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A GENERAL SCHEME FOR AGGREGATING EVIDENCE IN AUDITING: Propagation of Beliefs in Networks[#]

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ABSTRACT

In this paper, we discuss a general scheme for aggregating uncertain items of evidence using the belief-function framework and demonstrate the application of a computer system, PLEAS (PLanning and Evaluation Audit System) that automates the above scheme for planning and evaluation of an audit. Since the approach is based on a well established theory for combining evidence, we believe that the scheme will provide an effective and efficient audit. There are two sources for efficiency in our approach. One comes from the fact that the evidence at all levels of the account are properly combined. That is, the structure of evidence is properly incorporated into the scheme. Such a scheme is not currently available in the auditing literature. Also, the belief-function representation of uncertainty in the audit evidence leads to further efficiency. It should be noted that our approach also has limitations. Our computer system, PLEAS, is designed to be user friendly and allows the user (the auditor) to create an evidential network for an audit, input judgments about the degrees of support, and aggregate all the evidence.

A GENERAL SCHEME FOR AGGREGATING EVIDENCE IN AUDITING: Propagation of Beliefs in Networks

In this paper, we have two primary objectives. First, we discuss a general scheme for aggregating uncertain items of audit evidence using the belief-function framework. Second, we demonstrate the application of a computer system, PLEAS[1] (PLanning and Evaluation Audit System) that automates the above scheme for planning and evaluation of an audit. There are two important issues dealing with the aggregation of audit evidence. One deals with the representation of uncertainty in the evidence and the other deals with the structure of the evidence. This paper deals with both the issues.

Regarding the first issue, Shafer and Srivastava (1990) contend that uncertainty in the audit evidence cannot be fully described in terms of probability; the belief-function theory provides a better framework (see the discussion in Section I). Since the belief-function framework is less known, we provide a brief introduction to the theory in Appendix A. For more details see Shafer and Srivastava (1990), and Shafer (1976). An interesting aspect of the theory is that when belief functions are probability measures, a belief-function treatment becomes a Bayesian treatment (Akresh, Loebbecke and Scott 1988, and Shafer and Srivastava 1990).

Regarding the second issue that deals with the structure of audit evidence, we find that the existing literature treats it inadequately. For example, the audit-risk formulas of SAS 47 (AICPA 1983) and the CICA (Canadian Institute of Chartered Accountants 1984, Leslie 1984, Kinney, Jr. 1984) do not consider any structure in the audit evidence (Cushing and Loebbecke 1983, Graham 1985a - 1985e, Kinney, Jr. 1989, see also the discussions in Section I).

It is important to note that a well-defined methodology for aggregating evidence should improve audit efficiency and effectiveness. Our attempt is to meet this objective by formalizing the evidence aggregation process in the belief-function formalism. There are two sources for efficiency in our approach. One comes from the fact that the evidence at all levels of the account are properly combined (see the discussion below, also see Figure 7). Such a scheme is not currently available in the literature; evidence at the financial statement level and at the account level cannot be objectively aggregated with the evidence at the assertion level. Also, the belief-function representation of uncertainty in the audit evidence leads to further efficiency. This is because of the fact that, in the belief-function formalism, a positive but weak evidence means that the auditor has a belief in support of the assertion, say 0.4, and no support for the negation of the assertion which means that the remaining 0.6 degree of belief is unassigned representing the ignorance. If a new item of evidence is gathered then part of the remaining belief, in this case part of 0.6, or all of it could be assigned to the state that the assertion is met. But, using SAS 47 interpretation of support (which is one minus the risk) the auditor is forced to think, in the present case, that there is 0.6 degree of support for the negation of the assertion and thus he needs a lot more positive evidence to counter balance the support for the negation of the assertion.

In our approach, the structure of the audit evidence is represented by a network of variables; variables being the financial statements as a whole, the accounts constituting the financial statements, and the audit objectives of the accounts (see Figures 1 and 5). Networks are formed when one item of evidence is relevant to more than one audit objective or more than one account. An item of evidence that bears on a given variable is directly connected to that variable. When an item of evidence bears on more than one audit objective of an account or more than one account then it is connected to all the variables that it relates to. For example, Procedure 17 in Figure 5, confirmations of accounts receivable, which provides assurance to both existence and valuation objectives of the account, is connected to the variables corresponding to the existence and valuation objectives in the network. Aggregating such items of evidence becomes a task of propagating uncertainties through the network. We will use numerical examples to illustrate the propagation process in the paper.

We would like to emphasize that, in this paper, we are mainly concerned with (1) the process of creating the network that represents the structure of audit evidence, and (2) the technical details of propagating uncertainties in such a network for aggregation of evidence using the belief-function formalism. It is important to note that our approach has also certain limitations. These limitations are discussed in Section IV.

As mentioned earlier, our system, PLEAS, uses the belief-function framework to aggregate audit evidence. Also, it allows the auditor to develop a comprehensive audit-risk model specific to the engagement. In an earlier article by Shafer, Shenoy and Srivastava (1988) when the system was not yet developed, a conceptual discussion on the features of the system as to how it could be used for audit decisions were presented. The system is now developed and through this paper we want to demonstrate its application and make this system available[2] to other researchers.

The paper is divided into five sections. Section I discusses the problems with the probability interpretation of risks in SAS 47 and reviews the lack of attention to the structure of audit evidence. Section II shows the essential steps involved in aggregating all the evidence, i.e., propagating belief functions in a network, using a simplified auditing example. Section III illustrates how PLEAS can be used for planning and evaluation of an audit. Section IV lists the major limitations with our approach and describes directions for future research. Finally, Section V summarizes the results. Appendix A presents a brief discussion on the belief-function theory. Appendix B discusses the basic concepts related to Markov trees and how to construct such a tree from an evidential network.

I. BACKGROUND

The probability interpretation of the risks in the audit risk model of SAS 47 (AICPA 1983) leads to several inconsistencies. For example, if an auditor decides not to depend on any inherent factors in conducting an audit then, according to SAS 47, he or she would assess the maximum value of inherent risk to be one. But probability of one means certainty, i.e., it is certain that there are errors in the account. Actually, the Bayesian model (Leslie 1984) does give a value of one for the total audit risk when inherent risk is assumed to be one. But this is not what the auditor had in mind when deciding not to depend on inherent factors. This situation is best described in the belief-function framework as lack of evidence or ignorance (Shafer and Srivastava 1990).

To emphasize the problem with the probability interpretation, we present one more example. Suppose the auditor believes, on the basis of inherent factors, that the account is fairly stated but at the same time does not want to put too much weight on inherent factors. As suggested by SAS 47, he may assign a value less than the maximum, say 0.7, for the inherent risk in the model. This number when interpreted in probability terms means that, based on inherent factors, there is 30% chance that the account is not materially misstated but 70% change that the account is misstated. This suggests that the evidence is negative. This is contrary to the auditor's intuition. It is even worse if we set inherent risk at

0.5. What does it mean? Does it mean that the auditor is ignorant about the state of the account, as the usual interpretation in probability, or does it mean he has more knowledge about the account being not materially misstated than when the inherent risk was 0.7? A belief-function interpretation would be that, in the first case, 0.3 represents the degree of support from inherent factors that the account is not materially misstated and 0.7 uncommitted belief, while in the second case, 0.5 represents the support and the remaining 0.5 uncommitted belief.

It is well known that the audit evidence has structure (e.g., see Arens and Loebbecke 1991). In general, there are items of evidence that support the account at various levels: at the financial statement level, at the account level and at the audit objective or the management assertion level of the account . Some items of evidence may support more than one audit objective or assertion of an account . The structure depends on relevance of the evidence to the objective. For example, information about the competence and integrity of management will affect the entire financial statement. In contrast, a comparison of the previous year's accounts payable with the current period's accounts payable provides assurance to the accounts payable balance as a whole, while confirmations of receivables provide assurance only to the valuation and existence objectives of the account.

The audit-risk formulas of SAS 47 (AICPA 1983) and the CICA (Canadian Institute of Chartered Accountants 1984, Leslie 1984, Kinney, Jr. 1984) do not consider any structure in the audit evidence (Cushing and Loebbecke 1983, Graham 1985a - 1985e, Kinney, Jr. 1989). In fact, Graham (1985a - 1985e) points out, for example, that “overall assessments of audit risk for the financial statements taken as a whole are usually impractical for audit planning and engagement control," since inherent and control risks can vary from one account to another and from one class of transactions to another. Graham further suggests that the audit-risk model should be decomposed into components that determine audit risk for management assertions related to each account balance (accounts receivable, inventory, etc.) or class of transactions (purchases, sales, etc.). It should be mentioned that Boritz and Jensen (1985) discussed the hierarchical structure in audit evidence and proposed ways to combine such evidence using probability numbers to represent risks.

Recently the AICPA through SAS 55 (AICPA 1988) has recognized the importance of the structure of evidence in assessing control risk, but it has not provided guidelines for integrating the concept into the audit-risk model. Boritz and Wensley (1990) have used the structure of evidence to develop a computer system for audit planning and evaluation, but their system uses heuristic rules rather than formal theory to combine uncertainties. Such heuristics have been seen to fail in complex systems (Buchanan and Shortliffe, 1984).

Leslie, Aldersley, Cockburn, and Reiter (1986) recognize the importance of the structure of audit evidence and emphasize the point that assurances from various items of evidence need to be assessed at the management assertion level of the account and then combined. They consider the relationship between various accounts (e.g., accounts receivable depends on sales and cash receipts) in the aggregation process, but they assume the different items of evidence for various accounts independent[3]. Such an assumption, in general, is not true. For example, if controls related to a cycle are weak then the analytical reviews performed using the accounting numbers generated by the control system will not be as reliable, i.e. if the controls are weak then the results of analytical reviews would be unreliable.

II. AGGREGATION OF AUDIT EVIDENCE (i,e., PROPAGATION OF BELIEFS IN A NETWORK)

Aggregation of evidence is equivalent to propagation of beliefs in an evidential network. We will describe the propagation scheme using a numerical example and then demonstrate how it can be used in general to aggregate audit evidence. The scheme for propagating belief functions through an evidential network is based on the theoretical work of Mellouli (1987), Shafer, Shenoy and Mellouli (1987), and Shenoy and Shafer (1986, 1990). These authors show how an evidential network can be embedded in a *Markov* tree, and how local computations (see Step Four in Section II) can be used to propagate beliefs through such a tree (see the discussion in Appendix B on Markov tree). It should be noted that construction of a Markov tree from an evidential network is done only to facilitate the computational process. It does not affect the evidential relationships. Knowledge or evidence that relates variables in a network are represented by nodes in the corresponding tree. The local computation technique is more efficient when relatively few variables are affected by each item of evidence, i.e., when the nodes are all relatively small (see also Zarley, Hsia and Shafer 1988; and Shenoy 1989, 1990).

In general, the propagation scheme involves four main steps. The first step is to draw the network representing the connections among variables and items of evidence, based on the experience and judgment of the auditor. In practice, the auditing profession can develop a template for a given industry and the audit partner along with the manager can customize it for a specific audit. PLEAS can be used by the auditor to develop the template and also to customize it (see Section III).

The second step involves expressing the impact of each item of evidence in terms of a belief function over the variables. The auditor performs each procedure and evaluates its outcome. Based on the outcome he or she assesses[4] the level of support for different values of the variables to which the procedure is relevant.

The third step is to construct a Markov tree from an evidential network. The basic concepts related to a Markov tree and how to construct such a tree from an evidential network are discussed in Appendix B using simple auditing examples. When each item of evidence bears directly on only one variable in the network, the network is already a tree. Propagation of beliefs in such a tree will be similar to the process discussed in step four below. However, when one item of evidence bears on more than one variable then the network has to be converted into a Markov tree. We will prefer a Markov tree whose nodes are as small as possible. Our system, PLEAS, uses an algorithm for finding Markov trees that has been developed by Kong (1986). This algorithm is discussed in Appendix B.

The fourth step deals with the propagation of belief functions in the Markov tree developed in step three. Propagating beliefs is equivalent to propagating **m**-values where **m**-values represent the probability mass assigned to a subset or a set of values of the variables in the node (see Appendix A for more details). We will use numerical examples to illustrate the propagation.

There are two concepts that are important for understanding the propagation process: *vacuous extension*[5] and *marginalization.*[6] Vacuous extension of a belief function or **m**-values is needed when the belief function or the **m**-values are being sent from a smaller node (fewer variables) to a larger node (more variables). In contrast, a belief function or **m**-values are to be marginalized whenever they are being sent from a larger node to a smaller node.

At each node, **m**-values received from its neighbors are combined with the **m**-values defined at the node without double counting to obtain overall **m**-values at the node. To avoid double counting, **m**-values received from all the neighbors except from the one to which the resulting **m**-values will be propagated are combined and sent to that node. Dempster's rule is used to combine these **m**-values (see Appendix A).

In general, we will use **m**' with an appropriate subscript to denote the **m**-values being sent from one node to another node. Subscripts of **m**'s will be determined as follows. Letters will represent nodes, and an arrow in middle will represent the direction in which the **m**-values are being sent. For example, $\mathbf{m}'_{AR\to R}$ implies that **m**-values from AR are being sent to the node R.

Example

Consider a network of only three variables: accounts receivable (AR) as one variable and the two audit objectives, existence (E) and valuation (V), as the other two variables. Figure 1 represents such a network where each variable node is represented by a rounded rectangle and each item of evidence by a rectangle. The fourth node is a relational node, R, which represents the relationship between the three variable nodes. We assume, for simplicity, that E and V are the only relevant objectives for AR. Also, we assume that all the variables are binary, i.e., AR, E and V take only two values[7]. The accounts receivable is either fairly stated (*ar*) or not fairly stated (~*ar*). The existence objective is either met (*e*) or not met (~*e*). Similarly, the valuation objective is either met (*v*) or not met (~*v*). Further, we assume the relationship between AR, and, E and V to be an 'and' relationship, i.e., accounts receivable is fairly stated if and only if the existence and valuation objectives have been met (*ar* is true if and only if *e* and *v* are true).

Step One: Construction of Evidential Network

As the first step, we draw the evidential network. In the present example, we have three nodes: AR, E, and V. We know that nodes E and V are related to node AR by an 'and' relationship. Thus, we connect E and V to AR by a node R that represents the 'and' relationship (see Figure 1). Next, we connect all the items of evidence to the variable nodes on which they directly bear. In our example, we have four items of evidence — one evidence for each variable node and one that directly bears on both E and V. For simplicity, we are considering one audit procedure or a set of audit procedures as one item of evidence. Such an assumption is not necessary in general; one could consider each procedure as one item of evidence so long as the auditor is able to assess the impact of each audit procedure on each variable node it directly bears on.

Figure 1 here

The following four procedures (or set of procedures) are considered in this case as items of evidence (Arens and Loebbecke, 1991):

Evidence 1 - Analytical Procedures: (i) Review accounts receivable trial balance for large and unusual receivables. (ii) Calculate ratios indicated in carry-forward working papers and follow up any significant changes from prior years.

Evidence 2 - Test of Transactions for sales being valid and cash receipts being complete: (i) Trace recorded sales from the sales journal to the file of supporting documents, which includes a duplicate sales invoice, bill of lading, sales order, and customer order. (ii) Obtain the prelisting of cash receipts, compare prelisting with the duplicate deposit slip and also trace amounts to the cash receipts journal, testing for names, amounts, and dates.

Evidence 3 - Test of Transactions for sales and cash receipts being properly valued: (i) Trace selected duplicate sales invoices numbers from the sales journal to: a) Duplicate sales invoices, and check for the total amount recorded in journal, date, customer name and account classification. Check the pricing, extensions and footings. b) Bill of lading, duplicate sales order, and customer order and test for customer name, product description, quantity, and date. (ii) Perform a proof of cash receipts.

Evidence 4 - Test of Details of Balance: Confirm accounts receivable using positive confirmations above a given amount and perform alternative procedures for all confirmations not returned on the first and second request.

Step Two: Impact of Evidence on Variables

The second step involves expressing the impact of each item of evidence in terms of a belief function over the variables on which the item of evidence directly bears. Suppose that the analytical procedures performed (Evidence 1) suggests that AR is reasonable, i.e., fairly presented, and there is nothing to indicate that the account is materially misstated. Such an assessment can be expressed in terms of belief functions as $\mathbf{Bel}_{AR}(ar) = 0.4$ and $\mathbf{Bel}_{AR}(\sim ar) = 0$ where the subscript stands for the node at which the belief functions or **m**-values are defined. $\mathbf{Bel}_{AR}(ar) = 0.4$ means that, based on the analytical procedures alone, the auditor believes to a degree 0.4 that AR is fairly presented. One can write the above beliefs in terms of **m**-values as $\mathbf{m}_{AR}(ar) = 0.4$, $\mathbf{m}_{AR}(\sim ar) = 0$, and $\mathbf{m}_{AR}(\{ar,\sim ar\}) = 0.6$. Since **m**-values are used in the propagation process, we will represent each assessment in terms of **m**-values.

Assume that Evidence 2 and Evidence 3 provide the levels of support for the corresponding audit objectives given in Table 1. Also suppose the auditor finds, based on Evidence 4, that all the accounts involved in confirmations are valid and valued properly. However, the auditor assigns different level of assurance to different objectives. The auditor assigns 0.95 degree of support to existence objective being met and only 0.8 to the valuation objective being met. This assessment can be written in terms of belief functions as $\mathbf{Bel}_{E,V}(e) = 0.95$, $\mathbf{Bel}_{E,V}(\sim e) = 0$, $\mathbf{Bel}_{E,V}(v) = 0.8$, and $\mathbf{Bel}_{E,V}(\sim v) = 0$, or in terms of **m**-values[8] as: $\mathbf{m}_{E,V}(e\&v)=0.8$, $\mathbf{m}_{E,V}(\{e\&v,e\&\sim v\})=0.15$, $\mathbf{m}_{E,V}(\Theta_{E,V})=0.05$, with **m**-values for all other subsets of the frame $\Theta_{E,V}$ zero, where $\Theta_{E,V} = \{e\&v, \sim e\&v, e\&\sim v, \sim e\&\sim v\}$ (see Table 1).

Table 1 here

The belief function at node R representing the 'and' relationship between AR, and E and V can be expressed by $\mathbf{Bel}_R(\Theta_R) = 1$ or $\mathbf{m}_R(\Theta_R) = 1$ where $\Theta_R = \{ar\&e\&v, \sim ar\&\sim e\&v, \sim ar\&e\&\sim v, \sim ar\&\sim e\&\sim v\}$. This means that out of eight possible sets of values: $\{ar\&e\&v, ar\&\sim e\&v, ar\&e\&\sim v, ar\&\sim e\&\sim v, \sim ar\&e\&v, \sim ar\&\sim e\&v, \sim ar\&e\&\sim v, \sim ar\&\sim e\&\sim v\}$ only the four states $\{ar\&e\&v, \sim ar\&\sim e\&v, \sim ar\&e\&\sim v, \sim ar\&\sim e\&\sim v\}$ are allowed by 'and' relationship.

Step Three: Converting a Network into a Markov Tree

Since the concepts of a Markov tree are not as familiar to the readers of this article, we have discussed this step in a more detail in Appendix B. We use the algorithm developed by Kong (1986) to illustrate the Markov tree construction process from an evidential network. Figure A-1 represents a Markov tree constructed from the network in Figure 1. It should pointed out that there can be several Markov trees constructed from one evidential network. These trees are equivalent; they give the same final beliefs when all the items of evidence are aggregated but they may differ in terms of the number of nodes in the tree and the number of variables in each node. We use Kong's approach to construct a Markov tree that has nodes with as few variables as possible.

Step Four: Propagation of Beliefs or m-values through a Markov Tree

We will use the Markov tree in Figure A-1 to illustrate the propagation process. There are five nodes in the Markov tree, three leaf nodes (i.e., terminal nodes) and two connecting nodes. The connecting nodes are bigger (more variables) than the leaf nodes. As discussed in Appendix B, since a belief function is defined at each node, propagation of a belief function from a leaf node to a connecting node will be done by *vacuously extending* the belief onto the frame of the connecting node. While propagation from a connecting node to a leaf node will be done by *marginalizing* the belief onto the frame of the leaf node. There are three directions of propagation in the present example. We discuss two of these cases below.

Figure 2 here

Case 1: Propagation From Existence and Valuation Nodes To Accounts Receivable Node

This situation is depicted in Figure 2. In order to achieve the objective we need to complete the following steps, as discussed earlier:

(1) Propagate **m**-values from nodes E and V to node (E,V) by vacuously extending $\mathbf{m}_E$ and $\mathbf{m}_V$ onto the frame $\Theta_{E,V}$. This step yields the following non-zero values of $\mathbf{m}'_{E\rightarrow E,V}$ and $\mathbf{m}'_{V\rightarrow E,V}$:

$$\mathbf{m}'_{E\rightarrow E,V}(\{e\&v, e\&{\sim}v\}) = \mathbf{m}_E(e) = 0.9,\ \ \mathbf{m}'_{E\rightarrow E,V}(\Theta_{E,V}) = \mathbf{m}_E(\Theta_E) = 0.1,$$

and

$$\mathbf{m}'_{V\rightarrow E,V}(\{e\&v, {\sim}e\&v\}) = \mathbf{m}_V(v) = 0.8,\ \ \mathbf{m}'_{V\rightarrow E,V}(\Theta_{E,V}) = \mathbf{m}_V(\Theta_V) = 0.2.$$

(2) Combine $\mathbf{m}'_{E\rightarrow E,V}$, $\mathbf{m}'_{V\rightarrow E,V}$, and $\mathbf{m}_{E,V}$, all at (E,V) node, using Dempster's rule (A-6). The following non-zero **m**-values result[9]:

$$\mathbf{m}''_{E,V}(e\&v) = 0.956,\ \ \mathbf{m}''_{E,V}(\{e\&v, e\&{\sim}v\}) = 0.039,$$

$$\mathbf{m}''_{E,V}(\{e\&v, {\sim}e\&v\}) = 0.004,\ \ \mathbf{m}''_{E,V}(\Theta_{E,V}) = 0.001.$$

(3) Propagate the above **m**-values to node R by vacuously extending to the frame Θ_R. This yields the following non-zero **m**-values:

$$\mathbf{m}'_{E,V\to R}(ar\&e\&v) = \mathbf{m}'_{E,V}(e\&v) = 0.956,$$
$$\mathbf{m}'_{E,V\to R}(\{ar\&e\&v,\sim ar\&e\&\sim v\}) = \mathbf{m}'_{E,V}(\{e\&v,\ e\&\sim v) = 0.039,$$
$$\mathbf{m}'_{E,V\to R}(\{ar\&e\&v,\sim ar\&\sim e\&v\}) = \mathbf{m}'_{E,V}(\{e\&v,\sim e\&v\}) = 0.004,$$
$$\mathbf{m}'_{E,V\to R}(\Theta_R) = \mathbf{m}'_{E,V}(\Theta_{E,V}) = 0.001.$$

(4) Combine the above **m**-values with $\mathbf{m}_R$ defined exclusively at R. Since $\mathbf{m}_R(\Theta_R) = 1$, the above values will remain unchanged after the combination:

$$\mathbf{m}''_R(ar\&e\&v) = 0.956,\ \ \mathbf{m}''_R(\{ar\&e\&v,\sim ar\&e\&\sim v\}) = 0.039,$$
$$\mathbf{m}''_R(\{ar\&e\&v,\sim ar\&\sim e\&v\}) = 0.004,\ \ \mathbf{m}''_R(\Theta_R) = 0.001.$$

(5) Propagate the above **m**-values to AR by marginalizing onto the frame of AR. This process yields the following non-zero **m**-values:

$$\mathbf{m}'_{R\to AR}(ar) = \mathbf{m}''_R(ar\&e\&v) = 0.956,$$
$$\mathbf{m}'_{R\to AR}(\{ar,\sim ar\}) = \mathbf{m}''_R(\{ar\&e\&v,\sim ar\&e\&\sim v\}) + \mathbf{m}''_R(\{ar\&e\&v,\sim ar\&\sim e\&v\})$$
$$+ \mathbf{m}''_R(\Theta_R) = 0.039 + 0.004 + 0.001 = 0.044.$$

(6) Combine the above **m**-values with $\mathbf{m}_{AR}$ (see Figure 2 for its values) defined at node AR. This yields the total **m**-values at AR:

$$\mathbf{m}^t_{AR}(ar) = \mathbf{m}_{AR}(ar)\mathbf{m}'_{R\to AR}(ar) + \mathbf{m}_{AR}(ar)\mathbf{m}'_{R\to AR}(\{ar,\sim ar\})$$
$$+ \mathbf{m}_{AR}(\{ar,\sim ar\})\mathbf{m}'_{R\to AR}(ar)$$
$$= 0.4\text{x}0.956 + 0.4\text{x}0.044 + 0.6\text{x}0.956 = 0.9736,$$
$$\mathbf{m}^t_{AR}(\sim ar) = 0,$$
$$\mathbf{m}^t_{AR}(\{ar,\sim ar\}) = \mathbf{m}_{AR}(\{ar,\sim ar\})\mathbf{m}'_{R\to AR}(\{ar,\sim ar\}) = 0.6\text{x}0.044 = 0.0264.$$

The above six steps complete the propagation of **m**-values from all the nodes to AR node. The total belief for various values of AR are given as (use A-4):

$$\mathbf{Bel}^t_{AR}(ar) = 0.9736,\ \ \mathbf{Bel}^t_{AR}(\sim ar) = 0,\ \text{and}\ \ \mathbf{Bel}^t_{AR}(\{ar,\sim ar\}) = 1.$$

Case 2: Propagation From Accounts Receivable and Valuation Nodes to Existence Node

Propagation of **m**-values from AR and V nodes to E node (see Figure 3) is similar to the propagation process from E and V to AR as discussed above. In this case, we need to complete the following steps:

(1) Propagate $\mathbf{m}_{AR}$ defined at AR to R by vacuously extending them onto frame Θ_R. This yields $\mathbf{m}'_{AR\to R}$. The non-zero values of $\mathbf{m}'_{AR\to R}$ are given below:

$$\mathbf{m}'_{AR\to R}(ar\&e\&v) = \mathbf{m}_{AR}(ar) = 0.4, \text{ and } \mathbf{m}'_{AR\to R}(\Theta_R) = \mathbf{m}_{AR}(\Theta_{AR}) = 0.6.$$

(2) Combine $\mathbf{m}'_{AR\to R}$ with $\mathbf{m}_R(\Theta_R)$ =1 and propagate the resulting **m**-values from R to (E,V) node. For the purpose of propagation, we marginalize the combined **m**-values onto frame $\Theta_{E,V}$. This process yields the following non-zero **m**-values:

$$\mathbf{m}'_{R\to E,V}(e\&v) = 0.4, \text{ and } \mathbf{m}'_{R\to E,V}(\Theta_{E,V}) = 0.6.$$

(3) Propagate $\mathbf{m}_V$ from V to (E,V) by vacuously extending onto frame $\Theta_{E,V}$ as done in step one in the previous case. This yields the following non-zero **m**-values:

$$\mathbf{m}'_{V\to E,V}(\{e\&v,{\sim}e\&v\}) = \mathbf{m}_V(v) = 0.8, \text{ and } \mathbf{m}'_{V\to E,V}(\Theta_{E,V}) = \mathbf{m}_V(\Theta_V) = 0.2.$$

(4) Combine the three **m**-values, $\mathbf{m}_{E,V}$, $\mathbf{m}'_{R\to E,V}$, and $\mathbf{m}'_{V\to E,V}$, all at node (E,V). This yields the following nonzero **m**-values[10]:

$$\mathbf{m}''_{E,V}(e\&v) = = 0.952,\ \mathbf{m}''_{E,V}(\{e\&v,e\&{\sim}v\}) = 0.018,$$

$$\mathbf{m}''_{E,V}(\{e\&v,{\sim}e\&v\}) = 0.024,\ \mathbf{m}''_{E,V}(\Theta_{E,V}) = 0.006.$$

(5) Propagate $\mathbf{m}''_{E,V}$ to node E by marginalizing onto frame Θ_E. This process gives $\mathbf{m}'_{E,V\to E}$:

$$\mathbf{m}'_{E,V\to E}(e) = \mathbf{m}''_{E,V}(e\&v) + \mathbf{m}''_{E,V}(\{e\&v,e\&{\sim}v\}) = 0.952 + 0.018 = 0.97,$$

$$\mathbf{m}'_{E,V\to E}(\Theta_E) = \mathbf{m}''_{E,V}(\{e\&v,{\sim}e\&v\}) + \mathbf{m}''_{E,V}(\Theta_{E,V}) = 0.024 + 0.006 = 0.03.$$

(6) Combine $\mathbf{m}'_{E,V\to E}$ with $\mathbf{m}_E$ to obtain the total **m**-values, $\mathbf{m}^t_E$, at node E:

$$\mathbf{m}^t_E(e) = \mathbf{m}_E(e)\left[\mathbf{m}'_{E,V\to E}(e) + \mathbf{m}'_{E,V\to E}(\Theta_E)\right] + \mathbf{m}_E(\Theta_E)\mathbf{m}'_{E,V\to E}(e)$$

$$= 0.9\ (0.97 + 0.03) + 0.1\text{x}0.97 = 0.997,$$

$$\mathbf{m}^t_E({\sim}e) = 0,$$

$$\mathbf{m}^t_E(\Theta_E) = \mathbf{m}_E(\Theta_E)\mathbf{m}'_{E,V\to E}(\Theta_E) = 0.1\text{x}0.03 = 0.003.$$

These steps complete the propagation process from AR and V to E. The total belief function at E is given by (use A-4):

$$\mathbf{Bel}_E^t(e) = 0.997,\ \ \mathbf{Bel}_E^t(\sim e) = 0, \text{ and } \mathbf{Bel}_E^t(\Theta_E) = 1.$$

The above results imply that when all the items of evidence at various nodes in Figure 3 are aggregated then the total belief that the existence objective is met, i.e., 'e' is true is 0.997 and the belief in $\sim e$ is zero. Propagation from AR and E to V is similar to this case and thus will not be discussed in this paper. Our system is programmed in such a way that it gives us the total belief at all the nodes of the network after considering all possible propagations.

Figure 3 here

III. AN EXAMPLE OF PLANNING AND EVALUATION OF AN AUDIT USING 'PLEAS'

Let us consider the simple network given in Figure 4 to illustrate audit planning and evaluation using 'PLEAS'. The network in Figure 4 relates accounts receivable (AR) being fairly presented to its three audit objectives: Existence, Completeness, and Valuation, through an 'and' node. We are assuming, for simplicity, that all other audit objectives have been met. Each audit objective of AR is connected to a set of sub-objectives, again, through an 'and' node. We are limiting the number of sub-objectives to two in each case.

Figure 4 here

Figure 5 represents the network in Figure 4 with items of evidence represented by rectangular boxes. The shaded boxes imply that the procedures related to those items of evidence have not been performed yet or judgment about the level of support is not yet input into the system. The connection or link between various procedures (or evidence) and variable nodes is based on the illustration of Arens and Loebbecke (1991: 391-393). We have mapped their relationship in our network in Figure 5. It should noted that the main purpose of our system is to allow the user to draw a network that he or she feels relevant in the situation. Let us say that Figure 5 represents the audit team's judgment about how audit objective and evidence should be related.

The next step is to analyze how much support we need or can obtain from each item of evidence. We will disuss two scenarios. One, where all the evidence is positive, i.e., each item of evidence supports directly the variable or set of variables being met. The other, where inherent factors represent a negative item of evidence, i.e., based on inherent factors the auditor feels that the account may be materially misstated. We will show, in the two cases, what level of support is needed from each item of evidence to obtain a satisfactory level of overall support, say 0.95, that the account is fairly stated.

Figure 5 here

Scenario One: All Positive Evidence

As shown in Figure 5, the total support that AR is fairly stated is 0.679 for a level of support of 0.3 and 0.4 from inherent factors, and analytical procedures, respectively, at the account level and a level of support of 0.6 from each of the test of control procedures[12]. Next, if we assume that each substantive test of transactions is performed at a level of support 0.8 then the total degree of support for AR that it is fairly stated is increased to 0.933 (see Figure 6). The overall support for various sub-objectives varies from 0.960 to 0.997, a relatively higher level of support than that for the accounts receivable.

Figure 6 here

Finally, as shown in Figure 7, when confirmations and related alternative procedures (Procedure 17) are performed at a level of support 0.7, the overall support for AR being fairly stated becomes 0.952, an acceptable level of support. Thus, for planning purposes, the auditor can use the above information to decide the extent, timing and nature of procedures based on what he or she already knows about the client and the account, and also based on what other procedures have already been performed. The extent, timing and nature of a procedure will determine the level of support the auditor might achieve from it[13]. The system allows the auditor to use evidence gathered at all levels of the account and also permits him or her to include all possible types of dependency in the network. Such an analysis has not been possible with SAS 47 or CICA formulas.

Figure 7 here

It is interesting to note that without considering inherent factors and analytical procedures at the account level the total support for AR being fairly stated is only 0.906 (see Figure 8). Although inclusion of inherent factors has been suggested in both SAS 47 and CICA formulas, a proper treatment has been lacking in both these approaches.

Figure 8 here

Scenario Two: Inherent Factors Negative

In this case, we assume that the auditor, based on the prior years' experience and other relevant inherent factors, believes to degree 0.3 that the accounts receivable will be materially misstated. In such a situation the auditor must do extensive work at both the levels: (1) tests of transactions, and (2) tests of details of balance. Of course, the auditor may feel reluctant to depend too much on internal controls. Such a situation is depicted in Figure 9 where the level of support from the controls are assumed to be only 0.5. It is observed that with this negative evidence based on inherent factors, the auditor needs to perform substantive tests of transactions and Procedure 17 at a level of support 0.9 to obtain an overall support of 0.95 for AR that it is not materially misstated.

Figure 9 here

Next, suppose we want to evaluate an audit where inherent factors were negative and the auditor found some errors. In such a case, the auditor must evaluate the situation in terms of the level of support that the evidence provides for or against the account being fairly stated and input the assessment into the system and 'evaluate' the network. If the aggregated support for AR being fairly stated is below the desired level, 0.95, then the auditor must take appropriate action in terms modifying the account or correcting the error. Such an action should provide a revised level of support for the account being fairly stated from that item of evidence. Again, the new assessment should be entered into the system and the network be evaluated to obtain the overall support for or against the account being fairly stated.

IV. LIMITATIONS AND FUTURE DIRECTIONS FOR RESEARCH

Beside the advantages of the present approach in representing uncertainties and handling the structure of audit evidence as discussed in Section I, we have several limitations that should be discussed. First, we have assumed that an account is either materially misstated or not materially misstated and thus we do not distinguish between material misstatement due to overstatement or understatement. This limitation will make the audit process less efficient. For example, if there were two accounts, one was materially overstated and the other was materially understated for the same amount, and the auditor feels that the combination of the two accounts is fairly stated because of the off-setting errors, the present approach will suggest that the combination is materially misstated and hence leads to inefficiency. However, one can consider such a situation by treating the information about off-setting errors as a separate item of evidence at the financial statement level and input the judgment about the fairness of the overall accounts in combination.

Second, we have assumed each audit objective to be equally important. This assumption will also make the audit process less efficient because the auditor will have to obtain a high level of assurance for a less important audit objective. Third, we have not explicitly shown how to treat immaterial errors in different objectives or accounts that may become material when considered in combination. This limitation will make the audit process less effective because the present approach will suggest that the financial statements are fairly stated whereas the auditor feels contrary. However, this situation can be treated by considering the information, that all the immaterial errors found by the auditor add to a material amount, as a separate item of evidence at the appropriate level. For example, if there were immaterial errors accumulated from various accounts that add to a material amount then this information can be treated as a separate item of evidence at the financial statement level. The auditor can input the judgment as to whether the overall financial statements are fairly presented in light of all the immaterial errors. Further research is needed in the areas discussed above to make the audit process more efficient and effective.

Also, we would like to caution the users of PLEAS, especially in evaluating an audit, that assessment of the level of support from various items of evidence be determined conservatively. Since these assessments are based on subjective judgments and in the absence of any guidelines, one might be tempted to overstate these values. Empirical research is needed to establish the relationship between the level of support from an item of evidence and the extent, timing and nature of the evidence before our system can be used as <u>a stand-alone device</u> for evaluation of an audit in practice. However, the auditor can use PLEAS to augment his or her overall judgment about the audit for both planning and evaluation.

There are many other issues that need to be addressed. These include the following: (1) what level of support or belief is obtained from a given statistical result of a test procedure for different variables, (2) how one can integrate statistical findings with non-statistical findings in belief-function framework, and (3) how one can combine belief aggregation with cost of evidence gathering to obtain the most effective and efficient audit strategy. All of these topics require further research. The above problems are not unique to the present approach but are also present in the existing approach of SAS 47.

V. SUMMARY AND CONCLUSION

We have discussed a general scheme for aggregating uncertain items of audit evidence using the belief-function framework. In general, the structure of audit evidence can be represented by a network of variables and evidence; variables being the financial statements as a whole, the accounts constituting the financial statements, and the audit objectives of the accounts. A network is formed when an item evidence supports more than one account or more than one objective of an account. It should be pointed out that inclusion of the structure of audit evidence in the aggregation process makes the audit process more efficient. Also, use of a structured approach for aggregating evidence should make the process more effective.

We have used our computer system, PLEAS, to demonstrate how it can be used for audit planning and evaluation. PLEAS is a friendly system, it allows the user to interactively draw an evidential network and input judgment about the uncertainties associated with each item of evidence. The system then aggregates all the evidence using the belief-function framework as discussed in this paper and provides an overall belief on all the variables of the evidential network. The overall belief that the objective is met or the account is fairly stated or the financial statements are fairly presented would help the auditor decide whether to give an unqualified opinion, qualify the opinion, modify the opinion or propose adjustments to the financial statements and give unqualified opinion.

We have used a simple example to illustrate an application of PLEAS, but the system can handle bigger networks that includes several accounts. Similar to audit objectives all the accounts would be connected to a relational node and then connected to the financial statement node. The system is more efficient with Macintosh Operating System 7 on a Macintosh IIci with eight megabyte memory. It takes about 2 minutes each to evaluate the networks in Figures 5 -9. But with System 6, a Mac. IIci will take about 15 - 17 minutes. In collaboration with one of the big six accounting firms, we are using PLEAS to develop templates for their audit in one given industry. It is very encouraging to note that the system takes only about six minutes to evaluate a big evidential network for accounts receivable with 13 nodes and 21 audit procedures. Further work is in progress on the application of PLEAS on a real audit. However, the users should be aware of the limitations of our approach as discussed in Section IV.

At the present, PLEAS handles only computations in the belief-function framework. We plan to extend the system's computing capability and user interface to include the Bayesian framework, which may be more useful to users accustomed to thinking in Bayesian terms.

FOOTNOTE

1. Shafer, Shenoy and Srivastava (1988) have used AUDITOR'S ASSISTANT (AA) for this system. However, since AA is commonly accepted as acronym for Arthur Andersen, we will use hereafter PLEAS for the system.

2. A copy of the system and documentation can be obtained by writing to the author and sending a 3.5" diskette.

3. Leslie et. al. (1986) assume error rates to be Poison distributed and also implicitly assume various items of evidence to be independent of each other as evidenced from their approach of summing various assurance factors (which in fact are parameters of Poisson distributions) to obtain the overall assurance.

4. Although it an empirical question, our experience with talking to several experienced partners of various accounting firms suggest that the auditors feel that it is more intuitive to think in terms of belief functions than to think of probabilities when dealing with audit evidence. For example, suppose the auditor has been involved with the client for several years and has very positive feeling about the management, and also does not see any problem with the industry. Hence, a positive feeling about the fair presentation of the overall financial statements. But, he is not willing to put too much weight on this evidence (or combination of evidence). It is more intuitive for him to say that he believes that the financial statements are fairly presented with a low level of assurance, say 0.3, and for the remaining belief of 0.7 he does not know what to do; it is definitely not in support of the

material misstatement. He will have to gather more evidence to reallocate 0.7. Representing the above evidence in probability terms does not quite express the auditor's feeling.

5. *Vacuous Extension:* Whenever a set of **m**-values is propagated from a smaller node (fewer variables) to a bigger node (more variables), the **m**-values are said to be *vacuously extended* onto the frame of the bigger node. As an illustration, suppose we have the following **m**-values on existence (E) node with frame $\Theta_E = \{e, \sim e\}$.

$$\mathbf{m}_E(e) = 0.9,\ \mathbf{m}_E(\sim e) = 0,\ \mathbf{m}_E(\{e,\sim e\}) = 0.1$$

We want to vacuously extend them to a bigger node consisting of existence (E) and valuation (V) objectives. The entire frame of this combined node is obtained by multiplying the two individual frames, $\Theta_E = \{e,\sim e\}$ and $\Theta_V = \{v,\sim v\}$. The resulting frame is $\Theta_{E,V} = \Theta_E \times \Theta_V = \{(e\&v), (e\&\sim v), (\sim e\&v), (\sim e\&\sim v)\}$. The vacuous extension of the above **m**-values from frame $\Theta_E = \{e,\sim e\}$ to frame $\Theta_{E,V}$ is as follows:

$$\mathbf{m}(\{(e\&v), (e\&\sim v)\}) = \mathbf{m}_E(e) = 0.9$$

$$\mathbf{m}(\{(e\&v), (e\&\sim v), (\sim e\&v), (\sim e\&\sim v)\}) = \mathbf{m}_E(\{e,\sim e\}) = 0.1$$

and **m**-values for other subsets of $\Theta_{E,V}$ are zero.

6. *Marginalization:* Propagating **m**-values from a bigger node to a smaller node is called *marginalization*. Let us consider the above example of Footnote 5 with the following **m**-values at $\Theta_{E,V}$ which is the frame of the combined nodes of existence and valuation:

$$\mathbf{m}_{E,V}(e\&v) = 0.8,$$

$$\mathbf{m}_{E,V}(\{(e\&v), (e\&\sim v)\}) = 0.1,$$

$$\mathbf{m}_{E,V}(\{(e\&v), (e\&\sim v), (\sim e\&v), (\sim e\&\sim v)\}) = 0.1,$$

all other **m**-values are zero.

Let us first marginalize onto the frame $\Theta_E = \{e,\sim e\}$. Similar to marginalization of probabilities, we will sum all the **m**-values over the elements of frame $\Theta_V = \{v,\sim v\}$ for a given set of elements of frame $\Theta_E = \{e,\sim e\}$, i.e.,

$$\mathbf{m}(e) = \mathbf{m}_{E,V}(e\&v) + \mathbf{m}_{E,V}(\{(e\&v), (e\&\sim v)\}) = 0.8 + 0.1 = 0.9,$$

$$\mathbf{m}_{E,V}(\sim e) = 0,$$

$$\mathbf{m}(\{e,\sim e\}) = \mathbf{m}_{E,V}(\{(e\&v), (e\&\sim v), (\sim e\&v), (\sim e\&\sim v)\}) = 0.1.$$

Marginalizing onto the frame $\Theta_V = \{v,\sim v\}$ yields the following values:

$$\mathbf{m}(v) = \mathbf{m}_{E,V}(e\&v) = 0.8,$$

$$\mathbf{m}_{E,V}(\sim v) = 0,$$

$$\mathbf{m}(\{v,\sim v\}) = \mathbf{m}_{E,V}(\{(e\&v),(e\&\sim v)\}) + \mathbf{m}_{E,V}(\{(e\&v),(e\&\sim v),(\sim e\&v),(\sim e\&\sim v)\})$$

$$= 0.1 + 0.1 = 0.2.$$

7. As a convention, we will use capital letters for the names of the variables in the network and small letters in script for their values. In general, the belief-function framework permits more than two values of a variable.

8. The given set of **m**-values yields the desired belief functions. For example, if we marginalize these **m**-values onto the frame $\{e,\sim e\}$ we get $\mathbf{m}_{E,V}(e) = 0.95$, $\mathbf{m}_{E,V}(\sim e) = 0$, and $\mathbf{m}_{E,V}(\{e,\sim e\}) = 0.05$ which yields $\mathbf{Bel}_{E,V}(e) = 0.95$, and $\mathbf{Bel}_{E,V}(\sim e) = 0$. Also, when we marginalize the **m**-values onto the frame of $\{V,\sim V\}$ we obtain $\mathbf{m}_{E,V}(V) = 0.8$, $\mathbf{m}_{E,V}(\sim V) = 0$, and $\mathbf{m}_{E,V}(\{V,\sim V\}) = 0.2$ which yields $\mathbf{Bel}_{E,V}(V) = 0.8$, and $\mathbf{Bel}_{E,V}(\sim V) = 0$.

9. Since the renormalization constant K in Dempster's rule (A-6) for this case is one, the combined non-zero **m**-values are obtained as follows:

$$\mathbf{m}''_{E,V}(e\&v) = \sum_{x\cap y\cap z=(e\&v)} \mathbf{m}_{E,V}(x)\,\mathbf{m}'_{E\to E,V}(y)\,\mathbf{m}'_{V\to E,V}(z)$$

$$= \mathbf{m}_{E,V}(e\&v)\left[\mathbf{m}'_{E\to E,V}(\{e\&v,e\&\sim v\}) + \mathbf{m}'_{E\to E,V}(\Theta_{E,V})\right]$$

$$\text{x}\left[\mathbf{m}'_{V\to E,V}(\{e\&v,\sim e\&v\}) + \mathbf{m}'_{V\to E,V}(\Theta_{E,V})\right]$$

$$+ \mathbf{m}_{E,V}(\{e\&v,e\&\sim v\})\left[\mathbf{m}'_{E\to E,V}(\{e\&v,e\&\sim v\})+\mathbf{m}'_{E\to E,V}(\Theta_{E,V})\right]\mathbf{m}'_{V\to E,V}(\{e\&v,\sim e\&v\})$$

$$+ \mathbf{m}_{E,V}(\Theta_{E,V})\mathbf{m}'_{E\to E,V}(\{e\&v,e\&\sim v\})\mathbf{m}'_{V\to E,V}(\{e\&v,\sim e\&v\})$$

$$= 0.8\,[0.9+0.1][0.8+0.2] + 0.15\,[0.9+0.1]0.8 + 0.05\text{x}0.9\text{x}0.8 = 0.956,$$

$$\mathbf{m}''_{E,V}(\{e\&v,e\&\sim v\}) = \mathbf{m}_{E,V}(\{e\&v,e\&\sim v\})\left[\mathbf{m}'_{E\to E,V}(\{e\&v,e\&\sim v\} + \mathbf{m}'_{E\to E,V}(\Theta_{E,V})\right]\mathbf{m}'_{V\to E,V}(\Theta_{E,V})$$

$$+ \mathbf{m}_{E,V}(\Theta_{E,V})\mathbf{m}'_{E\to E,V}(\{e\&v,e\&\sim v\})\mathbf{m}'_{E\to E,V}(\Theta_{E,V})$$

$$= 0.15\,[0.9 + 0.1]0.2 + 0.05\text{x}0.9\text{x}0.2 = 0.039,$$

$$\mathbf{m}''_{E,V}(\{e\&v,\sim e\&v\}) = \mathbf{m}_{E,V}(\Theta_{E,V})\,\mathbf{m}'_{E\to E,V}(\Theta_{E,V})\mathbf{m}'_{V\to E,V}(\{e\&v,\sim e\&v\})$$

$$= 0.05\text{x}0.1\text{x}0.8 = 0.004,$$

$$\mathbf{m}''_{E,V}(\Theta_{E,V}) = \mathbf{m}_{E,V}(\Theta_{E,V})\,\mathbf{m}'_{E\to E,V}(\Theta_{E,V})\mathbf{m}'_{V\to E,V}(\Theta_{E,V})$$

$$= 0.05\text{x}0.1\text{x}0.2 = 0.001.$$

10. Here, again, since the renormalization constant K in Dempster's rule (A-6) is one, the combined non-zero **m**-values are obtained as follows:

$$\mathbf{m}''_{E,V}(e\&v) = \sum_{x\cap y\cap z=(e\&v)} \mathbf{m}_{E,V}(x)\,\mathbf{m}'_{R\to E,V}(y)\,\mathbf{m}'_{V\to E,V}(z)$$

$$= \mathbf{m}_{E,V}(e\&v)\left[\mathbf{m}'_{R\to E,V}(e\&v)+\mathbf{m}'_{R\to E,V}(\Theta_{E,V})\right]$$

$$x\left[\mathbf{m}'_{V\to E,V}(\{e\&v,\sim e\&v\})+\mathbf{m}'_{V\to E,V}(\Theta_{E,V})\right]$$

$$+\ \mathbf{m}_{E,V}(\{e\&v, e\&\sim v\})\mathbf{m}'_{R\to E,V}(e\&v)\left[\mathbf{m}'_{V\to E,V}(\{e\&v,\sim e\&v\})+\mathbf{m}'_{V\to E,V}(\Theta_{E,V})\right]$$

$$+\ \mathbf{m}_{E,V}(\{e\&v, e\&\sim v\})\mathbf{m}'_{R\to E,V}(\Theta_{E,V})\mathbf{m}'_{V\to E,V}(\{e\&v,\sim e\&v\})$$

$$+\ \mathbf{m}_{E,V}(\Theta_{E,V})\mathbf{m}'_{R\to E,V}(e\&v)\left[\mathbf{m}'_{V\to E,V}(\{e\&v,\sim e\&v\})+\mathbf{m}'_{V\to E,V}(\Theta_{E,V})\right]$$

$$= 0.8\ [0.4+0.6][0.8+0.2] + 0.15\ x0.4\ [0.8+0.2]$$

$$+\ 0.15x0.6x0.8 + 0.05x0.4\ [0.8 + 0.2] = 0.952,$$

$$\mathbf{m}''_{E,V}(\{e\&v,e\&\sim v\}) = \mathbf{m}_{E,V}(\{e\&v,e\&\sim v\})\ \mathbf{m}'_{R\to E,V}(\Theta_{E,V})\mathbf{m}'_{V\to E,V}(\Theta_{E,V})$$

$$= 0.15x0.6x0.2 = 0.018,$$

$$\mathbf{m}''_{E,V}(\{e\&v,\sim e\&v\}) = \mathbf{m}_{E,V}(\Theta_{E,V})\ \mathbf{m}'_{R\to E,V}(\Theta_{E,V})\mathbf{m}'_{V\to E,V}(\{e\&v,\sim e\&v\})$$

$$= 0.05x0.6x0.8 = 0.024,$$

$$\mathbf{m}''_{E,V}(\Theta_{E,V}) = \mathbf{m}_{E,V}(\Theta_{E,V})\ \mathbf{m}'_{R\to E,V}(\Theta_{E,V})\mathbf{m}'_{V\to E,V}(\Theta_{E,V})$$

$$= 0.05x0.6x0.2 = 0.006.$$

11. It is assumed here that accounts receivable's completeness objective will be met if all cash receipts are valid and all existing sales are recorded. One could also include some other variables in this relationship such as the trial balance being complete or subsidiary ledger being complete. But, for simplicity of presentation, we are assuming those variables to have been met without any uncertainty.

12. The assessment of the level of support from a given item of evidence is a matter of professional judgment. The user has full freedom to choose whatever number he or she feels appropriate in the situation. We feel that inherent factors alone will not give much support to the account that it is fairly stated. That is the reason we choose a conservative value of 0.3. Similarly, we choose a small value of 0.4 for the analytical procedures and 0.6 for test of controls.

13. Further research is needed to determine the impact of timing, nature and extent of a procedure on the level of support one would obtain.

14. We assume here the existence of simple probabilities rather than conditional probabilities. That is, based on his or her experience, the auditor may determine the probabilities of a set of analytical procedures being effective or not being effective in detecting errors. Some may worry about the conditional probabilities, e.g., probabilities for the states that the set of procedures is effective and it detect errors given that errors exist or the set of procedure is not effective and it does not detect errors given that errors exist. If such a detailed knowledge is available then a Bayesian treatment will be appropriate for such situations rather than a belief-function treatment.

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Figure 1

The network of variables and evidence. Θ_X represents the frame for node X.
$\Theta_R = \{ar\&e\&v, \sim ar\&\sim e\&v, \sim ar\&e\&\sim v, \sim ar\&\sim e\&\sim v\}$

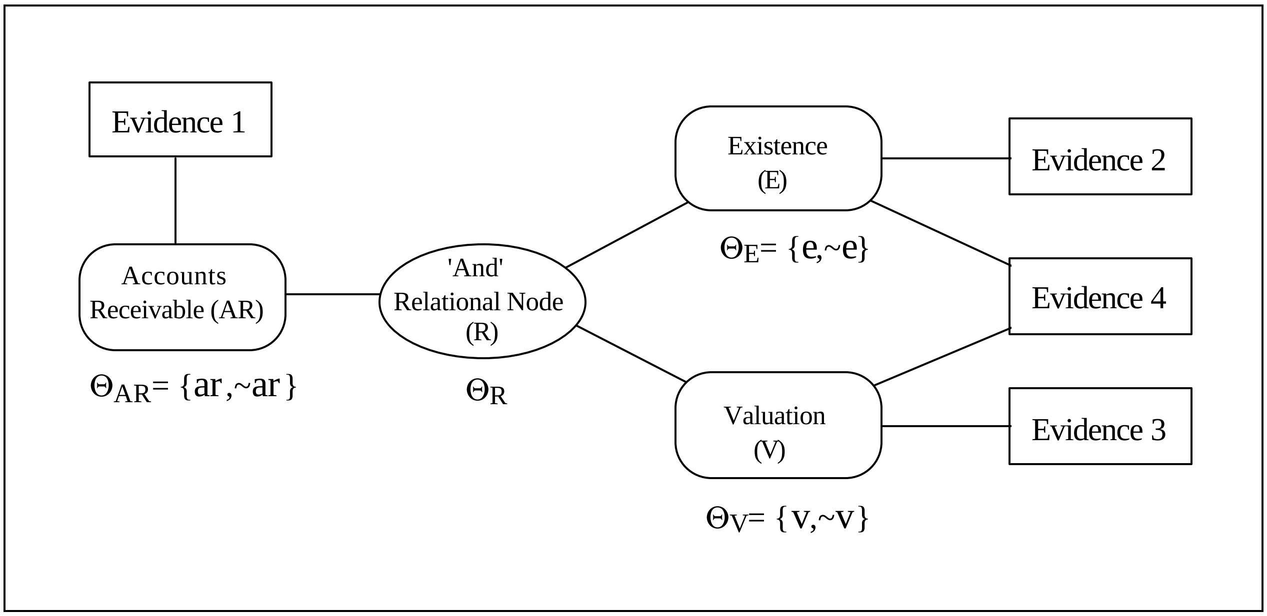


Figure 2

Propagation of **m**-values from E (Existence) and V (Valuation) nodes to AR (Accounts Receivable) node using Figure A-1(5-a). **m**-values defined at each node are given next to the node (see Table 1 for details).

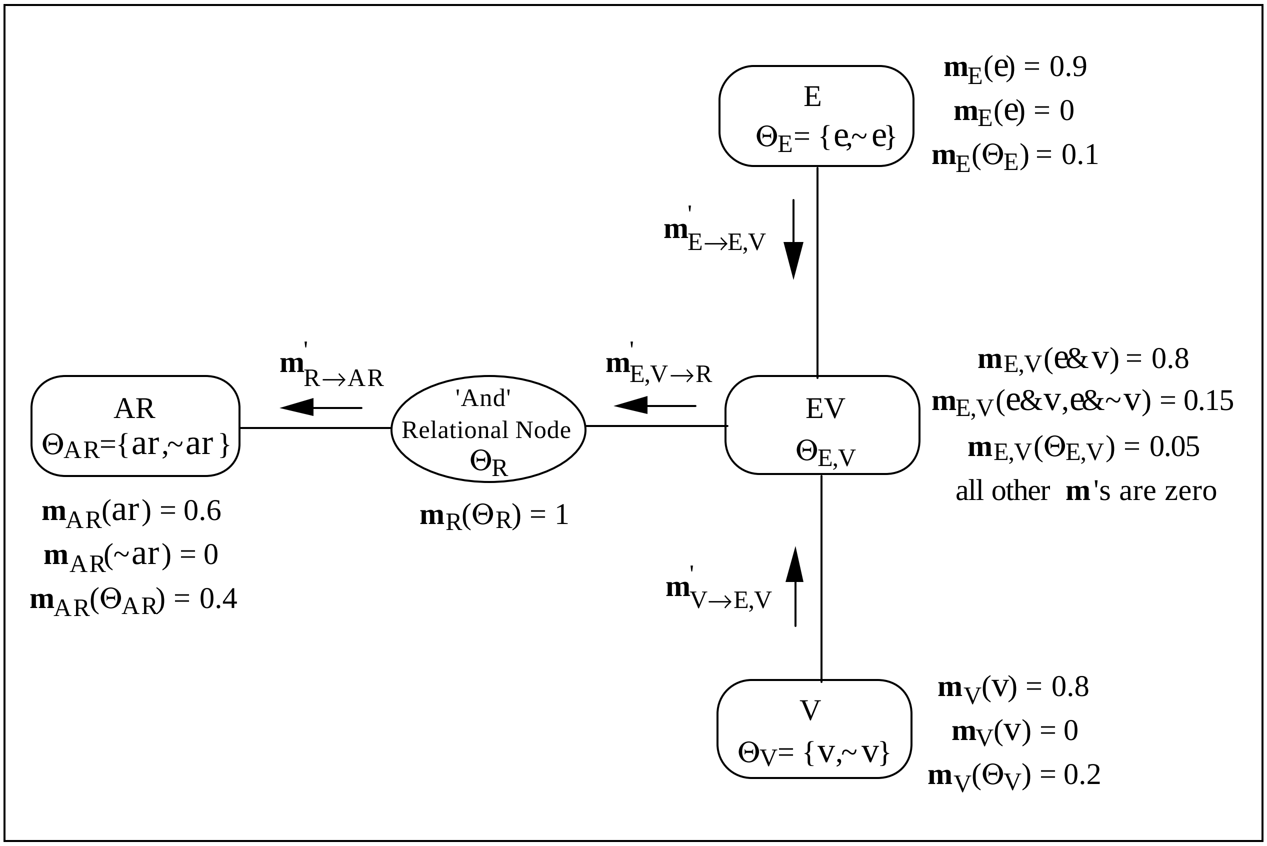


Figure 3

Propagation of **m**-values from AR (Accounts Receivable) and V (Valuation) nodes to E (Existence) node. **m**-values defined at each node are given next to the node.

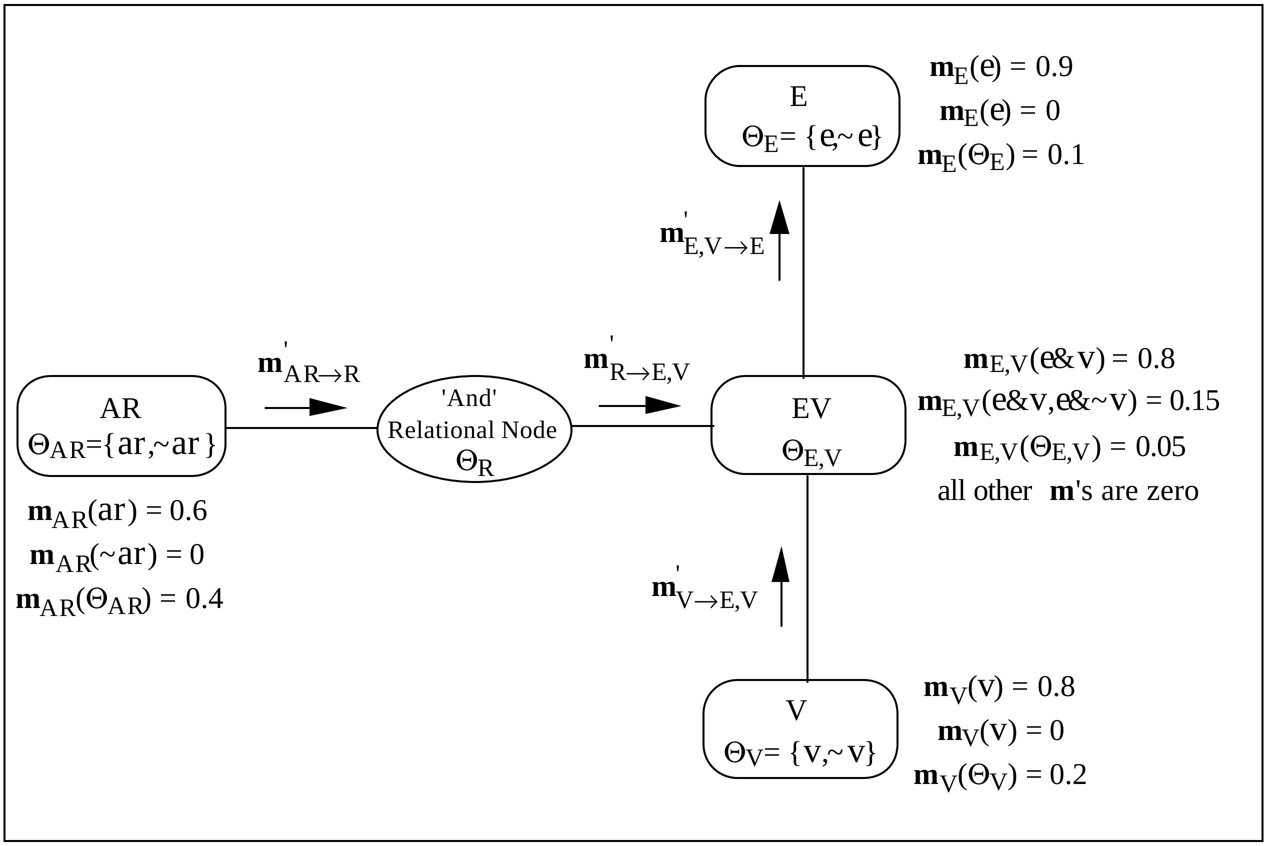


Figure 4

Network of variables created by using PLEAS for accounts receivable (AR). A rounded box represents a variable node and the symbol '&' enclosed in a circle represents an 'and' relationship between the variable on its left and the variables on its right. For example, 'AR complete' node is related to 'Cash Receipts Valid' and 'Sales Complete' through an 'and' node implying that AR will be complete if and only if cash receipts are valid and sales are complete.[11]

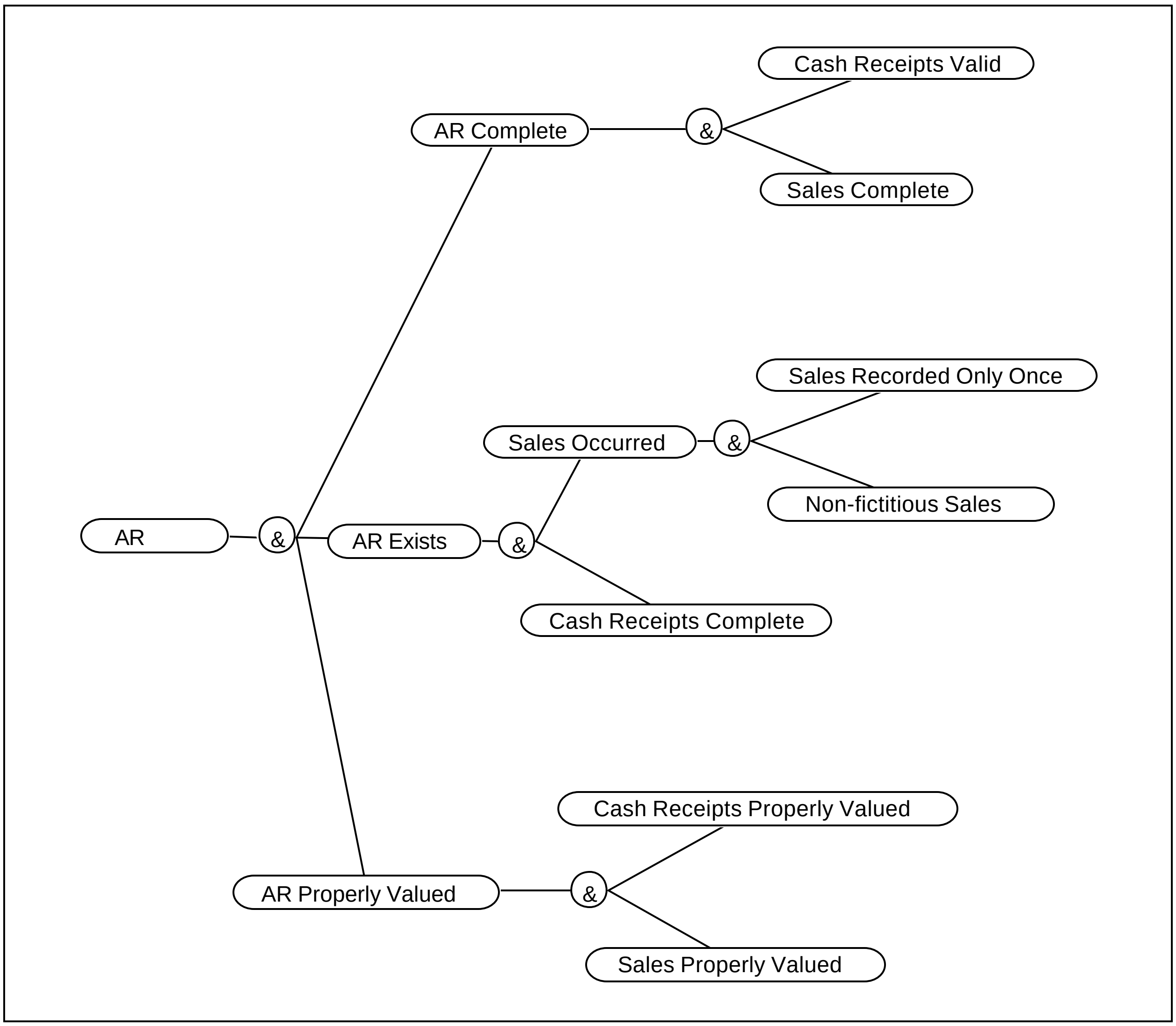
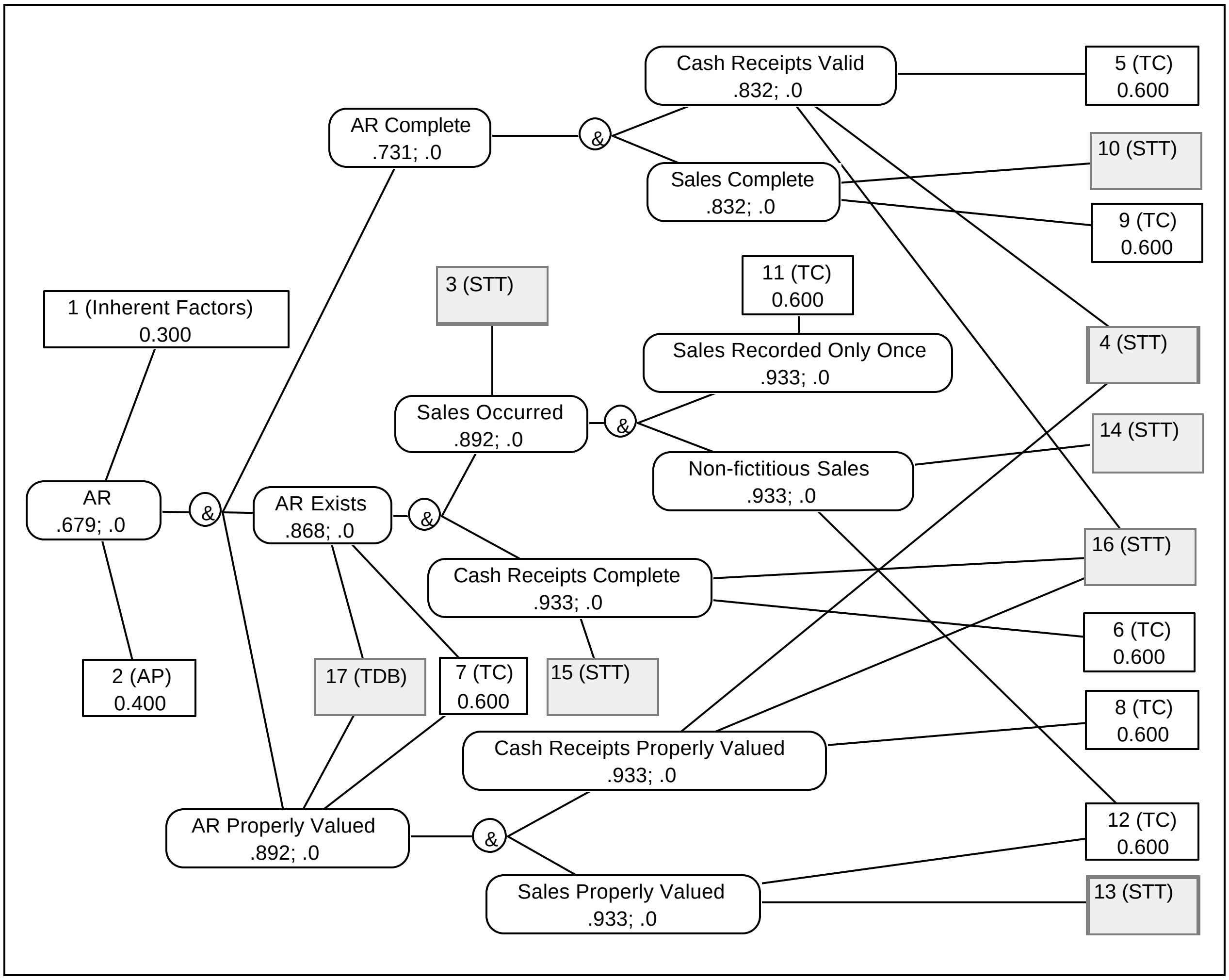


Figure 5

Evidential network with rectangular boxes as items of evidence*. This network is the result of aggregating all the evidence using PLEAS (i.e., in the system's terminology, after 'evaluating' the network).



* The top number inside a rectangular box represents the procedure number in Table 2 and the lower number represents the 'belief' or the level of support for the corresponding variable (or set of variables) that it is met. A shaded box implies that the procedure is not yet performed. The first number in a variable node represents the overall level of support for the objective that it is met and the second number represents the overall support for the objective that it is not met. The procedures used here have been selected from Arens and Loebbecke (1991). TC, STT, and TDB stand for test of controls, substantive test of transactions, and test of details of balance, respectively.

Figure 6

Evidential network for accounts receivable (AR) as the main node. All the procedures are performed except 17 (TDB) and the corresponding 'beliefs' are entered into the system and aggregated.

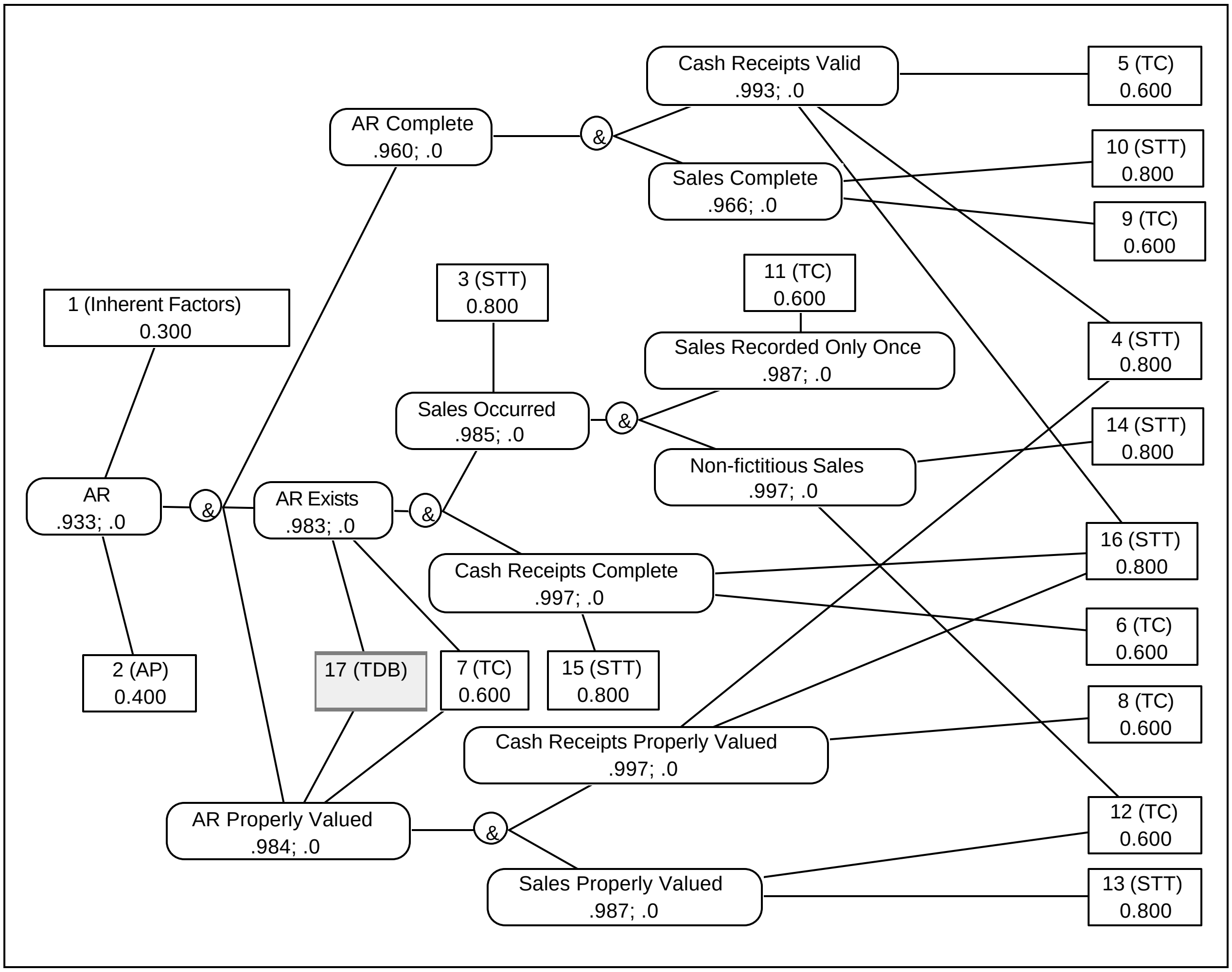


Figure 7

Evidential network for accounts receivable (AR) as the main node. All the procedures have been performed and the corresponding 'beliefs' have been aggregated by the system.

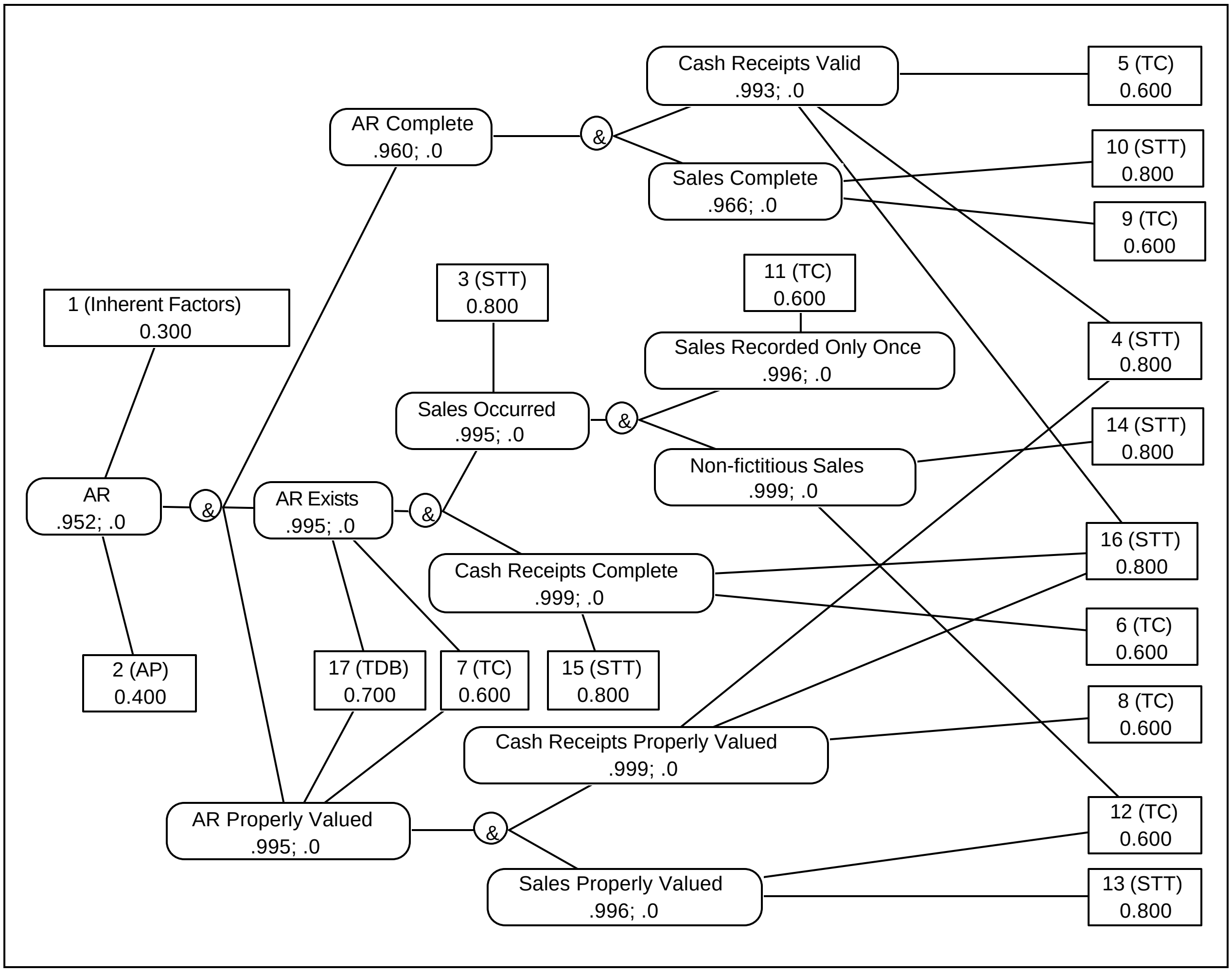


Figure 8

Evidential network for accounts receivable (AR) as the main node. All the procedures, except first two, have been performed and the corresponding 'beliefs' have been aggregated by the system.

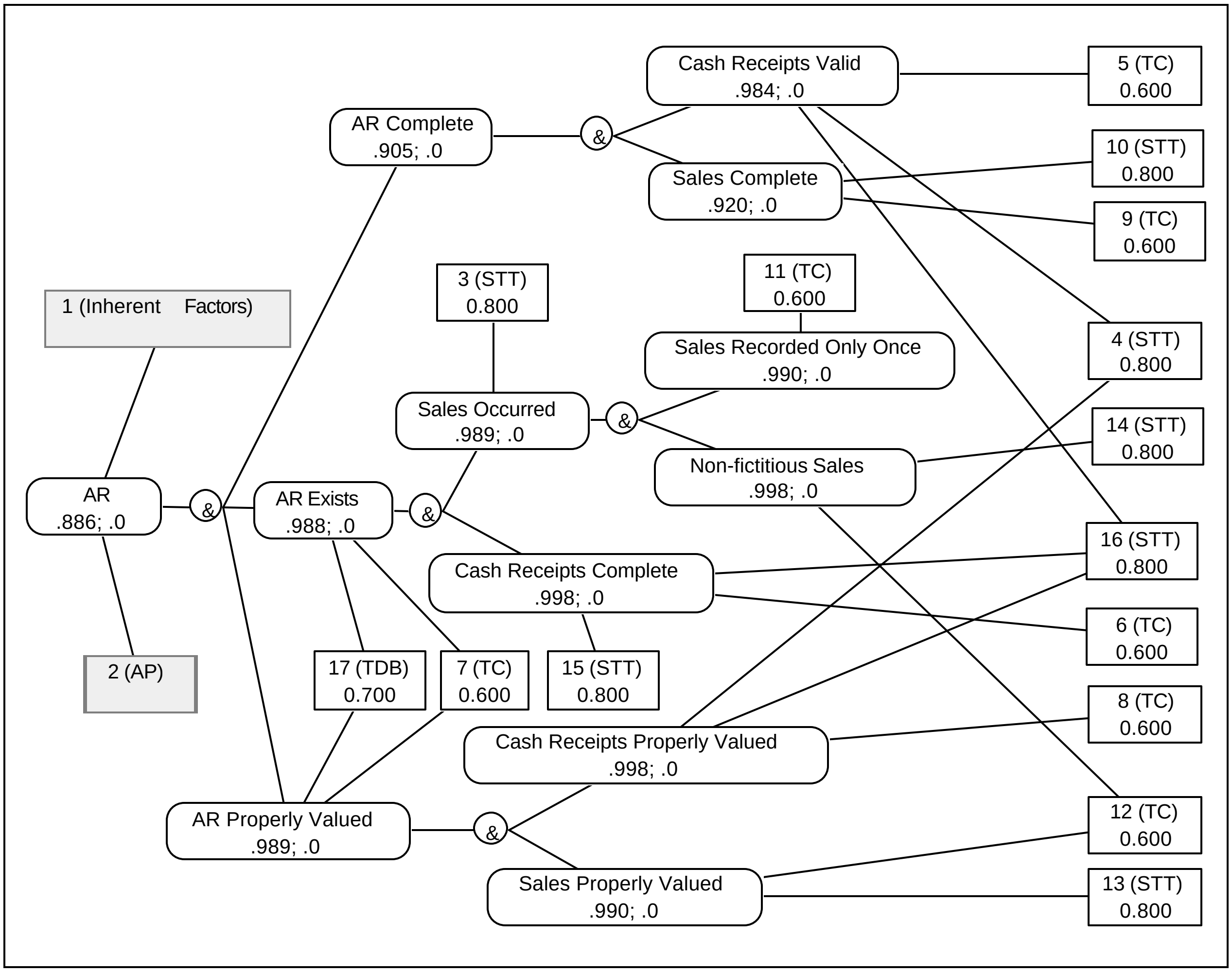
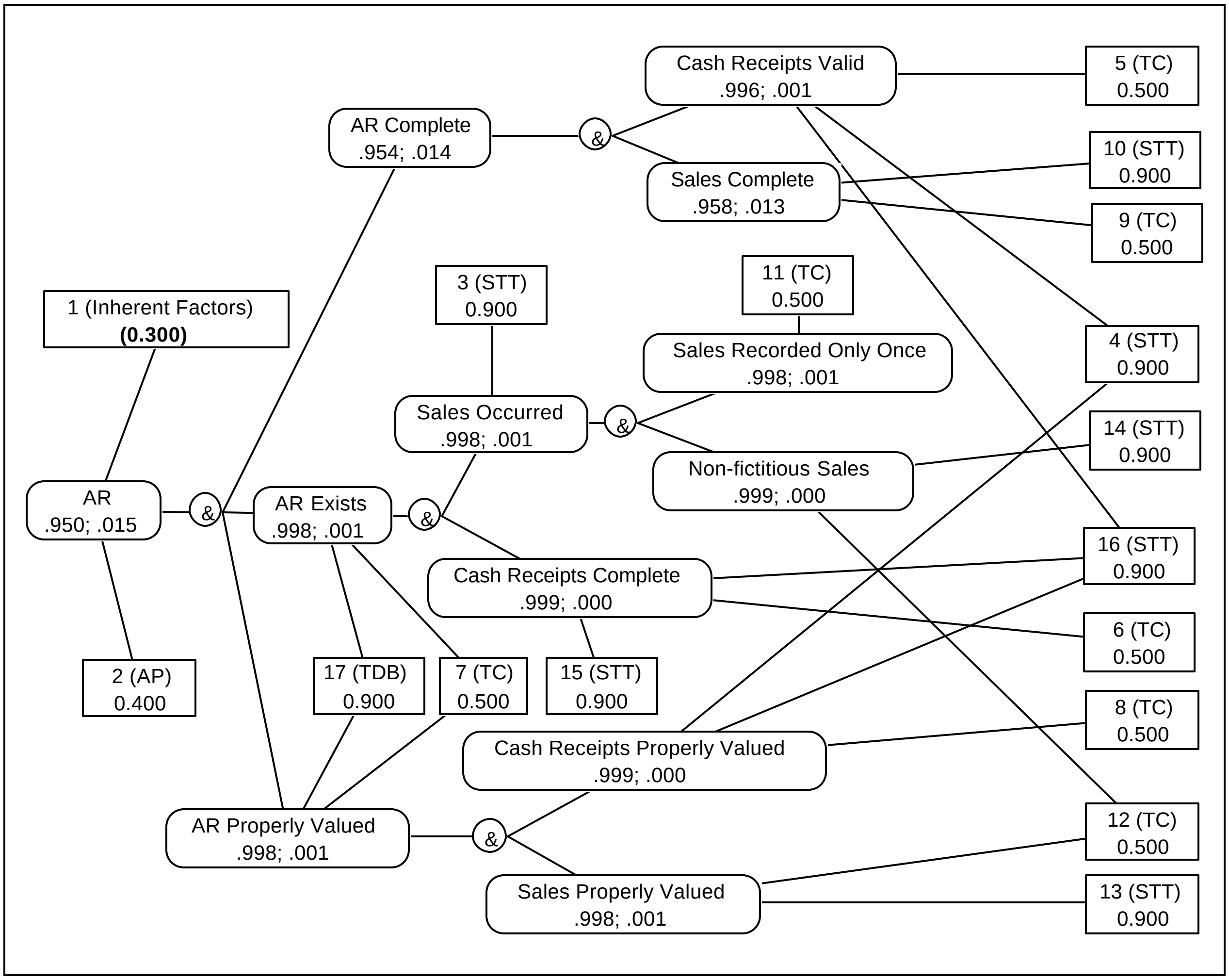


Figure 9

Evidential network for accounts receivable as the main node when inherent factors suggest that the account may be materially misstated with a degree 0.3 as shown by bold numbers in parenthesis in the box*.



*The current version of PLEAS just displays the input number without parentheses. We have added the parentheses to remind that the entry is in support of the negation.

Figure A-1

Markov Tree for the Network in Figure 1.

Nodes: N = {{AR}, {E}, {V}, {E,V}, {AR, E,V} }

Edges : E = {{{AR},{AR, E,V} }, {{E,V},{AR, E,V} }, {{E,V},{E} }, {{E,V},{V} }}

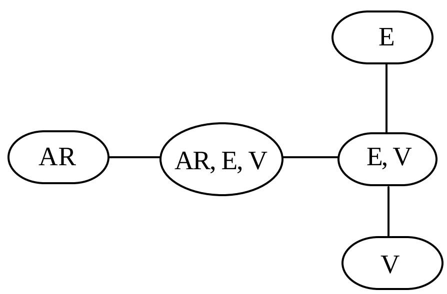


Table 1

Assessed Values of Belief Functions and the Corresponding **m**-values
At Various Nodes in Figure 1

Item of Evidence	Node	Frame Θ	Belief Function	**m**-values
Evidence 1	AR	$\Theta_{AR} = \{ar, \sim ar\}$	$\mathbf{Bel}_{AR}(ar) = 0.4$ $\mathbf{Bel}_{AR}(\sim ar) = 0$	$\mathbf{m}_{AR}(ar) = 0.4$ $\mathbf{m}_{AR}(\sim ar) = 0$ $\mathbf{m}_{AR}(\Theta_{AR}) = 0.6$
Evidence 2	E	$\Theta_E = \{e, \sim e\}$	$\mathbf{Bel}_E(e) = 0.9$ $\mathbf{Bel}_E(\sim e) = 0$	$\mathbf{m}_E(e) = 0.9$ $\mathbf{m}_E(\sim e) = 0$ $\mathbf{m}_E(\Theta_E) = 0.1$
Evidence 3	V	$\Theta_V = \{v, \sim v\}$	$\mathbf{Bel}_V(v) = 0.8$ $\mathbf{Bel}_V(\sim v) = 0$	$\mathbf{m}_V(v) = 0.8$ $\mathbf{m}_V(\sim v) = 0$ $\mathbf{m}_V(\Theta_V) = 0.2$
Evidence 4	E,V	$\Theta_{E,V} = \{e,\sim e\} x \{v,\sim v\}$ $=\{e\&v, \sim e\&v,$ $e\&\sim v, \sim e\&\sim v\}$	$\mathbf{Bel}_{E,V}(v) = 0.8$ $\mathbf{Bel}_{E,V}(\sim v) = 0$ $\mathbf{Bel}_{E,V}(e) = 0.95$ $\mathbf{Bel}_{E,V}(\sim e) = 0$	$\mathbf{m}_{E,V}(e\&v)=0.8$ $\mathbf{m}_{E,V}(\{e\&v, e\&\sim v\})=0.15$ $\mathbf{m}_{E,V}(\Theta_{E,V}) = 0.05$ all other **m**-values are zero
Categorical Relationship	R	$\Theta_R=\{ar\&e\&v,$ $\sim ar\&\sim e\&v,$ $\sim ar\&e\&\sim v,$ $\sim ar\&\sim e\&\sim v\}$	$\mathbf{Bel}_R(\Theta_R) = 1$	$\mathbf{m}_R(\Theta_R) = 1$ all other **m**-values are zero.

Table 2

The Audit Procedures used in Figures 5 - 9 (Arens and Loebbecke, 1991: 391-393).

1 (Inherent Factors) - Prior years' experience with the account, related accounting system, and the control environment. Also, the knowledge about the competence and trustworthiness of accounting personnels working in the sales and collection cycle, and other relevant inherent factors.

2 (AP) - (i) Review accounts receivable trial balance for large and unusual receivables. (ii) Calculate ratios indicated in carry-forward working papers (not included here) and follow up any significant changes from prior years.

3 (STT) - Review the sales journal and ledger for unusual transactions and amounts.

4 (STT) - (i) Review the cash receipts journal and the ledgers for unusual transactions and amounts. (ii) Review the subsidiary ledger for miscellaneous credits.

5 (TC) - Observe for segregation of duties between receipt and recording of cash and also preparation of independent bank reconciliation statement.

6 (TC) - Observe whether a restrictive endorsement is used on cash receipts.

7 (TC) - Observe whether monthly statements are mailed.

8 (TC) - Observe whether the accountant reconciles bank account.

9 (TC) - Account for a sequence of shipping documents.

10 (STT) - Trace selected shipping documents to duplicate sales invoice and the sales journal for assurance that each one has been billed and included in the journal.

11 (TC) - Account for a sequence of sales invoices in the sales journal.

12 (TC) - For selected duplicate invoice numbers from the sales journal, examine underlying documents for indication of internal certification that the total amount recorded in the journal, date, customer name, pricing, extension, and footings have been checked.

13 (STT) - Trace selected duplicate invoice numbers from the sales journal to (a) Duplicate sales invoice, and test for the total amount recorded in the journal, date, customer name. Check the pricing, extensions, and footings. (b) Bill of lading, and test for customer name, product description, quantity, and date. (c) Duplicate sales order, and test for customer name, product description, quantity, date, and internal approval. (d) Customer order, and test for customer name, product description, quantity, date, and credit approval by the credit manager.

14 (STT) - Trace recorded sales from the sales journal to the file of supporting documents, which includes a duplicate sales invoice, bill of lading, sales order, and customer order.

15 (STT) - Obtain the prelisting of cash receipts, and trace amounts to the cash receipts journal, testing for name, amount, and date.

16 (STT) - Compare the prelisting of cash receipts with the duplicate deposit slip, testing for names, amounts, and dates. Trace the total from the cash receipts journal to the bank statement, testing for dates, amounts of deposit,and delay in deposit.

17 (TDB) - Confirm accounts receivable using positive confirmations above a given amount and perform alternative procedures for all confirmations not returned on the first and second request.

APPENDIX A

BELIEF-FUNCTION THEORY

The belief-function theory is based on mathematical probability, just as the Bayesian theory is, but it allows us to bring probability statements to bear on questions of interest in a less direct way (see the discussion below). Belief functions have antecedents in the seventeenth century work of George Hooper and James Bernoulli (Shafer 1976). This section is an elementary introduction to those aspects of the theory that we use in this paper. For a more comprehensive and detailed introduction, see Shafer (1976).

Belief function degree of belief can arise in two ways. First, a decision maker may make direct subjective judgments about degrees of support for certain propositions based on certain evidence. Second, a decision maker may derive degree of belief for one question or topic from probabilities for another. The first approach is the most common, but it can be facilitated by thinking of the direct judgment as an analogy to the more structured second approach. We will discuss both approaches after explaining the technical ideas of frame, compatibility relation, and **m**-value.

Frames and Compatibility Relations

We call an exhaustive and mutually exclusive set of possible answers to a question a *frame*. (Some readers may prefer the name *sample space*; but it may be better to reserve this name for the set of outcomes in an experiment with well-defined probabilities.) We will often use the symbol Θ to represent the frame in which we are interested. In the case of a yes-no question, the frame has only two elements;

$$\Theta = \{\text{yes,no}\},$$

or

$$\Theta = \{a = \text{account 'A' is fairly stated}, \sim a = \text{account 'A' is not fairly stated}\},$$

etc. But in general, a frame may be a very large set, for its question may have many possible answers.

In auditing problems, we typically work with many different frames. The basic question and hence the basic frame of interest may be very simple (Is the financial statement fairly stated? Yes or no.). But it may be necessary to bring in many subsidiary and related questions. If we need to consider these questions together, we may end up working with very large and complex frames.

Suppose we have probabilities not for the frame Θ that interests us but instead for some related frame, say S. We might, for example, have probabilities for whether an analytical procedure (or a set of analytical procedures) is effective in detecting errors or irregularities related to an account[14]. But we might not have probabilities for the more basic question of real interest, the question of whether the account for which the analytical procedure is being performed is fairly stated. If the relation between the two frames can be expressed by saying that certain elements of the two frames are incompatible with each other, then our probabilities for S can give rise to belief-function degrees of belief for Θ. For example, if we judge that the account will be fairly stated if the analytical procedure performed is effective and no errors or irregularities have been detected, then our probability for the analytical procedure being effective will lead to a degree of belief that the account is fairly stated.

The knowledge that only certain elements of Θ are not compatible with certain elements of S can be represented formally by listing, for each element s of S, those elements of Θ that are compatible with s. We may call such a list the *compatibility relation* between Θ and S.

Consider the following auditing example. Suppose an auditor is interested in determining whether account 'A' is fairly stated. The auditor begins with a frame Θ consisting of two possible states; $\Theta = \{a, \sim a\}$, where a and $\sim a$ are defined above.

The auditor performs a certain analytic procedure (or a set of analytical procedures) that is relevant to the account. Thinking about the relevance of this procedure leads the auditor to consider the frame $S = \{s_1, s_2\}$, where

s_1 = the procedure is effective in detecting material errors,

and

s_2 = the procedure is not effective in detecting material errors.

If the auditor performs the procedure and finds no evidence of material error, then he or she has established a compatibility relationship between these two frames. The element s_1 is compatible with a; when the procedure is effective and no error is detected, the account is fairly stated. On the other hand, s_2 is compatible with both a and $\sim a$; when the procedure is ineffective and no error is detected, the account might be in material error and might not be.

In general, we write $\Gamma(s)$ for the subset of Θ consisting of elements with which the element s of S is compatible. In our example,

$$\Gamma(s_1) = \{a\} \text{ and } \Gamma(s_2) = \{a,\sim a\} = \Theta.$$

In general, $\Gamma(s)$ can be any subset of Θ, except that it cannot be empty. (Were there are no elements of Θ compatible with s, s would be impossible, whence its probability would be zero, and we could omit it from our formulation of the problem.). Different s may have the same $\Gamma(s)$.

Basic Probability Assignments (m-values)

In simple terms, uncertainties assigned to a subset of elements of a frame Θ denotes the **m**-value (or the *basic probability assignment*) for the subset. The basic difference between **m**-values and probabilities is that probabilities are assigned to individual elements of a frame whereas **m**-values are assigned to a subset of elements of the frame. The sum of all the **m**-values for all the subsets of the frame Θ is one. Formally, a basic probability assignment is a function **m** that assigns a number **m**(B) to each subset B of Θ such that **m**($\emptyset$) = 0 and

$$\sum_{B\subseteq\Theta} \mathbf{m}(B) = 1.$$

As we mentioned above, there are two ways to obtain **m**-values on a frame: (1) they may be assigned directly by the decision maker based on his or her subjective judgment, and (2) they may be derived from a compatibility relationship between a frame with known probabilities and the frame of interest. The two approaches are discussed below.

m-values Based on Subjective Judgement

The decision maker may assign **m**-values to the subset of elements of the frame of interest directly based on the subjective judgment. For example, suppose the auditor has performed some analytical procedures appropriate to account 'A' and finds no discrepancy or errors in the account. Based on this observation, he or she feels that the evidence is positive and provides a medium level of support, say, 0.7, to 'a' that the account is fairly stated. However, at the same time, the auditor feels that there is nothing to indicate that the account is not fairly stated, i.e., materially misstated ($\sim a$). Such a statement implies that 0.7 degree of support is assigned to 'a', 0 to '$\sim a$', and the remaining 0.3 is uncommitted and assigned to the entire frame $\{a, \sim a\}$, i.e.,

$$\mathbf{m}(a) = 0.7,\ \mathbf{m}(\sim a) = 0, \text{ and } \mathbf{m}(a, \sim a) = 0.3.$$

m-values Based on Compatibility Relationship

In the case where the probabilities for elements of S together with the compatibility relation between S and Θ are known then this knowledge determines belief-function degrees of belief for Θ. The basic idea is that each probability $\mathbf{P}(s)$ should contribute to a degree of belief in the subset $\Gamma(s)$ of Θ consisting of elements with which s is compatible. If several s have the same $\Gamma(s)$, say $\Gamma(s)$ is equal to B for several s, then the probabilities of all these s will contribute to our degree of belief that the answer to the question considered by Θ is somewhere in B. Formally, for each subset B of Θ, $\mathbf{m}$(B) is the total probability for all the s whose Γ(s) is equal to B:

$$\mathbf{m}(B) = \sum_{\Gamma(s)=B} \mathbf{P}(s). \tag{A-1}$$

The mapping **m** defined in this way is always a *basic probability assignment*. That is to say, we will necessarily have

$$\sum_{B \subseteq \Theta} \mathbf{m}(B) = 1, \tag{A-2}$$

and

$$\mathbf{m}(\emptyset) = 0, \tag{A-3}$$

where ϕ represents the empty set.

Let us return to our auditing example and assign probabilities to the elements of S. Suppose, based on his or her experience with the procedure (before applying it to this particular case), the auditor feels that the procedure is 70% reliable, i.e., it detects errors in 70% of the cases and 30% not reliable, i.e., it does not detects errors in 30% of the cases (see Footnote 14 for a discussion on conditional probabilities). Formally, this means that the auditor's probabilities are $\mathbf{P}(s_1) = 0.7$ and $\mathbf{P}(s_2) = 0.3$.

Applying (A-1) with these probabilities, we obtain the following values of **m** when the auditor has performed the procedure and not found any errors.

$$\mathbf{m}(a) = \mathbf{P}(s_1) = 0.7, \text{ and } \mathbf{m}(\Theta) = \mathbf{P}(s_2) = 0.3.$$

Since there is no element in S that is compatible only with $\sim a$,

$$\mathbf{m}(\sim a) = 0.$$

Of course, **m**(ϕ) is zero.

If the mapping Γ maps every point in S to a point in Θ rather than to a larger subset—i.e., each element of S is compatible with only one element of Θ—then the **m**-values are simply probabilities for the elements of Θ. In this special case, the **m**-value for each point in Θ is its probability, and the **m**-values for larger subsets of Θ are all zero. In our example, however, one of the points in S is mapped to a larger subset. Thus the **m**-values are not exactly probabilities; the 70% probability is assigned to the point *a*, but the other 30% is assigned to the whole frame Θ rather than to ~*a*.

In this example, the frame Θ is very small. In other cases, where the frame is much larger, the belief-function structure may still be quite simple, because the **m**-values may be zero for most subsets of the frame. In general, we call the subsets for which the **m**-values are not zero *focal elements*. The **m**-values for the focal elements must add to one. Aside from the requirement that the empty set cannot be a focal element, there is no restriction on what subsets can be a focal elements. Two focal elements can overlap or be disjoint, or one can contain the other.

Belief Functions

We have explained the basic probability assignment (the **m**-values), which is one way of representing the mathematical information in a belief function, but we have not yet explained the belief function itself. We reserve the term *belief function* for the function that expresses, for each subset of the frame, our *total* belief in that subset.

In general, our total degree of belief in a subset A of Θ will be more than **m**(A). This **m**-value is the total probability for s that are compatible with everything in A and nothing outside of A. But in order for its probability to contribute to belief in A, it is enough for s to be compatible with some of the elements of A and nothing outside of A. So to get a total degree of belief in A, we should add to **m**(A) the **m**(B) for all subsets B of A. In symbols:

$$\mathbf{Bel}[A] = \sum_{B \subseteq A} \mathbf{m}(B) \qquad \text{(A-4)}$$

We call a function **Bel** defined in this way a *belief function*. It follows from this definition that **Bel**[Θ] =1 and that **Bel**[∅] = 0, where ∅ represents the empty set.

Applying definition (A-4) to our example, we find the degrees of belief

$$\mathbf{Bel}[a] = \mathbf{m}(a) = 0.7, \mathbf{Bel}[\sim a] = \mathbf{m}(\sim a) = 0,$$

and

$$\mathbf{Bel}[\Theta] = \mathbf{m}(a) + \mathbf{m}(\sim a) + \mathbf{m}(\{a,\sim a\}) = 1.0.$$

The above results imply that, based on the analytical procedure alone, the auditor has 0.7 degree of support that the account is fairly stated and no support that the account is materially misstated. Such an interpretation has an intuitive appeal. For example, based on the auditor's knowledge of using the analytical procedure with various clients in the client's industry, the auditor may feel with 70% confidence that the account is fairly stated. The fact that this level of confidence is less than 100% does not indicate any evidence for the account being misstated; it indicates only that the positive evidence is too weak to provide certainty. In general, a zero belief in belief-function theory means there is no evidence to support the proposition—i.e., it represents lack of evidence. In contrast, a zero probability in probability theory means that the proposition cannot be true—i.e., it represents an impossibility.

In the above auditing example, there are only two focal elements. One of them, $\{a\}$, was a proper subset of the frame Θ; the other was Θ itself. This type of belief function is very common, and it is convenient to have a name for it. We call a belief function that has at most one proper subset of the frame as a focal element a *simple support function*, and we call the proper subset that is a focal element the *focus* of the simple support function. Thus the belief function **Bel** in our example is a simple support function with $\{a\}$ as its focus.

Plausibility Functions

Given a belief function **Bel**, we can define another interesting function that we call the *plausibility function* for **Bel**. The plausibility function for **Bel** is denoted by **PL**, and it is defined by

$$\mathbf{PL}[A] = 1 - \mathbf{Bel}[\sim A], \tag{A-5}$$

where ~A is the complement of A—the set of points in the frame that are not in A.

Intuitively, the plausibility of A is the degree to which A is plausible in the light of the evidence—the degree to which we do not disbelieve A or assign belief to its negation ~A. Complete ignorance or lack of opinion about A is represented by **Bel**[A] = 0 and **PL**[A] = 1.

In general, **Bel**[A] = **PL**[A] for every subset A of our frame Θ. If we believe A, then we think A is plausible, but the converse is not necessarily true. A zero plausibility for a proposition means that we are sure that it is false (like a zero probability in the Bayesian theory), but a zero degree of belief for a proposition means only that we see no reason to believe the proposition.

The plausibility function can be useful in explaining the significance of audit evidence. In some cases, the plausibility of a negative statement provides a non-frequentist interpretation of the auditing concept of risk. In the example above, the auditor had direct evidence that a is true with 0.7 degree of belief, and we assigned the remaining 0.3 to the entire frame {a,~a}. The plausibility for ~a is **PL**(~a) = 0.3 (see A-5). This is a measure of how risky we feel it would be to stop with this evidence. On the other hand, **PL**(a) = 1 indicates that a is maximally plausible; we have no evidence against it.

Suppose the auditor decides to perform audit procedures that may provide 0.7 degree of support for a. In terms of the plausibility function, this means the auditor intends to permit a plausibility of error of **PL**(~a) = 0.3. Thus, there are two ways we can talk about audit planning with the belief-function theory. We can say that the audit procedure should provide a desired level of support, say 0.7 for a. Or we can say that the audit procedure should limit the plausibility or risk of error, say to 0.3 for ~a. If the achieved support is higher than the planned support, or the achieved risk is lower than the planned risk, so much the better.

Belief functions differ from Bayesian probability in representation of ignorance. In Bayesian theory, ignorance is represented by assigning equal probability to all the outcomes. In the belief-function framework, ignorance is represented by a *vacuous* belief function. This belief function assigns an **m**-value of one to the entire frame Θ and an **m**-value of zero to all its proper subsets. This results in **Bel**[A] = 0 and **PL**[A] = 1 for every proper non-empty subset A of Θ.

Dempster's Rule of Combination

Dempster's rule is the basic rule for combining independent items of evidence in the belief-function framework. Dempster's rule is similar to Bayes' rule. In fact, Bayes' rule is a special case of Dempster's rule (Shafer, 1976). For simplicity of exposition we will discuss Dempster's rule of combination for only two items of evidence. Shafer (1976) has discussed the general rule for n items of evidence.

Consider two independent items of evidence with $\mathbf{m}_1$ and $\mathbf{m}_2$ representing the **m**-values on a frame Θ, then the combined **m**-value for a subset A of frame Θ using Dempster's rule is given by:

$$\mathbf{m}(A) = K^{-1}\Sigma\{\mathbf{m}_1(B_1)\mathbf{m}_2(B_2)|B_1 \cap B_2 = A, A? \phi\} \quad \text{(A-6)}$$

where $K = 1 - \Sigma\{\mathbf{m}_1(B_1)\mathbf{m}_2(B_2)|B_1 \cap B_2 = \phi\}$. The second term in K represents the conflict between the two items of evidence. If the conflict term is one, i.e., when the two items of evidence exactly contradict each other then K = 0 and, in such a situation, the two items of evidence are not combinable. In other words, Dempster's rule cannot be used when K = 0.

Concurrent Evidence

Consider the two items of evidence to be combined are the inherent factors and the analytical procedures both bearing directly on account 'A'. The items of evidence are assumed to be positive in nature and independent. The following **m**-values represent the corresponding supports at the level of account 'A'.

From Inherent Factors:

$$\mathbf{m}_1(a) = 0.4, \mathbf{m}_1(\sim a) = 0, \text{ and } \mathbf{m}_1(\{a, \sim a\}) = 0.6 \quad \text{(A-7)}$$

From Analytical Procedures:

$$\mathbf{m}_2(a) = 0.7, \mathbf{m}_2(\sim a) = 0, \text{ and } \mathbf{m}_2(\{a, \sim a\}) = 0.3 \quad \text{(A-8)}$$

where

a = Account 'A' is fairly presented

and

$\sim a$ = Account 'A' is not fairly presented

Now we use Dempster's rule in (A-6) to combine the two items of evidence. In this case, since there is no conflict, i.e., K = 1, the combined **m**-values at the level of account 'A' according to (A-6) are:

$$\mathbf{m}(a)=\mathbf{m}_1(a)\mathbf{m}_2(a) + \mathbf{m}_1(a)\mathbf{m}_2(\{a,\sim a\}) + \mathbf{m}_1(\{a,\sim a\})\mathbf{m}_2(a)$$

$$= 0.4x0.7 + 0.4x0.3 + 0.6x0.7 = 0.82,$$

$$\mathbf{m}(\sim a) = \mathbf{m}_1(\sim a)\mathbf{m}_2(\sim a) + \mathbf{m}_1(\sim a)\mathbf{m}_2(\{a,\sim a\}) + \mathbf{m}_1(\{a,\sim a\})\mathbf{m}_2(\sim a) = 0.0,$$

and

$$\mathbf{m}(\{a,\sim a\}) = \mathbf{m}_1(\{a,\sim a\})\mathbf{m}_2(\{a,\sim a\}) = 0.6\text{x}0.3 = 0.18$$

The corresponding beliefs and plausibilities can be obtained using (A-4) and (A-5):

$$\mathbf{Bel}[a] = \mathbf{m}(a) = 0.82,\ \mathbf{Bel}[\sim a] = \mathbf{m}(\sim a) = 0,$$

$$\mathbf{PL}[a] = 1 - \mathbf{Bel}[\sim a] = 1,\ \mathbf{PL}[\sim a] = 1 - \mathbf{Bel}[a] = 0.18,$$

and

$$\mathbf{Bel}[\Theta] = \mathbf{m}(a) + \mathbf{m}(\sim a) + \mathbf{m}(\{a,\sim a\}) = 1.0.$$

Conflicting Evidence

In this example, let us assume that the inherent factors provide the same level of assurance as in the previous example but the results of the analytical procedures performed suggest that the account could be materially misstated with degree 0.4. The corresponding **m**-values are:

From Inherent Factors:

$$\mathbf{m}_1(a) = 0.4,\ \mathbf{m}_1(\sim a) = 0, \text{ and } \mathbf{m}_1(\{a, \sim a\}) = 0.6 \qquad \text{(A-9)}$$

From Analytical Procedures:

$$\mathbf{m}_2(a) = 0,\ \mathbf{m}_2(\sim a) = 0.4, \text{ and } \mathbf{m}_2(\{a, \sim a\}) = 0.6 \qquad \text{(A-10)}$$

From (A-6), one obtains:

$$K = 1 - \Sigma\{\mathbf{m}_1(B_1)\mathbf{m}_2(B_2)|B_1 \cap B_2 = \phi\} = 1 - \mathbf{m}_1(a)\mathbf{m}_2(\sim a) - \mathbf{m}_1(\sim a)\mathbf{m}_2(a)$$

$$= 1 - 0.4\text{x}0.4 - 0 = 0.84 \qquad \text{(A-11)}$$

and

$$\mathbf{m}(a)=K^{-1}[\mathbf{m}_1(a)\mathbf{m}_2(a) + \mathbf{m}_1(a)\mathbf{m}_2(\{a,\sim a\}) + \mathbf{m}_1(\{a,\sim a\})\mathbf{m}_2(a)]$$

$$= (0.84)^{-1}[0.4\text{x}0 + 0.4\text{x}0.6 + 0.6\text{x}0] = 0.2857,$$

$$\mathbf{m}(\sim a) = K^{-1}[\mathbf{m}_1(\sim a)\mathbf{m}_2(\sim a) + \mathbf{m}_1(\sim a)\mathbf{m}_2(\{a,\sim a\}) + \mathbf{m}_1(\{a,\sim a\})\mathbf{m}_2(\sim a)]$$

$$= (0.84)^{-1}[0\text{x}0.4 + 0\text{x}0.6 + 0.6\text{x}0.4] =0.2857,$$

and

$$\mathbf{m}(\{a,\sim a\}) = K^{-1}[\mathbf{m}_1(\{a,\sim a\})\mathbf{m}_2(\{a,\sim a\})] = (0.84)^{-1}[0.6\text{x}0.6] = 0.4286$$

The corresponding beliefs and plausibilities can be obtained using (A-4) and (A-5):

$$\mathbf{Bel}[a] = \mathbf{m}(a) = 0.2857,\ \mathbf{Bel}[\sim a] = \mathbf{m}(\sim a) = 0.2857,$$

$$\mathbf{PL}[a]=1- \mathbf{Bel}[\sim a] = 1- 0.2857 = 0.7143,$$

and

$$\mathbf{PL}[\sim a] = 1 - \mathbf{Bel}[a] = 1 - 0.2857 = 0.7143.$$

APPENDIX B

MARKOV TREE

A Markov tree is a topological tree, whose nodes are subsets of variables, with the property that when a variable belongs to two distinct nodes, then every node lying on the path between these two nodes contains the variable (Shenoy 1991). The properties of Markov trees and how to construct such trees have been studied and discussed by Kong (1986) and Mellouli (1987). Also, Markov trees are discussed in the computer science literature under the name "join trees" (e.g., see Maier 1983).

Let us consider the evidential network in Figure 1. We have three variables: AR, E, and V. And, as one can see, based on the evidence we have direct beliefs bearing on the following set of variables: {AR}, {E}, {V}, and {E,V}. Furthermore, we have a belief function bearing on {AR,E,V} that defines 'and' relationship among the three variables AR, E, and V (see Table 1). Thus, based on the evidence we have a total of five subsets of variables, {AR}, {E}, {V}, {E,V}, and {AR,E,V}, each with a belief function associated with it. Let H denote the set of subsets that have belief functions associated. In our case, H = {{AR}, {E}, {V}, {E,V}, {AR,E,V}}. We will show below how to construct a Markov tree from H using an algorithm proposed by Kong (1986).

In order to describe the algorithm, we need to present a formal definition of a Markov tree. A Markov tree is characterized by a set of nodes N and a set of *edges* E where each edge is a two-element subset of N such that:

(i) (N,E) is a tree.

(ii) If N and N' are two distinct nodes in N, and {N, N'} is an edge, i.e., $\{N, N'\} \in E$, , then $N \cap N' \neq \emptyset$.

(ii) If N and N' are distinct nodes of N, and X is a variable in both N and N', then X is in every node on the path from N to N'.

Let us define X to be the set of variables in H. In our case, X = {AR,E,V}. Shenoy (1991) describes the Markov tree construction process in terms of pseudo-Pascal as given below:

u:= X	{Initialization}

H_0:=H	{Initialization}
N:=∅	{Initialization}
E:=∅	{Initialization}

For i = 1 **to** n **do** (n is the number of variables in X)

begin

Pick a variable from set u and call it X_i

u:= u - $\{X_i\}$

$G_i := \cup\{N \in H_{i-1} | X_i \in N\}$.

$F_i := G_i - \{X_i\}$.

$N := N \cup \{N \in H_{i-1} | X_i \in N\} \cup \{F_i\} \cup \{G_i\}$.

$E := E \cup \{\{N, G_i\} | N \in H_{i-1}, N \neq G_i, X_i \in N\} \cup \{\{F_i, G_i\}\}$

$H_i := \{N \in H_{i-1} | X_i \notin N\} \cup \{F_i\}$

end.

In the above algorithm, every sequence gives a Markov tree. Our objective in constructing a Markov tree is to have the nodes in the tree as small as possible. The reason for constructing such a Markov tree is that computations for propagating beliefs in such a tree becomes very efficient. Kong (1986) has proposed the *one-step-look-ahead* heuristic that selects a variable X_i in each iteration which yields a G_i with the least number of variables. Such an approach provides a "good" Markov tree, i.e., a tree whose nodes will contain as few variables as possible. Our system PLEAS uses the *one-step-look-ahead* heuristic to construct a Markov tree.

Let us describe the above algorithm step-by-step using our example in Figure 1. We have the following initial values:

u = {AR, E, V}
H_0 = {{AR},{E},{V},{E,V},{AR,E,V}}
N = ∅,
E = ∅.

Iteration one, i =1:

Pick a variable from set u = {AR,E,V} and determine the union of all the nodes in H_0 = {{AR},{E},{V},{E,V},{AR,E,V}} that contain the variable. Repeat this process for all the variables in u. Start the construction sequence with the variable for which the number of variables in the union is the smallest. We call this variable X_1 and the set G_1. In our case, if we perform the above procedure for AR then we find that the union is {AR,E,V}. Similarly, for E and V we find the union to be {AR,E,V}. Since all the variables in u give the same union, we can use any variable to start. Let us start with AR, thus the first variable in the construction sequence is X_1= AR, and the corresponding union of the nodes that contain AR is G_1={AR,E,V}. Thus we have:

$$u = \{AR,E,V\} - \{X_1\} = \{E,V\}$$

$$G_1 = \{AR,E,V\}$$

$$F_1 = G_1 - \{X_1\} = \{AR,E,V\} - \{AR\} = \{E,V\}$$

$$N = \emptyset \cup \{N \in H_0 | X_1 \in N\} \cup \{F_1\} \cup \{G_1\}$$

$$= \{\{AR\},\{E,V\},\{AR,E,V\}\}$$

$$E = \emptyset \cup \{\{N, G_1\} | N \in H_0, N \neq G_1, X_1 \in N\} \cup \{\{F_1, G_1\}\}$$

$$= \{\{\{AR\},\{AR,E,V\}\}, \{\{AR,E,V\},\{E,V\}\}\}$$

$$H_1 := \{N \in H_0 | X_1 \notin N\} \cup \{F_1\} = \{\{E\},\{V\},\{E,V\}\}.$$

Iteration two, i = 2:

In this iteration, we pick a variable from set u = {E,V} and determine the union of all the nodes in H_1 = {{E},{V},{E,V}} that contain the variable. We repeat this process for all the variables in u = {E,V}, find the variable for which the union has the smallest number of variables, and start the construction process with this variable. In our case, we find that the union is {E,V} for both E and V. Thus, we can start either with E or with V as our second variable to complete this iteration. Let us pick E, i.e., X_2 = E. Then we have:

$$u = \{E,V\} - \{X_2\} = \{V\},$$

$G_2 = \{E,V\}$,

$F_2 = G_2 - \{X_2\} = \{E,V\} - \{E\} = \{V\}$,

$N = N \cup \{N \in H_1 | X_2 \in N\} \cup \{F_2\} \cup \{G_2\}$
$= \{\{AR\},\{E\},\{V\},\{E,V\},\{AR,E,V\}\}$,

$E = E \cup \{\{N, G_2\} | N \in H_1, N \neq G_2, X_2 \in N\} \cup \{\{F_2, G_2\}\}$
$= \{\{\{AR\},\{AR,E,V\}\}, \{\{AR,E,V\},\{E,V\}\}, \{\{E\},\{E,V\}\}, \{\{V\},\{E,V\}\}\}$,

$H_2 = \{N \in H_1 | X_2 \notin N\} \cup \{F_2\} = \{V\}$.

Iteration 3, i = 3:

$X_3 = V$,

$u = \{V\} - \{X_3\} = \{\varnothing\}$,

$G_3 = \{V\}$,

$F_3 = G_3 - \{X_3\} = \{V\} - \{V\} = \{\varnothing\}$,

$N = N \cup \{N \in H_2 | X_3 \in N\} \cup \{F_3\} \cup \{G_3\}$
$= \{\{AR\},\{E\},\{V\},\{E,V\},\{AR,E,V\}\}$,

$E = E \cup \{\{N, G_3\} | N \in H_2, N \neq G_3, X_3 \in N\} \cup \{\{F_3, G_3\}\}$
$= \{\{\{AR\},\{AR,E,V\}\}, \{\{AR,E,V\},\{E,V\}\}, \{\{E\},\{E,V\}\}, \{\{V\},\{E,V\}\}\}$.

$H_3 = \{N \in H_2 | X_3 \notin N\} \cup \{F_3\} = \{\varnothing\}$.

Now the Markov tree construction process is complete. A graphical representation of this tree is given in Figure A-1.