shafer and Srivastava 1990). We believe that an explicit consideration of the risk of material misstatement due to management fraud in the audit risk model will enable the auditor to conduct an efficient and effective audit. Efficiency is improved because the auditor can determine the quantity of audit procedures necessary to achieve the desired level of audit risk. Intuitive estimates can be refined by determining the impact of additional fraud related procedures on total audit risk. Effectiveness is improved because the auditor has greater assurance that sufficient evidence has been gathered regarding all types of errors and irregularities. This should help reduce litigation risk on audits where the risk of irregularities has been recognized.

A simpler evidential structure than Srivastava and Shafer (1992) has been modeled to facilitate comparisons with the SAS 47 model (AICPA 1983), and to demonstrate the importance and implications of the risk of fraud in the model. We develop the model at the audit objective level and assume that the three variables (Audit Objective, Errors and Defalcations, and Fraud) considered in the evidential structure are binary. Srivastava and Shafer (1992) make similar assumptions for the variables used in their models.

It is well known that the two types of misstatements, one due to errors, and the other due to irregularities, require significantly different approaches for their detection (e.g., see Loebbecke, Eining, and Willingham 1989, hereafter LEW), hence audit evidence on the absence of errors may not provide any assurance on the absence of material irregularities. Random errors are unintentional and hence not concealed. Such errors can be effectively detected through proper sampling procedures. However, such procedures have limited usage in detecting material irregularities (LEW). Since material irregularities (due to defalcation or management fraud) are intentional, the perpetrators try to conceal it, hence these are difficult to detect based on sampling or other routine audit procedures. Thus, assurance on the absence of material irregularity can be obtained through different audit procedures which may not provide assurance on the absence of errors. This distinction in the nature of audit evidence and audit procedures for detecting errors and irregularities has to be explicitly specified in the audit evidence structure. Failure to do so may result in misstating the achieved audit risk.

In summary, SAS 47 provided the conceptual framework for addressing audit risk which was further expanded in SAS 53. Subsequently, many researchers have operationalized these ideas using probability theory (e.g. Kinney 1984, Boritz 1990, Sennetti 1990, Dutta and Srivastava 1993) and belief-function framework (e.g. Srivastava and Shafer 1992). However, these articles did not explicitly model the impact on audit risk due to management fraud. It is generally accepted that different types of audit procedure are required to detect management fraud. LEW and Bell et.al. (1991) provide a list of such procedures but provide no formal model for audit risk computation. This article develops a mathematical model of audit risk using the belief function framework while explicitly considering the impact of management fraud. It extends LEW by developing a model and by mathematically demonstrating the consequences of not considering the possibility of management fraud while planning an audit. It extends Srivastava and Shafer (1992) research by explicitly modeling the impact of management fraud and by allowing for mixed items of evidence.

The remainder of the paper is divided into six sections and two appendices. Section II discusses the structure of the audit evidence used in the audit risk formulation. Section III provides a primer to the theory of belief function and develop the notations. Section IV derives the audit risk formulae under the belief-function framework and discusses its implications. Section V discusses the situation where we have mixed items of evidence. Section VI concludes with a brief summary. Appendix A derives the audit risk formula under the belief-function framework. Appendix B derives the audit risk formula under the probability framework.

II. STRUCTURE OF AUDIT EVIDENCE

This section discusses the structure of audit evidence with explicit consideration of risk of material irregularities. A schematic diagram of audit evidence pertinent to an assertion of an account is presented in Figure 1. The nodes in the Figure denote the variables[ii] ('O' for Audit objective, 'E' for errors and defalcations, and 'F' for management fraud) and the arcs denote the relevance relationships between the variables. The variables are assumed to be binary, that is, they each take two values[iii]. The evidence obtained through audit procedures are also denoted as nodes and are joined through arcs to other pertinent nodes.

An auditor evaluates various types of evidence to ascertain whether all assertions of an account are met, i.e., there is no material misstatement in the account due to the violation of any assertion. For example, in the case of accounts receivable, one of the important assertion is that the account balance is properly valued. The auditor conducts audit procedures, such as the confirmations of accounts receivables, to obtain audit evidence pertaining to proper valuation of account receivables. In general, a given audit objective (i.e., a given assertion) is satisfied if and only if the following two conditions have been satisfied: (1) there are no material misstatements due to random errors; (2) there are no material misstatements due to irregularities.

In Figure 1, the above condition is represented by an 'and' relationship, between the variable 'Audit Objective (O)' and the other two variables: 'E', and 'F'. That is, the variable 'O' takes the value 'o' only when variables 'E'" and 'F' take the values 'ne' and 'nf', respectively[iv]. In other words, the audit objective is met if and only if there are no material misstatements due to random errors and defalcations (ne), and no material misstatement due to fraud (nf). We represent such a relationship in our diagram by connecting the audit objective 'O' to the two sub-objectives, 'E' and 'F' through an 'and' relationship.

FIGURE 1 HERE

Inherent factors could be relevant to either errors or irregularities, or to both The factors relevant to only one (errors or fraud) node is represented by drawing an arc from these factors to the respective node (see Figure 1). Further, the factors relevant to both nodes is represented by drawing an arc from these factors to the node 'O'. An example of inherent factors at 'O' which influence both 'E' and 'F' is a bad attitude of management towards internal controls; it will permit random errors to pass through the system without being detected and corrected, and also raises the possibility of management fraud. An example of inherent factors at 'E' is implementation of a new accounting system; the employees may make unintentional errors because they are not able to fully understand the system. There are several examples of inherent factors that relate only to

'F'. LEW list several such factors. They suggest that the possibility of fraud is very high when management has the attitude and motivation for it and when the condition is right for fraud. For example, likelihood of fraud will be high if management is dishonest (attitude), the business is losing money (motivation), and management is using subjective judgments to determine the balance of an account that is material to the financial statements (condition).

We consider analytical procedures only at the audit objective level because they indicate unusual situations irrespective of the cause. That is, when the actual amount or the ratio is significantly different from the expectation, the auditor could only deduce that the account may contain a material misstatement, but in most situations they can not determine the reason. Similar to Srivastava and Shafer (1992), we consider all items of evidence related to the accounting system, control procedures, and the traditional tests of transactions to be relevant to 'E' only, as the standard audit evidence do not provide any assurance on the absence of fraud.

Similar to inherent factors, we consider 'test of details' (TD) at all the three levels in Figure 1. An example of a procedure at 'O' is the physical examination of all the inventory items. This procedure will relate to both 'E' and 'F'. An example of TD at 'E' is the confirmations obtained for the existence of inventory at a warehouse controlled by someone other than the client and not related to the client. In the case where a management fraud is suspected because the sales for the period have been excessively high compared to previous years and the industry average, the procedures to test whether the sales are genuine, or are simply a transfer of inventory from a company facility to another location, is an example of a procedure that relates to 'F'.

III. A PRIMER TO THE THEORY OF BELIEF FUNCTIONS

In this section we present the basics of the theory of belief functions (for more details see Shafer 1976) and their correspondence to auditing terms. We also develop the notations to be used in the remainder of the paper.

We refer to the set of all possible values of a variable as the variable's 'frame of discernment' (Θ). In this article, there are three variables, random errors (denoted as E), fraud

(F), and audit objective (O). Each of the variables have two values, denoted by the lower case letters. For example, two values of variable E are 'e' and 'ne' denoting the presence and absence, respectively, of material errors due to random errors and defalcations. Values of other variables are listed in Table 1.

TABLE 1 HERE

A basic probability assessment (or m-value) in belief function framework is a function m: such that

$$m(A) \geq 0 \ \forall \ A \in 2^{\Theta}, m(\phi) = 0, \text{ and } \Sigma\{m(A) \mid A \in 2^{\Theta}\} = 1. \qquad (1)$$

Intuitively, m(A) represents the degree of belief assigned exactly to the set A. The m-values generalizes the probability mass function of standard probability theory. Whereas a probability mass function is restricted to simple values of variables, an m-value is allowed to assign masses to sets of values, without assigning any mass to the individual values contained in the sets. The notation used to denote m-values is 'm' subscripted with the variable it pertains to (See Table 1). Associated with the m-value are two other functions, the belief function and the plausibility function. The belief function is defined as

$$\text{Bel}(A) = \Sigma\{m(B) \mid B \subseteq A\} \qquad (2)$$

Note that Bel (ϕ) = 0 and Bel (Θ) = 1, for any basic probability assignment function.

The plausibility function is defined as

$$\text{Pl}(A) = \Sigma\{m(B) \mid B \cap A \neq \phi\} \qquad (3)$$

Pl(A) represents the total belief that could be assigned to A. Note that Pl(A) = 1 - Bel(~A), where ~A is the complement of A. Also note that Pl(A) µ Bel(A).

In an auditing context, SAS 47 suggests that the audit risk at the objective level (AR_O) be divided into three component risks:

- Inherent risk (denoted as IR and subscripted with the variable it pertains to) is the susceptibility of an audit objective or account balance to error assuming that there were no related internal controls.

- Control risk (denoted as CR and subscripted with the variable it pertains to) is the risk that the internal controls will not be able to prevent or detect errors in an account balance or pertaining to an audit objective.
- Detection risk (denoted as DR and subscripted with the variable it pertains to) is the risk that an auditor's procedure will not detect a material error in an account balance or pertaining to an audit objective.

The risks in an audit context is akin to the plausibility function in the belief function framework (See Shafer and Srivastava 1990 for details). This is so because the inability to assert no material misstatement is an implicit acknowledgment of the risk of material misstatement. Thus, for an account objective susceptible to material misstatement, $m_{IO}(o) = 0$. Hence, $Pl_{IO}(no) = 1 - m_{IO}(o) = 1$, which signifies that the plausibility of material misstatement due to inherent factors at O (IR_O) equals one. Similarly, when no audit evidence is collected at the objective level, which would alleviate concerns regarding errors at the objective level, $m_{DO}(o) = 0$. Hence, the plausibility of material misstatement after test of details, Pl_{DO} equals one.

In an audit, the auditors first assesses the inherent risk. Next, they evaluate the internal controls and assess the control risks. Based on the assessments of inherent risk and control risk, audit procedures (such as analytical procedures, test of details, etc.) are designed to detect errors and material misstatements. The scope of the audit procedures conducted determine the detection risk. These independent assessments are then aggregated to determine the audit risk. Appendix A develops formulas for aggregation of risk for confirming items of evidence. Appendix B develops formulas for aggregation of risk for mixed items of evidence.

IV. AUDIT RISK FORMULA

The audit risk formula for the evidential structure given in Figure 1 is derived in Appendix A. The details of the symbols used in the paper are given in Table 1. From (A-30) we have the total audit risk at the audit objective level as:

$$AR_O^t = AR_O\left(AR_E + AR_F - AR_E AR_F\right), \tag{4}$$

where

$$AR_O = IR_O APR_O DR_O;\ \ AR_E = IR_E CR_E DR_E;\ \text{ and }\ AR_F = IR_F DR_F. \tag{5}$$

We can write (4) as:

$$AR_O^t = IR_O APR_O DR_O\left(IR_E CR_E DR_E + IR_F DR_F - IR_E CR_E DR_E IR_F DR_F\right). \tag{6}$$

Equation (6) represents the audit risk formula at the audit objective level. This formula has some interesting features as discussed below.

First, under the assumption of no fraud, that is, $IR_F = 0$, the formula reduces to:

$$AR_O^t = IR_O APR_O DR_O IR_E CR_E DR_E, \tag{7}$$

or

$$AR_O^t = \left(IR_O IR_E\right) CR_E APR_O \left(DR_O DR_E\right), \tag{8}$$

which is similar to the SAS 47 model, the product of four types of risks: inherent risk, control risk, analytical procedure risk, and detection risk. In fact, if there is no risk of material misstatements due to fraud ($IR_F = 0$), i.e., variable 'F' can be eliminated from the evidential network, then there is no difference between the objective 'O' and the variable 'E'. Thus, 'E' collapses into the audit objective 'O', and there is only one estimate for each type of risk. In this special case (8) is exactly like the SAS 47 model. This is because the SAS 47 model does not explicitly recognizes management fraud separate from random errors. SAS 47 approach does not pose a problem when all audit procedures provide assurance on both random errors and fraud to the same extent. However, as discussed by LEW, the design of audit procedures to detect fraud are different than those to detect random errors. Hence, fraud should be modeled distinct from random errors to ensure that sufficient evidence is obtained to minimize the risk of random error as well as that of material irregularities due to fraud.

Second, under the assumption that auditors do not perform any procedures to detect fraud, i.e., $IR_I = 1$ and $DR_I = 1$, then (6) reduces to:

$$AR_O^t = AR_O = IR_O APR_O DR_O. \tag{9}$$

This is an important result. This suggests that irrespective of the amount of competent evidential matter is accumulated by the auditor for random errors and defalcations, if no evidential matter is collected related to fraud then the total audit risk will always be equal to the audit risk at the audit objective level which is the product of the three risks at the objective level. The audit risk formula is further discussed below using numerical examples. Table 2 presents the calculations for the total audit risk at the audit objective level for various values of the input risks using (6).

TABLE 2 HERE

Case 1: Rows 1-3 of Table 2 illustrates that the achieved audit risk may be acceptable even when no audit procedures are conducted to detect fraud provided there is no risk of fraud. The auditor plans the audit assuming that there is no possibility of fraud ($IR_F = 0$, and $DR_F = 1$). In this situation, traditionally, the auditor will study the inherent factors that govern random errors and employees' defalcations. Based on the inherent factors for such errors, the auditor will evaluate the accounting system and the controls to see how much reliance can be placed on the system. Next, the auditor will determine the level of support to be obtained from the test of details that he or she should perform to achieve the desired level of total audit risk or the support in favor of the objective.

Let us assume that the auditor has assessed (1) a low level of support from the analytical procedures for the objective ($m_{PO}(o) = 0.3$, or $APR_O = 0.7$), and (2) at the 'Errors and Defalcations' level, a low level of support from the inherent factors ($m_{IE}(ne) = 0.1$-0.3, or $IR_E = 0.9$-0.7) and a medium level of support from the accounting system and control procedures ($m_{CE}(ne) = 0.4$, or $CR_E = 0.6$). If we assume that the auditor plans the audit at about a 0.05 level of total audit risk then a medium to high level of support from the test of details performed for errors and defalcations (say, $m_{DE}(ne) = 0.85$, or $DR_E = 0.15$) would provide a 0.044 to 0.057 level of audit risk (see the last column of rows 1-3 in Table 2).

Case 2: Rows 4-6 of Table 2 illustrate that the achieved audit risk will be unacceptable when no audit procedures are conducted to detect fraud and there is a possibility of fraud. Here, the auditor recognizes the possibility of fraud but does not perform any procedures to detect it ($IR_F = 1$, $DR_F = 1$). The total audit risk in this case is 0.7, an unacceptable value. This is an important result because it illustrates that the audit risk is governed by both the risk of undetected errors as well as irregularities. The auditor can lower this value if he or she obtains evidence which provide assurance on the absence of fraud. Rows 7-9 show the impact of inherent factors that relate to irregularity on the overall audit risk. In the present example, we are assuming that inherent factors are conducive to fraud, hence a high level of inherent risk on fraud.

Case 3: Rows 10-12 illustrate that the impact of the risk of fraud on the achieved audit risk can be mitigated by performing audit procedures designed to detect fraud. In our example, we assume $DR_F = 0.3$ which implies that the specific procedures performed for detecting fraud provide 0.7 level of assurance[v] that there is no fraud. Also, we assume that $DR_E = 1.0$ and $DR_O = 0.1$. These values are based on the assumption that the test of details that was performed just for 'E' is now being re-designed to collect evidence on both 'E' and 'F'. This process simply shifts the level at which the auditor determines the extent and nature of the test from just 'E' to both 'E' and 'F'. This shift is achieved by setting $DR_E = 1.0$. This indicates that the auditor is not performing the procedure for 'E' alone but is performing the procedure and considering its impact at 'O'.

In this example, we assume that the auditor performs an extensive test of details and feels that the evidence provides 0.9 level of assurance that the objective is met, i.e., there is no material misstatement due to any of the possible causes: errors, defalcations, or material irregularity due to management fraud. The total audit risk in this case is fairly low, between 0.034 to 0.042 (see the last column of rows 10-12 in Table 2). These are acceptable results since the desired audit risk is 0.05. The last scenario is similar to the current professional practice for dealing with fraud as described by LEW. Further this is consistent with the guidelines established in SAS 53 (AICPA 1988) but have not been incorporated in any mathematical model of audit risk.

V. MIXED ITEMS OF EVIDENCE

Here, we want to show the impact of negative items of evidence on the total audit risk or on the total belief that the objective is met. An example of such an item of evidence would be the inherent factors at 'F' as described earlier where the auditor knows that (1) management is dishonest, (2) the business is losing money, and (3) management is using subjective judgments to determine the account balance which is material to the financial statements. The auditor may use this knowledge to assign a low level of support, say 0.3, to the assertion that there could be a material misstatement due to fraud and assign no support to the assertion that there is 'no fraud.' As discussed in Footnote 2, the support from this evidence can be written as: $m_{IF}(nf) = 0$, $m_{IF}(f) = 0.3$, $m_{IF}(\{nf, f\}) = 0.7$. Here, we have a belief in 'no material misstatement due to fraud' equal to zero, i.e., Bel(nf) = 0 which yields plausibility for 'material misstatement due to fraud' to be one (Pl(f) = 1). A plausibility value of one simply means that given the evidence, a material misstatement due to fraud is plausible with degree 1.0 but the direct evidence that there is a material misstatement due to fraud has a belief value of 0.3.

TABLE 3 HERE

Table 3 shows the impact of negative items of evidence on the audit risk and the belief that the assertion or the objective is met. Consider the following scenario:

1. The analytical procedures related to the objective suggest that there is positive support that there is no material misstatement. Let us suppose that the auditor assumes a low, say 0.3, level of support, i.e., $m_{PO}(o) = 0.3$, $m_{PO}(no) = 0$, and $m_{PO}(\{o, no\}) = 0.7$.

2. The inherent factors related to errors and defalcations are good. For example, the auditor thinks the client has a good internal control system for the objective. The auditor, knowing this information, decides that he can associate a positive but low, say 0.2, degree of support that there is no material misstatement due to errors and/or defalcations and zero support that there is a material misstatement due to 'E.' This feeling can be written in terms of m-values as: $m_{IE}(ne) = 0.2$, $m_{IE}(e) = 0$ $m_{IE}(\{ne, e\}) = 0.8$.

3. The auditor has performed tests of controls and tests of transactions and feels that he or she can get a positive but low, say 0.4, degree of support that there is no material misstatement due to random errors and/or defalcations. This feeling can be expressed in terms of the following m-values: $m_{CE}(ne) = 0.4$, $m_{CE}(e) = 0$, and $m_{CE}(\{ne, e\}) = 0.6$.

4. Since the assurances from analytical procedures, the inherent factors, and from the accounting systems and controls, as stated above, are low, the auditor plans the test of details for a high level, say 0.9, degree of support, i.e., $m_{DE}(ne) = 0.9$, $m_{DE}(e) = 0$, and $m_{DE}(\{ne, e\}) = 0.1$.

It is interesting to note that if we use the SAS 47 model without worrying about the risk of fraud then we get a value of 0.022 for the audit risk which is a much better value than 0.05, the usual acceptable risk. However, when we consider the risk of fraud then the situation changes drastically. Even if the auditor does not collect any evidence related to fraud but simply considers that a material misstatement due to fraud is plausible (i.e., $m_{IF}(nf) = 0$, $m_{IF}(f) = 0$, and $m_{IF}(\{nf, f\}) = 1$, or $Pl(f) =1 = IR_{IF}(f)$), the total audit risk goes up to 0.7 for exactly the same items of evidence as described above (see row 1 of Panels A and B of Table 3, in particular column 12 of Panel B). This is an important result. As discussed earlier, it simply states that ignoring the risk of fraud in the SAS 47 model would make the auditor accept an unacceptable audit result.

Let us consider the same items of evidence as describe in conditions 1-4 above but with a possibility of fraud. Suppose the auditor performs certain test of details at 'F' to ensure that there is no material misstatement due to fraud. The evidence is positive and moderately strong, say $m_{IF}(nf) = 0.8$, $m_{IF}(f) = 0$, and $m_{IF}(\{nf, f\}) = 0.2$. In this case, the total audit risk AR_O^t is reduced significantly to 0.17 from an earlier value of 0.7 but it still may not be acceptable to the auditor (see row 2 of Panel A and B in Table 3).

Consider now the LEW approach where if the auditor feels that there is a risk of fraud then he or she not only performs a direct test of details to detect fraud but also becomes relatively more skeptical regarding the other audit evidence and expands the other tests of details which are usually performed for errors and defalcations. These expanded procedures will now be used to look for fraud as well as errors and defalcations. In our approach, this process means that we set

$m_{DE}(ne) = 0$, $m_{DE}(e) = 0$, and $m_{IE}(\{ne, e\}) = 1$, i.e., there are no tests of details for only 'E' because the test has shifted to 'O.' Suppose that the auditor performs extensive tests of details and concludes that the evidence strongly supports, say with 0.9 degree of assurance, that there is no material misstatement due to fraud, errors, or defalcations (i.e., $m_{DO}(o) = 0.9$, $m_{DO}(no) = 0$, and $m_{DO}(\{o, no\}) =0.1$). This situation corresponds to row 3 of Table 3. The total audit risk in this case is 0.04 (see column 12 of row 3 of Panel B in Table 3). This seems like an acceptable value for audit risk. Also, it is intuitively consistent with what auditors are already doing to detect fraud as discussed by LEW. With this model, auditors are able to fully incorporate their intuitive approaches in determining total audit risk.

Let us consider a case where all the conditions listed above in 1-4 are the same but where the auditor feels that there is a good possibility of management fraud since all three factors, condition, attitude and motivation, suggested by LEW are present. This knowledge could be stated in terms of m-values as: $m_{IF}(nf) = 0$, $m_{IF}(f) = 0.3$, and $m_{IF}(\{nf, f\}) =0.7$. Row 4 of Panels A and B corresponds to this situation. The test of details at 'E' provides 0.9 degree of support for 'ne' but total audit risk is 0.77, a very high value because there are no tests performed by the auditor to detect fraud in the current situation. However, when the auditor performs specific tests of details for detecting fraud and either finds no material misstatements or makes adjustments to the account, total audit risk reduces significantly (see row 5). Row 6 describes the same situation except that the test of details are being performed for both 'E' and 'F' instead of only for 'E' and the tests are performed with more skepticism. Suppose the auditor finds no material problem of any kind and concludes that the evidence supports the objective with 0.9 degree of assurance (i.e., $m_{DO}(o) = 0.9$, $m_{DO}(no) = 0$, and $m_{DO}(\{o, no\}) =0.1$). The total audit risk in this case is 0.05, an acceptable level of risk.

If we compare row 3 with row 6, we find that the total audit risk in row 3 is smaller than the total audit risk in row 6, since in row 6 the auditor has 0.3 degree of support in favor of material misstatement, while in row 3 there is no such support for or against. This is an intuitive result. If there is direct evidence that fraud has occurred the auditor will perform not only extended tests of details for ‘E' and 'F' but also some specific procedures to detect fraud. Only after establishing that there is no material misstatement due to any or all of the reasons random errors, defalcations, or fraud, can the auditor assign a high level of assurance in favor of 'o.'

VI. SUMMARY AND CONCLUSION

This article has expanded the audit risk model described in SAS 47 to include the requirements of SAS 53 to assess the risks of both errors and irregularities and to plan the audit to provide reasonable assurance of finding both errors and irregularities. This has been done in the context of the belief-function framework in which audit risk is described in terms of plausibility. The audit risk model developed in this paper has incorporated the different types of risk analysis which must be considered when the likelihood of fraud exists.

If the auditor decides that there is additional risk in the audit due to potential management fraud, procedures will be expanded, new procedures may be added and skepticism about all of the evidence collected increases. This additional component of risk adds another branch to the audit risk model at the audit objective level. If this branch is not properly considered, this paper shows that total audit risk can be materially understated.

The formulas given here provide guidance for audit planning when an auditor faces the possibility of management fraud. Further, this paper illustrates the impact that somewhat contradictory evidence (mixed evidence) can have on the amount of evidence required to achieve the desired level of audit risk.

Other issues that were not addressed in the paper, include (1) the form of the audit risk model when the evidential structure is a network, (2) the impact of irregularities on the audit risk model at the account and financial statement levels, (3) empirical issues dealing with how auditors formulate their judgments about the strength of the evidence collected and (4) empirical issues focusing on whether auditors actually formulate their judgments in a manner which is accurately described by the belief-function formulas. All of these issues require further research.

APPENDIX A

DERIVATION OF PLAUSIBILITY (AUDIT RISK) FORMULAS

In this section, we will derive an audit risk formula for the simple evidential structure as given in Figure 1 in the belief -function framework. In Figure 1, we have three variables: Audit Objective (O), Errors and Defalcations (E), and Fraud (F). Variables 'E' and 'F' are connected to variable 'O' through an 'and' relationship.

Strength of Evidence or m-values for Various Items of Evidence

As mentioned earlier in the paper, we assume affirmative items of evidence for deriving the formula (see Footnote 2 for more details on affirmative and negative items of evidence). We will use the notation used by Srivastava and Shafer (1992, p. 258) in the derivation. However, for the convenience of the readers we have defined these symbols in Table 1. Since all the items of evidence are assumed to be affirmative, we have the following m-values for each item of evidence at different levels:

Audit Objective level (O):

$$m_{IO}(o) = 1 - IR_O,\ \ m_{IO}(no) = 0,\ \ m_{IO}(\{o, no\}) = IR_O, \tag{A-1}$$

$$m_{PO}(o) = 1 - APR_O,\ \ m_{PO}(no) = 0,\ \ m_{PO}(\{o, no\}) = APR_O, \tag{A-2}$$

$$m_{DO}(o) = 1 - DR_O,\ \ m_{DO}(no) = 0,\ \ m_{DO}(\{o, no\}) = DR_O, \tag{A-3}$$

Errors and Defalcations level (E):

$$m_{IE}(ne) = 1 - IR_E,\ \ m_{IE}(e) = 0,\ \ m_{IE}(\{ne, e\}) = IR_E, \tag{A-4}$$

$$m_{CE}(ne) = 1 - CR_E,\ \ m_{CE}(e) = 0,\ \ m_{CE}(\{ne, e\}) = CR_E, \tag{A-5}$$

$$m_{DE}(ne) = 1 - DR_E,\ \ m_{DE}(e) = 0,\ \ m_{DE}(\{ne, e\}) = DR_E. \tag{A-6}$$

Fraud level (F):

$$m_{IF}(nf) = 1 - IR_F,\ \ m_{IF}(f) = 0,\ \ m_{IF}(\{nf, f\}) = IR_F, \tag{A-7}$$

$$m_{DF}(nf) = 1 - DR_F,\ \ m_{DF}(f) = 0,\ \ m_{DF}(\{nf, f\}) = DR_F, \tag{A-8}$$

In order to combine all the above m-values for the total audit risk, we first combine the individual set of m-values bearing at each level, and then combine the resulting m-values. For the first step, we use Dempster's rule[vi], and for the second step we use Srivastava and Shafer (1992) results on combining m-values with an 'and' relationship. The following are the details of the two steps.

Step 1: Combination of m-values at Each Level

Audit Objective Level (O):

Given the set of m-values at 'O' in (A-1 - A-3), we find that the renormalization constant used in Dempster's rule is one, i.e., K=1, and the combined m-values are (see footnote 4):

$$m_O(o) = 1 - IR_O APR_O DR_O \tag{A-9}$$

$$m_O(no) = 0 \tag{A-10}$$

$$m_O(\{o, no\}) = IR_O APR_O DR_O \tag{A-11}$$

The easiest way to derive the above result is to first determine the combined m-value at the frame $\{o, no\}$ and the value of $m_O(no)$, and then use the property of m-values to determine the value of $m_O(o)$. From Dempster's rule, $m_O(\{o, no\})$ is obtained by multiplying the three m-values at the frame as given in (A-1 - A-3). That is:

$$m_O(\{o, no\}) = m_{IO}(\{o, no\}) m_{PO}(\{o, no\}) m_{DO}(\{o, no\}) = IR_O APR_O DR_O. \tag{A-12}$$

Since there is no support for 'no' from any of the items of evidence at 'O,' we know that $m_O(no) = 0$. Thus, using the property that all the m-values defined on a frame add to one, we obtain:

$$m_O(o) = 1 - m_O(no) - m_O(\{o, no\}) = 1 - IR_O APR_O DR_O. \tag{A-13}$$

The above result can also be obtained by two other ways: (1) by combining two items of evidence at a time, and (2) by combining all the items of evidence at the same time. These alternatives are easy to implement, and we will not discuss them in this paper.

Errors and Defalcations Level (E):

The m-values at this level are similar to the m-values at the audit objective level discussed in the previous subsection. Thus, we use the results of (A-12) and (A-13) to determine the combined m-values at 'E:'

$$m_E(ne) = 1 - IR_E CR_E DR_E, \tag{A-14}$$

$$m_E(e) = 0, \tag{A-15}$$

$$m_E(\{ne, e\}) = IR_E CR_E DR_E. \tag{A-16}$$

Fraud level (F):

At this level, we have two items of evidence with the m-values given in (A-7 - A-8). Since both the items of evidence are affirmative, again, we have no conflict between them and thus the renormalization constant in Dempster's rule is one. We obtain the following combined m-values at 'F:'

$$m_F(nf) = m_{IF}(nf)m_{DF}(nf) + m_{IF}(nf)m_{DF}(\{nf, f\}) + m_{IF}(\{nf, f\})m_{DF}(nf)$$

$$= 1 - IR_F DR_F, \tag{A-17}$$

$$m_F(f) = m_{IF}(f)m_{DF}(f) + m_{IF}(f)m_{DF}(\{nf, f\}) + m_{IF}(\{nf, f\})m_{DF}(f) = 0 \tag{A-18}$$

$$m_F(\{nf, f\}) = m_{IF}(\{nf, f\})m_{DF}(\{nf, f\}) = IR_F DR_F \tag{A-19}$$

Step 2: Combination of m-values at Audit Objective Level (O)

In the second step, we combine the m-values at 'O' obtained in (A-9 - A-11) with the m-values coming from 'E' and 'F' to 'O.' The reason for combing all the evidence is that we want to determine the impact of all these items of evidence at the audit objective level. The way we achieve this goal is to first propagate the m-values from 'E' and 'F' to 'O' and then combine them with the m-values at 'O.' Srivastava and Shafer have derived a general result for such a propagation. We use their results to obtain the m-values propagated from 'E' and 'F' to 'O' as follows (See Equations A-1 - A-3 in Srivastava and Shafer 1992, pp. 274-275):

$$m_{O \leftarrow E\&F}(o) = m_E(ne)m_F(nf) \tag{A-20}$$

$$m_{O \leftarrow E\&F}(no) = m_E(e)\big(m_F(nf) + m_F(f) + m_F(\{nf, f\})\big) + \big(m_E(ne) + m_E(\{ne, e\})\big)m_F(f) \tag{A-21}$$

$$m_{O \leftarrow E\&F}(\{o, no\}) = m_E(ne)m_F(\{nf, f\}) + m_E(\{ne, e\})m_F(nf) + m_E(\{ne, e\})m_F(\{nf, f\}) \tag{A-22}$$

Equations (A-20 - A-22) represent a general result of propagation of m-values from the two sub-objectives, 'E' and 'F' to the main objective 'O' where an 'and' relationship between 'O', and 'E' and 'F' exists. The above result makes intuitive sense. For example, (A-20) implies that the objective is met if and only if there is no material misstatement due to errors and defalcations and also due to fraud. In other words, o is true if and only if 'ne' is true and 'nf' is true.

Equation (A-21) implies that the objective is not met when (1) there is a material misstatement due to errors and defalcations but there may or may not be any material misstatement due to fraud, or (2) there is a material misstatement due to fraud but either there is no material misstatement due to 'E' or we do not know about it. Similarly, (A-22) implies that we have no knowledge whether 'o' or 'no' are true under the following two conditions: (1) when any one of the sub-objectives is met (i.e., either 'ne' is true or 'nf' is true) and about the other objective we have no knowledge, or (2) when we have no knowledge about both the sub-objectives.

Substituting the m-values from (A-14 - A-19) into (A-20 - A-22), we obtain the following values:

$$m_{O\leftarrow E\&F}(o) = (1 - IR_E CR_E DR_E)(1 - IR_F DR_F) \tag{A-23}$$

$$m_{O\leftarrow E\&F}(no) = 0 \tag{A-24}$$

$$m_{O\leftarrow E\&F}(\{o, no\}) = (IR_E CR_E DR_E) + (IR_F DR_F) - (IR_E CR_E DR_E)(IR_F DR_F) \tag{A-25}$$

Now we combine the m-values at 'O' defined in (A-9 - A-11) with the m-values in (A-23 - A-25) using Dempster's rule. Since the renormalization constant is again unity, the total m-values are easily obtained. The result is:

$$m^t_O(o) = 1 - (IR_O APR_O DR_O)\big[(IR_E CR_E DR_E) + (IR_F DR_F) - (IR_E CR_E DR_E)(IR_F DR_F)\big] \tag{A-26}$$

$$m^t_O(no) = 0 \tag{A-27}$$

$$m^t_O(\{o, no\}) = (IR_O APR_O DR_O)\big[(IR_E CR_E DR_E) + (IR_F DR_F) - (IR_E CR_E DR_E)(IR_F DR_F)\big] \tag{A-28}$$

Using the above m-values, we have the following value for the belief for 'o:'

$$Bel^t_O(o) = m^t_O(o) = 1 - (IR_O APR_O DR_O)\big[(IR_E CR_E DR_E) + (IR_F DR_F) - (IR_E CR_E DR_E)(IR_F DR_F)\big] \tag{A-29}$$

From the definition of the plausibility function, $Pl^t_O(no) = 1 - Bel^t_O(o)$, and the relationship that plausibility that the objective is not met is defined to be the audit risk associated with the objective, i.e., $Pl^t_O(no) = AR^t_O$, we obtain the formula for the total audit risk at the objective level in the belief-function framework as:

$$AR_O^t = (IR_O APR_O DR_O)\left[(IR_E CR_E DR_E) + (IR_F DR_F) - (IR_E CR_E DR_E)(IR_F DR_F)\right]. \qquad \text{(A-30)}$$

APPENDIX B

COMBINATION OF MIXED ITEMS OF EVIDENCE

In this appendix, we want to show how one can combine mixed items of evidence in an evidential diagram as given in Figure 1. We will follow almost the same procedure as discussed in Appendix A for the audit risk formula. However, there is one difference that we will not derive an algebraic formula in this case rather we will show the steps involved in combining such items of evidence. We will use these steps to illustrate our numerical examples.

As the first step, we combine the m-values at each level using Dempster's rule. If there are more than two items of evidence at a variable then it is much easier to combine two items at a time rather than combining all the evidence at the same time. This process can be easily programmed in a spreadsheet. As the second step, we propagate the m-values from 'E' and 'F' to 'O' and combine the resulting m-values with the m-values at 'O.' These steps complete the combination process of all the evidence (i.e., all the m-values) in the evidential diagram in Figure 1. Here are the details.

Step 1: Combining m-values directly bearing on the variable

Audit Objective level (O):

In the present case, at the objective level 'O,' we have three items of evidence: inherent factors (IF_O), analytical procedures (AP_O), and test of details (TD_O). The corresponding m-values are denoted by: $m_{IO}(o)$, $m_{IO}(no)$, and $m_{IO}(\{o, no\})$; $m_{PO}(o)$, $m_{PO}(no)$, and $m_{PO}(\{o, no\})$; $m_{DO}(o)$, $m_{DO}(no)$, and $m_{DO}(\{o, no\})$ (see Table 1 for definitions). Let us combine first m-values obtained from inherent factors and analytical procedures. The renormalization constant is given by

$$K_O^i = 1 - \mathbf{(}m_{IO}(o)m_{PO}(no) + m_{IO}(no)m_{PO}(o)\mathbf{)},$$

and the combined intermediate m-values (m_O^i's) as:

$$m_O^i(o) = (K_O^i)^{-1}\mathbf{[}m_{IO}(o)\mathbf{\{}m_{PO}(o) + m_{PO}(\{o, no\})\mathbf{\}} + m_{IO}(\{o, no\})m_{PO}(o)\mathbf{]},$$

$$m_O^i(no) = (K_O^i)^{-1}\mathbf{[}m_{IO}(no)\mathbf{\{}m_{PO}(no) + m_{PO}(\{o, no\})\mathbf{\}} + m_{IO}(\{o, no\})m_{PO}(no)\mathbf{]},$$

$$m_O^i(\{o, no\}) = (K_O^i)^{-1}m_{IO}(\{o, no\})m_{PO}(\{o, no\}).$$

Next, we combine m_O^i's with m_{DO}'s to obtain the overall m-values at 'O:'

$$K_O = 1 - \mathbf{(}m_O^i(o)m_{DO}(no) + m_O^i(no)m_{DO}(o)\mathbf{)}, \qquad \text{(B-1)}$$

$$m_O(o) = (K_O)^{-1}\mathbf{[}m_O^i(o)\mathbf{\{}m_{DO}(o) + m_{DO}(\{o, no\})\mathbf{\}} + m_O^i(\{o, no\})m_{DO}(o)\mathbf{]}, \qquad \text{(B-2)}$$

$$m_O(no) = (K_O)^{-1}\mathbf{[}m_O^i(no)\mathbf{\{}m_{DO}(no) + m_{DO}(\{o, no\})\mathbf{\}} + m_O^i(\{o, no\})m_{DO}(no)\mathbf{]}, \qquad \text{(B-3)}$$

$$m_O(\{o, no\}) = (K_O)^{-1}m_O^i(\{o, no\})m_{DO}(\{o, no\}). \qquad \text{(B-4)}$$

Errors and Defalcations level (E):

Similar to the audit objective level, we have three items of evidence at 'E:' inherent factor (IF_E), accounting systems and control procedures (CP_E), and test of details (TD_E). The corresponding m-values are denoted by: $m_{IE}(ne)$, $m_{IE}(e)$, and $m_{IE}(\{ne, e\})$; $m_{CE}(ne)$, $m_{CE}(e)$, and $m_{CE}(\{ne, e\})$; $m_{DE}(ne)$, $m_{DE}(e)$, and $m_{DE}(\{ne, e\})$ (see Table 1 for definitions). The steps involved in combining these m-values will be exactly the same as describe above.

Let us first combine m-values obtained from inherent factors, and accounting systems and control procedures. The renormalization constant is given by

$$K_E^i = 1 - \mathbf{(}m_{IE}(ne)m_{CE}(e) + m_{IE}(e)m_{CE}(ne)\mathbf{)},$$

and the combined intermediate m-values (m_E^i's) as:

$$m_E^i(ne) = (K_E^i)^{-1}\mathbf{[}m_{IE}(ne)\mathbf{\{}m_{CE}(ne) + m_{CE}(\{ne, e\})\mathbf{\}} + m_{IE}(\{ne, e\})m_{CE}(ne)\mathbf{]},$$

$$m_E^i(e) = (K_E^i)^{-1}\mathbf{[}m_{IE}(e)\mathbf{\{}m_{CE}(e) + m_{CE}(\{ne, e\})\mathbf{\}} + m_{IE}(\{ne, e\})m_{CE}(e)\mathbf{]},$$

$$m_E^i(\{ne, e\}) = (K_E^i)^{-1}m_{IE}(\{ne, e\})m_{CE}(\{ne, e\}).$$

Next, we combine m_E^i's with m_{DE}'s to obtain the combined m-values at 'E:'

$$K_E = 1 - \mathbf{(}m_E^i(ne)m_{DE}(e) + m_E^i(e)m_{DE}(ne)\mathbf{)}, \qquad \text{(B-5)}$$

$$m_E(ne) = (K_E)^{-1}\mathbf{[}m_E^i(ne)\mathbf{\{}m_{DE}(ne) + m_{DE}(\{ne, e\})\mathbf{\}} + m_E^i(\{ne, e\})m_{DE}(ne)\mathbf{]}, \qquad \text{(B-6)}$$

$$m_E(e) = (K_E)^{-1}\mathbf{[}m_E^i(e)\mathbf{\{}m_{DE}(e) + m_{DE}(\{ne, e\})\mathbf{\}} + m_E^i(\{ne, e\})m_{DE}(e)\mathbf{]}, \qquad \text{(B-7)}$$

$$m_E(\{ne, e\}) = (K_E)^{-1}m_E^i(\{ne, e\})m_{DE}(\{ne, e\}). \qquad \text{(B-8)}$$

Fraud level (F):

At this level, we have only two items of evidence: inherent factor (IR_F), and the test of details (TD_F). The corresponding m-values are denoted by: $m_{IF}(nf)$, $m_{IF}(f)$, and $m_{IF}(\{nf, f\})$; and

$m_{DF}(nf)$, $m_{DF}(f)$, and $m_{DF}(\{nf, f\})$ (see Table 1 for definitions). The renormalization constant is given by

$$K_F = 1 - \mathbf{(}m_{IF}(nf)m_{DF}(f) + m_{IF}(f)m_{DF}(nf)\mathbf{)}, \tag{B-9}$$

and the combined m-values are:

$$m_F(nf) = (K_F)^{-1}\mathbf{[}m_{IF}(nf)\mathbf{\{}m_{DF}(nf) + m_{DF}(\{nf, f\})\mathbf{\}} + m_{IF}(\{nf, f\})m_{DF}(nf)\mathbf{]}, \tag{B-10}$$

$$m_F(f) = (K_F)^{-1}\mathbf{[}m_{IF}(f)\mathbf{\{}m_{DF}(nf) + m_{DF}(\{nf, f\})\mathbf{\}} + m_{IF}(\{nf, f\})m_{DF}(f)\mathbf{]}, \tag{B-11}$$

$$m_F(\{nf, f\}) = (K_F)^{-1}m_{IF}(\{nf, f\})m_{DF}(\{nf, f\}). \tag{B-12}$$

Step 2: Propagation of m-values to 'O' from 'E' and 'F'

Here, we first propagate the m-values from 'E' and 'F' to 'O' and then combine the resulting m-values with the m-values due to the evidence directly bearing at 'O.' We have already discussed this process in Step 2 in Appendix A. The results are rewritten below for convenience.

$$m_{O\leftarrow E\&F}(o) = m_E(ne)m_F(nf) \tag{B-13}$$

$$m_{O\leftarrow E\&F}(no) = m_E(e)\Big(m_F(nf) + m_F(f) + m_F(\{nf, f\})\Big) + \Big(m_E(ne) + m_E(\{ne, e\})\Big)m_F(f) \tag{B-14}$$

$$m_{O\leftarrow E\&F}(\{o, no\}) = m_E(ne)m_F(\{nf, f\}) + m_E(\{ne, e\})m_F(nf) + m_E(\{ne, e\})m_F(\{nf, f\}) \tag{B-15}$$

Next, we combine the above m-values with the m-values at 'O' given in (B-2 - B-4). The result is:

$$K = 1 - \mathbf{(}m'_{O\leftarrow E\&F}(o)m_O(no) + m'_{O\leftarrow E\&F}(no)m_O(o)\mathbf{)}, \tag{B-16}$$

and

$$m^t_O(o) = (K)^{-1}\mathbf{[}m'_{O\leftarrow E\&F}(o)\mathbf{\{}m_O(o) + m_O(\{o, no\})\mathbf{\}} + m'_{O\leftarrow E\&F}(\{o, no\})m_O(o)\mathbf{]}, \tag{B-17}$$

$$m^t_O(no) = (K)^{-1}\mathbf{[}m'_{O\leftarrow E\&F}(no)\mathbf{\{}m_O(no) + m_O(\{o, no\})\mathbf{\}} + m'_{O\leftarrow E\&F}(\{o, no\})m_O(no)\mathbf{]}, \tag{B-18}$$

$$m^t_O(\{o, no\}) = (K)^{-1}m'_{O\leftarrow E\&F}(\{o, no\})m_O(\{o, no\}). \tag{B-19}$$

Based on the above m-values, the total belief that the objective is met and the plausibility (or the audit risk) that the objective is not met are given by:

$$Bel^t_O(o) = m^t_O(o) = (K)^{-1}\mathbf{[}m'_{O\leftarrow E\&F}(o)\mathbf{\{}m_O(o) + m_O(\{o, no\})\mathbf{\}} + m'_{O\leftarrow E\&F}(\{o, no\})m_O(o)\mathbf{]}, \tag{B-20}$$

$$Pl^t_O(no) = AR^t_O = 1 - Bel^t_O(o) = 1 - m^t_O(o). \tag{B-21}$$

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Figure 1

The Structure of Audit Evidence

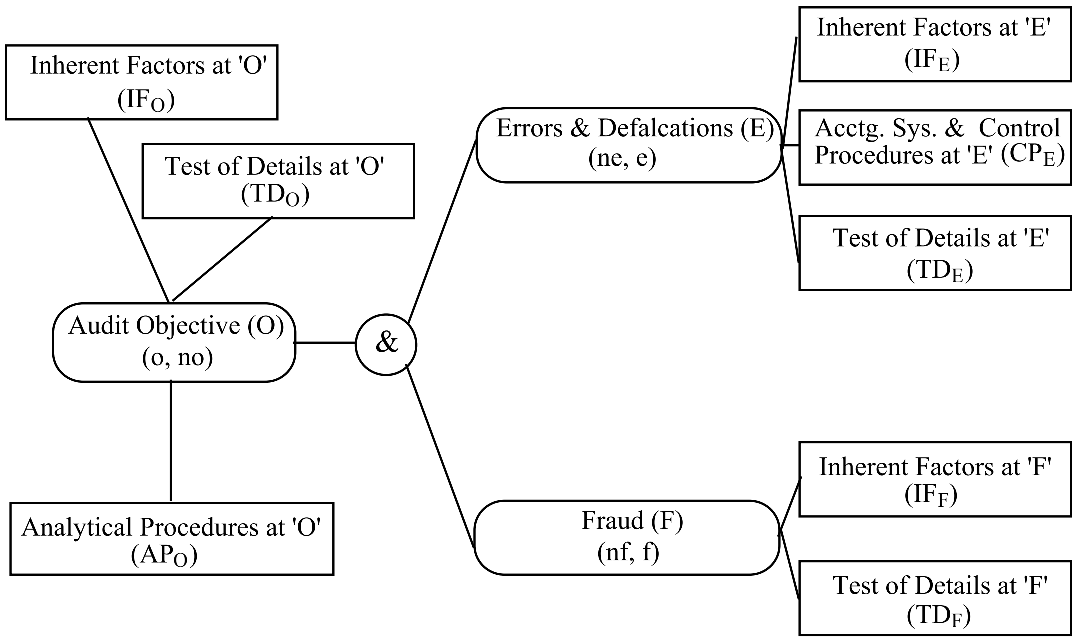

Table 1

List of Symbols and Their Definitions

Variables:

E	-	Random errors and defalcation
I	-	Fraud
O	-	Audit objective

Values of the variables:

e	-	There is a material misstatement due to random errors and/or defalcation.
ne	-	There is no material misstatement due to random errors or defalcation.
f	-	There is a material misstatement due to fraud.
nf	-	There is no material misstatement due to fraud.
o	-	The audit objective is met; there is no material misstatement.
no	-	The audit objective is not met; there is a material misstatement.

m-Functions, Belief Functions, and Plausibility Functions

$m_{CE}(.)$	-	m-values obtained from internal *controls* and accounting systems at 'E' for the proposition(s) in the argument.
$m_{DE}(.)$	-	m-values obtained from the test of *details* at 'E' for the proposition(s) in the argument.
$m_{IE}(.)$	-	m-values obtained from *inherent* factors at 'E' for the proposition(s) in the argument.
$m_{DF}(.)$	-	m-values obtained from the test of *details* at 'F' for the proposition(s) in the argument.
$m_{IF}(.)$	-	m-values obtained from *inherent* factors at 'F' for the proposition(s) in the argument.
$m_{DO}(.)$	-	m-values obtained from the test of *details* at the audit *objective* level for the proposition(s) in the argument.
$m_{IO}(.)$	-	m-values obtained from *inherent* factors at the audit *objective* level for the proposition(s) in the argument.
$m_{PO}(.)$	-	m-values obtained from analytical *procedures* at the audit *objective* level for the proposition(s) in the argument.
$Bel_x(.)$	-	A belief function, x represents various indices as described above in the case of m.
$Pl_x(.)$	-	A plausibility function, x again represents various indices as described in the case of m.

Plausibility Functions for Material Misstatements (i.e., Audit Risks*)

CR_E	-	Plausibility of material misstatement due to accounting systems and controls at 'E.'
DR_E	-	Plausibility of material misstatement due to test of details at 'E.'
IR_E	-	Plausibility of material misstatement due to inherent factors at 'E.'
DR_F	-	Plausibility of material misstatement due to test of details at 'F'.
IR_F	-	Plausibility of material misstatement due to inherent factors at 'F'.
APR_O	-	Plausibility of material misstatement due to analytical procedures at 'O.'
DR_O	-	Plausibility of material misstatement due to test of details at 'O.'
IR_O	-	Plausibility of material misstatement due to inherent factors at 'O.'

* Note that we have used the term "risk" for plausibility of material misstatement in the table and also in the text as used by Srivastava and Shafer (1992).

Table 2

Audit Risk Model at the Audit Objective (Assertion) Level in the Belief-Function Framework.

	Input Risks								**Audit Risk at Various Levels***			
	Error and Defalcation Level (E)			Fraud Level (F)		Assertion Level (O)			Error and Defalcation Level (E)	Fraud Level (F)	Assertion Level (O)	Total Audit Risk at Objective Level (O)
	IR_E	CR_E	DR_E	IR_F	DR_F	IR_O	APR_O	DR_O	AR_E	AR_F	AR_O	AR_O^t
1	**0.9**	0.6	0.15	0.0	1.0	1.0	0.7	1.0	0.081	0.0	0.7	0.057
2	**0.8**	0.6	0.15	0.0	1.0	1.0	0.7	1.0	0.072	0.0	0.7	0.050
3	**0.7**	0.6	0.15	0.0	1.0	1.0	0.7	1.0	0.063	0.0	0.7	0.044
4	**0.9**	0.6	0.15	1.0	1.0	1.0	0.7	1.0	0.081	1.0	0.7	0.7
5	**0.8**	0.6	0.15	1.0	1.0	1.0	0.7	1.0	0.072	1.0	0.7	0.7
6	**0.7**	0.6	0.15	1.0	1.0	1.0	0.7	1.0	0.063	1.0	0.7	0.7
7	**0.9**	0.6	0.15	**0.9**	1.0	**0.9**	0.7	1.0	0.081	0.9	0.63	0.57
8	**0.8**	0.6	0.15	**0.8**	1.0	**0.9**	0.7	1.0	0.072	0.8	0.63	0.51
9	**0.7**	0.6	0.15	**0.7**	1.0	**0.9**	0.7	1.0	0.063	0.7	0.63	0.45
10	**0.9**	0.6	**1.0**	**0.9**	**0.3**	0.9	0.7	**0.1**	0.54	0.27	0.06	0.042
11	**0.8**	0.6	**1.0**	**0.8**	**0.3**	0.9	0.7	**0.1**	0.48	0.24	0.06	0.038
12	**0.7**	0.6	**1.0**	**0.7**	**0.3**	0.9	0.7	**0.1**	0.42	0.21	0.06	0.034

*AR_E, AR_F, and AR_O are calculated using the definitions in (5), and AR_O^t is calculated from (4) or (6).

Table 3

Audit Risk Model at the Assertion (or Audit Objective) Level with Mixed Items of Evidence in the Belief-Function Framework.

Panel A: **Input Beliefs**

	Error and Defalcation Level (E)						**Fraud Level**				**Assertion Level**					
	Inherent Factors		Accounting System & Controls		Test of Details of Balance		Inherent Factors		Test of Details of Balance		Inherent Factors		Analytical Procedures		Test of Details of Balance	
	$m_{IE}(ne)$	$m_{IE}(e)$	$m_{CE}(ne)$	$m_{CE}(e)$	$m_{DE}(ne)$	$m_{DE}(e)$	$m_{IF}(nf)$	$m_{IF}(f)$	$m_{DF}(nf)$	$m_{DF}(f)$	$m_{IO}(o)$	$m_{IO}(no)$	$m_{PO}(o)$	$m_{PO}(no)$	$m_{DO}(o)$	$m_{DO}(no)$
1	0.2	0.0	0.4	0.0	**0.9**	0.0	0.0	0.0	**0.0**	0.0	0.0	0.0	0.3	0.0	**0.0**	0.0
2	0.2	0.0	0.4	0.0	**0.9**	0.0	0.0	0.0	**0.8**	0.0	0.0	0.0	0.3	0.0	**0.0**	0.0
3	0.2	0.0	0.4	0.0	**0.0**	0.0	0.0	0.0	**0.8**	0.0	0.0	0.0	0.3	0.0	**0.9**	0.0
4	0.2	0.0	0.4	0.0	**0.9**	0.0	0.0	**0.3**	**0.0**	0.0	0.0	0.0	0.3	0.0	**0.0**	0.0
5	0.2	0.0	0.4	0.0	**0.9**	0.0	0.0	**0.3**	**0.8**	0.0	0.0	0.0	0.3	0.0	**0.0**	0.0
6	0.2	0.0	0.4	0.0	**0.0**	0.0	0.0	**0.3**	**0.8**	0.0	0.0	0.0	0.3	0.0	**0.9**	0.0

Panel B: **Output Beliefs or the Combined Beliefs**

	Combined Beliefs based on the Items of Evidence at Error and Defalcation Level (E) From (B-6 - B-7)			Combined Beliefs based on the items of evidence at Fraud Level (F) From (B-10 - B-11)			Combined Beliefs based on the Items of Evidence at Objective Level (O) From (B-2 - B-4)			Combined Beliefs at the Audit Objective Level (O) based on all the Items of Evidence at all Levels From (B-17 - B-21)			
	$m_E(ne)$	$m_E(e)$	AR_E*	$m_F(nf)$	$m_F(f)$	AR_F	$m_O(o)$	$m_O(no)$	AR_O	$m^t_O(o)$	$m^t_O(no)$	AR^t_O	$Bel^t_O(o)$
1	0.952	0.0	0.048	0.0	0.0	1.0	0.30	0.0	0.70	0.30	0.0	**0.70**	**0.30**
2	0.952	0.0	0.048	0.8	0.0	0.2	0.30	0.0	0.70	0.83	0.0	**0.17**	**0.83**
3	0.520	0.0	0.480	0.8	0.0	0.2	0.93	0.0	0.07	0.96	0.0	**0.04**	**0.96**
4	0.952	0.0	0.048	0.00	0.30	1.00	0.30	0.0	0.70	0.23	0.23	**0.77**	**0.23**
5	0.952	0.0	0.048	0.74	0.08	0.26	0.30	0.0	0.70	0.79	0.06	**0.21**	**0.79**
6	0.520	0.0	0.480	0.74	0.08	0.26	0.93	0.0	0.07	0.95	0.006	**0.05**	**0.95**

* In this table and the paper, we are defining the audit risk to be the plausibility that the corresponding objective is not met, e.g., $AR_E = Pl_E(e) = 1 - Bel_E(ne) = 1 - m_E(ne)$.

[i] Currently, the proposal of the standard is available from the AICPA (See Exposure Draft Announcements in *Journal of Accountancy,* September 1996).

[ii] In the figure the variables are represented by rectangles with rounded corners and the audit evidence by rectangles.

[iii] As a notation, upper case letters are used for names of variables and lower case letters for their values. For example, the variable audit objective "O" has two values 'o' and 'no' meaning that the objective is met or not met, respectively. Similarly, the variable "F" (Fraud) has two values 'f' and 'nf', meaning fraud or no fraud, respectively.

[iv] For all other combination of values for variables "E" and "F", variable "O" takes the value of 'no'. That is, when either or both variables "E" and "F" take the values of 'e' or 'f', variable "O" takes the value of 'no'

[v] Here we have $m_{DF}(nf) = 0.7$, $m_{DF}(f) = 0$, and $m_{DF}(\{nf, f\}) = 0.3$. These values yield $Pl(f) = 1 - Bel(nf) = 1 - 0.7 = 0.3$. By definition then we have $DR_{DF} = Pl_{DF}(f) = 0.3$.

[vi] Using Dempster's rule (Shafer 1976), the combined m-value for a subset A of frame Θ is

$$m(A) = K^{-1}\Sigma\{m_1(B_1)m_2(B_2)|B_1 \cap B_2 = A, A \neq \varnothing\},$$

where K is the renormalization constant:

$$K = 1 - \Sigma\{m_1(B_1)m_2(B_2)|B_1 \cap B_2 = \varnothing\}$$

and m_1 and m_2 represent the m-values for two independent items of evidence on the frame Θ. The second term in K represents the conflict between the two items of evidence. If K=0, that is, if the two items of evidence exactly contradict each other, then Dempster's rule cannot be used to combine such items of evidence.