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Belief Function Approach to Evidential Reasoning in Causal Maps

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Abstract

The purpose of this chapter is to demonstrate the use of evidential reasoning approach under Dempster-Shafer (D-S) theory of belief functions to analyze revealed causal maps. Revealed causal mapping (RCM) technique, as applied in this chapter, is a qualitative method used to develop or extend understanding of a phenomenon within a specific context. The map can be used to develop models, either as grounded theory or evocative theory building. The example referenced in this study used interview data as the primary source in the RCM method. The participants from information technology (IT) organizations provided the concepts to describe the target phenomenon of Job Satisfaction; they also identified the associations between the concepts. The researchers used coding rules to aggregate similar concepts to produce a composite RCM. The researchers proposed potential evidence measures that could be used to evaluate the model. This chapter discusses the steps necessary to transform a causal map into an evidential diagram. The evidential diagram can then be analyzed using belief functions technique with survey data, thereby extending the research from a discovery and explanation stage to testing and prediction. An example is provided to demonstrate these steps. This chapter also provides the basics of Dempster-Shafer theory of belief functions and a step-by-step description of the propagation process of beliefs in tree like evidential diagrams.

Belief Function Approach to Evidential Reasoning in Causal Maps

I. Introduction

The main purpose of this chapter is to demonstrate the use of evidential reasoning approach under Dempster-Shafer (D-S) theory of belief functions (Shafer, 1976; see also, Srivastava & Datta, 2002; and Srivastava & Mock, 2000, 2002) to analyze revealed causal maps. The Revealed Causal Mapping (RCM) technique is used to represent the model of a mental map and to determine the constructs or variables of the model and their interrelationships from the data. RCM focuses on the cause/effect linkages disclosed by individuals intimately familiar with a phenomenon under investigation. The researcher deliberately avoids determining the variables and their associations *a priori*, allowing both to emerge during the discourse or from the textual analysis (Narayanan & Fahey, 1990). In contrast, other forms of causal mapping begin with a framework of variables based on theory, and the associations are provided by the participants in the study (cf. Bougon, et al., 1977).

While RCM helps determine the significant variables in the model and their associations, it does not provide a way to integrate uncertainties involved in the variables or to use the model to predict future behavior. The evidential reasoning approach provides a technique where one can take the RCM model, convert it into an evidential diagram, and then use it to predict how a variable of interest would behave under various scenarios. An evidential diagram is a model showing interrelationships among various variables in a decision problem along with relevant items of evidence pertaining to those variables that can be used to evaluate the impact on a given variable of all other variables in the diagram. In other words, RCM is a good technique to identify the significant constructs (i.e., variables) and their interrelationships relevant to a model, whereas evidential approach is good for making if-then analyses once the model is established.

There are two steps required in order to achieve our objective. One is to convert the RCM model to an evidential diagram with the variables taken from the RCM model and items of evidence identified for the variables from the problem domain. The second step is to deal with uncertainties associated with evidence. In general, uncertainties are inherent in RCM model variables. For example, in our case of IT professionals' job satisfaction, the variable "Feedback from Supervisors/Co-Workers" partly determines whether an individual will have a "high" or "low" level of satisfaction. However, the level of job satisfaction will depend on the level of confidence we have in our measure of the variable. The Feedback from Supervisors/Co-Workers may be evaluated through several relevant items of evidence such as interviews or surveys. In general, such items of evidence provide less than 100% assurance in support of, or negation of, the pertinent variable. The uncertainties associated with these variables are better modeled under Demspter-Shafer theory of belief functions than probabilities as empirically shown by Harrison, Srivastava and Plumlee (2002) in auditing and by Curley and Golden (1994) in psychology. We use belief functions to represent uncertainties associated with the model variables and use evidential reasoning approach to determine the impact of a given variable on another in the model. This combination of techniques adds the strength of prediction to the usefulness of descriptive modeling when studying behavioral phenomena. Evidential reasoning under Dempster-Shafer theory of belief functions thereby extends the impact of revealed causal mapping.

The chapter is divided into eight sections. Section II provides a brief description of the Revealed Causal Mapping (RCM) technique. Section III discusses the basic concepts of belief functions, and provides an illustration of Dempster's rule of combination of independent items of evidence. Section IV describes the evidential reasoning approach under belief functions. Section V describes a causal map developed through interviews and surveys of IT employees on their job satisfaction. Section VI shows the process of converting a RCM map to an evidential diagram under belief functions. Section VII presents the results of the analysis, and Section VIII provides conclusions and directions for future research.

II. Revealed Causal Mapping Technique

Revealed causal mapping is a form of content analysis that attempts to discern the mental models of individuals based on their verbal or text-based communications (Narayanan & Fahey, 1990; Nelson, et al., 2000; Darais, et al., 2003). The general structure of the causal map can reveal a wealth of information about cognitive associations, explaining idiosyncratic behaviors and reasoning.

The actual steps used to develop the IT Job Satisfaction revealed causal map in the present paper are outlined in Table 1. The research constructs were not determined *a priori*, but were derived from the assertions in the data. The sequence of steps directly develops the structure of the model from the data sample.

First, a key consideration in using RCM is the determination of source data (Narayanan & Fahey, 1990). Since this study assessed the job satisfaction of IT professionals, it was logical to gather data from IT workers in a variety of industries. Interviews and surveys were conducted with employees of IT departments, and responses were analyzed to produce the model presented later in the chapter.

Table 1. Steps for Revealed Causal Mapping Technique.

Step	Description
1	Identify source data.
2	Identify causal statements.
3	Create concept dictionary.
4	Aggregate maps.
5	Produce RCM and analyze maps.

Second, the researchers identified causal statements from the original transcripts or documents. The third step in the procedure is to combine concepts based on coding rules (Axelrod, 1976; Wrightson, 1976), producing a concept dictionary (see Appendix A). Synonyms are grouped to enable interpretation and comparison of the resultant causal maps. Care must be taken to ensure that synonyms are true to the original conveyance of the participant. For example, two interviewees might use different words that hold identical or very similar meanings such as "computer application" and "computer program". In mapping these terms, the links are not identical until the concepts are coded by the researcher. It is preferable for investigators to ere on the side of too many concepts, rather than inadvertently combine terms inappropriately for the sake of parsimony.

Next, the maps of the individual participants were aggregated by combining the linkages between the relevant concepts. The result of this step is a representative causal map for the sample of participants.

RCM produces dependent maps, meaning that the links between nodes indicate the presence of an association explicitly revealed in the data (Nadkarni & Shenoy, 2001). The absence of a line does not imply independence between the nodes, however. It simply means that a particular link was not stated by the participants. This characteristic of RCM demonstrates the close relationship of the graphical result (map) to the data set. Therefore, it is vital that the sample be representative of the population of interest. The following section introduces belief functions and the importance of evidential reasoning in managerial decision making.

III. Dempster-Shafer Theory of Belief Functions

Dempster-Shafer (D-S) theory of belief functions, which is also known as the belief-function framework, is a broader framework than probability theory (Shafer and Srivastava 1990). Actually, Bayesian framework is a special case of belief function framework. The basic difference between the belief-function framework and probability theory or Bayesian framework is in the assignment of uncertainties to a mutually exclusive and collectively exhaustive set of elements, say Θ, with elements, $\{a_1, a_2, a_3, \ldots a_n\}$. This set of elements, $\Theta = \{a_1, a_2, a_3, \ldots a_n\}$, is known as a *frame of discernment* in belief-function framework. In probability theory, probabilities are assigned to individual elements, i.e., to the singletons, and they all add to one. For example, for the frame, $\Theta = \{a_1, a_2, a_3, \ldots a_n\}$, with n mutually exclusive and collectively exhaustive set of elements, a_i*'s*, with i = 1, 2, 3, … n, one assigns a probability measure to each element, $1.0 \geq P(a_i) \geq 0$, such that $\sum_{i=1}^{n} P(a_i) = 1.0$. Under belief functions, however, the probability mass is distributed over the super set of the elements of Θ instead of just the singletons. Shafer (1976) calls this probability mass distribution the *basic probability assignment* function, whereas Smets calls it *belief masses* (Smets 1998, 1990a, 1990b). We will use Shafer's terminology of probability mass distribution over the superset of Θ.

Basic Probability Assignment Function (m-values)

In the present context, the *basic probability assignment function* represents the strength of evidence. For example, suppose that we have received feedback from a survey of the IT employees of a company on whether their work is challenging or not. On average, the employees believe that their work is challenging but they do not say this with certainty; they put a high level of comfort, say 0.85, on a scale 0 – 1.0 that their work is challenging. But, they do not say that their work is not challenging. This response can be represented through the *basic probability function*, m-values[1], on the frame, {'yes_{CW}', 'no_{CW}'}, of the variable 'Challenging Work (CW)' as: $m(yes_{CW}) = 0.85$, $m(no_{CW}) = 0$, and $m(\{yes_{CW}, no_{CW}\}) = 0.15$. These values imply that the evidence suggests that the work is challenging to a degree 0.85, it is not challenging to a degree zero (there is no evidence in support of the negation), and it is undecided to a degree 0.15.

Mathematically, the basic *probability assignment function* represents the distribution of probability masses over the superset of the frame, Θ. In other words, probability masses are assigned to all the singletons, all subsets of two elements, three elements, and so on, to the entire frame. Traditionally, these probability masses are represented in terms of m-values and the sum of all these m-values equals one, i.e., $\sum_{B \subseteq \Theta} m(B) = 1$, where B represents a subset of elements of frame Θ. The m-value for the empty set is zero, i.e., $m(\emptyset) = 0$.

In addition to the *basic probability assignment* function, i.e., m-values, we have one other function, Belief function, represented by Bel(.), that is of interest in the present discussion. As defined below, Bel(A), determines the degree to which we believe, based on the evidence, that A is true. This function is discussed further below.

[1] See the following references for more discussion on belief functions and their applications: Bovee et. al., 2003; Srivastava, 1993; Srivastava & Datta, 2002; Srivastava & Liu, 2003; and Srivastava & Mock, 2000.

Belief Functions

The function, Bel(B), defines the belief in B, a subset of elements of frame Θ, that it is true, and is equal to m(B) plus the sum of all of the m-values for the set of elements contained in B, i.e., $\text{Bel(B)} = \sum_{C \subseteq B} m(C)$. Let us consider the example described earlier to illustrate the definition. Based on the Survey Results, we have 0.85 level of belief that the employees have challenging work, zero belief that the employees do not have challenging work. This evidence can be mapped in the following belief functions by using the above definition:

$$\text{Bel}(\text{yes}_{CW}) = m(\text{yes}_{CW}) = 0.85,$$

$$\text{Bel}(\text{no}_{CW}) = m(\text{no}_{CW}) = 0.0,$$

$$\text{Bel}(\{\text{yes}_{CW}, \text{no}_{CW}\}) = m(\text{yes}_{CW}) + m(\text{no}_{CW}) + m(\{\text{yes}_{CW}, \text{no}_{CW}\})$$
$$= 0.85 + 0.0 + 0.15 = 1.0.$$

The above belief values imply that we have direct evidence from surveying the employees that the work is challenging to a degree 0.85, no belief that the work is challenging, and the belief that the work is either challenging or not challenging is 1.0. Note that in our example there is no state or element contained in ‘yes_{CW}’ or ‘no_{CW}’. Thus, m-values and Bel(.) for these elements are the same.

Dempster’s Rule of Combination

Dempster’s rule of combination is similar to Bayes’ rule in probability theory. It is used to combine various independent items of evidence pertaining to a variable or a frame of discernment. As mentioned earlier, the strength of evidence is expressed in terms of m-values.

Thus, if we have two independent items of evidence pertaining to a given variable, i.e., we have two sets of m-values for the same variable then the combined m-values are obtained by using Dempster's rule. For a simple case[2] of two items of evidence pertaining to a frame Θ, Dempster's rule of combination is expressed as:

$$m(B) = K^{-1}. \sum_{\substack{i,j \\ B_i \cap B_j = B}} m_1(B_i)m_2(B_j),$$

where m(B) represents the strength of the combined evidence and m_1 and m_2 represent the individual strengths of the two items of evidence. In other words, m(B) is the resultant m-value for the subset B of the frame Θ, m_1 and m_2 are the two sets of m-values associated with the two independent items of evidence. K is the renormalization constant given by:

$$K = 1 - \sum_{\substack{i,j \\ B_i \cap B_j = \emptyset}} m_1(B_i)m_2(B_j).$$

The second term in K represents the conflict between the two items of evidence. When K = 0, i.e., when the two items of evidence totally conflict with each other, these two items of evidence are not combinable.

A simple interpretation of Dempster's rule is that the combined m-value for a set of elements B is equal to the sum of the product of the two sets of m-values (from each item of evidence), $m_1(B_1)$ and $m_2(B_2)$, such that the intersection of B_1 and B_2 is equal to B and renormalize the m-values to add to one by eliminating the conflicts.

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[2] For three independent items of evidence, Dempster's rules can be written as:

$$m(B) = K^{-1}. \sum_{\substack{i,j,k \\ B_i \cap B_j \cap B_k = B}} m_1(B_i)m_2(B_j)m_3(B_k), \text{ where } K = 1 - \sum_{\substack{i,j,k \\ B_i \cap B_j \cap B_k = \emptyset}} m_1(B_i)m_2(B_j)m_3(B_k)\ .$$

One can easily generalize the above formula for n independent items of evidence (see Shafer 1976 for details).

Let us consider an example to illustrate Dempster's rule. Consider that we have the following sets of m-values from two independent items of evidence pertaining to a variable, say A, with two values, '*a*', and '~*a*', representing respectively, that A is true and is not true:

$$m_1(a) = 0.4,\ m_1(\sim a) = 0.1,\ m_1(\{a, \sim a\}) = 0.5,$$

$$m_2(a) = 0.6,\ m_2(\sim a) = 0.2,\ m_2(\{a, \sim a\}) = 0.2.$$

As mentioned earlier, the general formula of Dempster's rule yields the combines m-value for an element or a set of elements of the frame of discernment by multiplying the two sets of m-values such that the intersection of their respective arguments is equal to the element or set of elements desired in the combined m-value, and by eliminating the conflicts and renormalizing the resulting m-values such that the resulting m-values add to one. This reasoning yields the following expressions as a result of Dempster's rule for binary variables:

$$m(a) = K^{-1}[m_1(a)m_2(a) + m_1(a)m_2(\{a,\sim a\}) + m_1(\{a,\sim a\})m_2(a)],$$

$$m(\sim a) = K^{-1}[m_1(\sim a)m_2(\sim a) + m_1(\sim a)m_2(\{a,\sim a\}) + m_1(\{a,\sim a\})m_2(\sim a)],$$

$$m(\{a,\sim a\}) = K^{-1}m_1(\{a,\sim a\})m_2(\{a,\sim a\}),$$

and

$$K = 1 - [m_1(a)m_2(\sim a) + m_1(\sim a)m_2(a)].$$

As we can see above, m(a) is the result of the multiplication of the two sets of m-values such that the intersection of their arguments is equal to '*a*' and the renormalization constant, K, is equal to one minus the conflict terms. Similarly m(~*a*) and m({*a*,~*a*}) are the results of multiplying two sets of m-values such that the intersection of their arguments is equal to '~*a*' and ({*a*,~*a*}), respectively.

Substituting the values for the two m-values, we obtain:

$$K = 1 – [0.4x0.2 + 0.1x0.6] = 0.86,$$

$$m(a) = [0.4x0.6 + 0.4x0.2 + 0.5x0.6]/0.86 = 0.72093,$$

$$m(\sim a) = [0.1x0.2 + 0.1x0.2 + 0.5x0.2]/0.86 = 0.16279,$$

$$m(\{a,\sim a\}) = 0.5x0.2/0.86 = 0.11628.$$

Thus, the total beliefs after combining both items of evidence are given by

$$Bel(a) = m(a) = 0.72093,\ \ Bel(\sim a) = m(\sim a) = 0.16279,$$

and

$$Bel(\{a,\sim a\}) = m(a) + m(\sim a) + m(\{a,\sim a\}) = 0.72093 + 0.16279 + 0.11628 = 1.0.$$

The above values of beliefs in '*a*' and '~*a*' represent the combined beliefs from two items of evidence. Belief that '*a*' is true from the first item of evidence is 0.4; from the second item of evidence it is 0.6, where as the combined belief that '*a*' is true based on the two items of evidence is 0.72093; a stronger belief as a result of the combination. The combined belief would have been much stronger if we did not have the conflict.

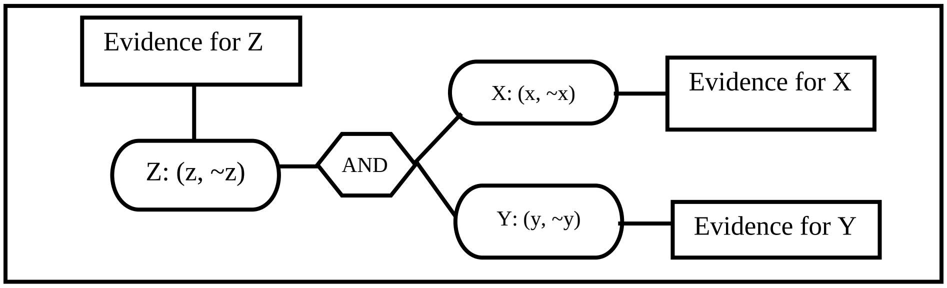
IV. Evidential Reasoning Approach

Strat (1984) and Pearl (1990) have used the term "evidential reasoning" for decision making under uncertainty. Under this approach one needs to develop an evidential diagram (as shown in Figure 4 in the next section, see also Srivastava & Mock, 2000 for other examples) containing all the variables involved in the decision problem with their interrelationships and the items of evidence pertaining to those variables. Once the evidential diagram is completed, the decision maker can determine the impact of a given variable on all other variables in the diagram by combining the knowledge about the variables. In other words, under the evidential reasoning approach, if we have knowledge about one or more variables in the evidential diagram, then we can *make predictions about the other variables* in the diagram given that we know how these variables are interrelated. Usually, the knowledge about the states of these variables is only partial, i.e., there is uncertainty associated with what we know about these variables. As mentioned earlier, we use Dempster-Shafer theory of belief functions to model these uncertainties.

In the present case, variables in the evidential diagram represent the ‘constructs’ of the model obtained through the Revealed Causal Mapping (RCM) process, and the interrelationships represent how one variable or multiple of variables influence a given variable. Such relationships among the variables can be defined either in terms of categorical relationships such as, ‘AND’, and ‘OR’, or in terms of uncertain relationships, such as a combination of ‘AND’ and ‘OR’, or some other relationships as discussed in the next section.

In order to illustrate the evidential reasoning approach, let us first construct an evidential diagram using a simple hypothetical decision problem involving three variables, X, Y, and Z (see Figure 1). Let us assume for simplicity that these variables are binary, i.e., each variable has two values: either the variable is true (x, y, and z) or false (~x, ~y, and ~z). Also, let us assume that variable Z is related to X and Y through the ‘AND’ relationship. This relationship implies that Z is true (z) if and only if X is true (x) and Y is true (y), but it is false (~z) when either X is true (x) and Y is false (~y), or X is false (~x) and Y is true (y), or both X and Y are false (~x, ~y). Now we draw a diagram consisting of the three variables, X, Y, and Z, represented by rounded boxes and connect them with a relational node represented by the hexagonal box. Further, connect each variable with the corresponding items of evidence represented by rectangular boxes. Figure 1 depicts the evidential diagram for the above case.

Figure 1: Example of an Evidential Network*



*Rounded boxes represent variables (constructs), hexagonal box represents a relationship, and rectangular boxes represent items of evidence pertinent to the variables they are connected.

As mentioned earlier, an evidential reasoning approach helps us infer about one variable given what we know about the other variables in the evidential diagram. For example, in Figure 1, we can predict about the state of Z given what we know about the states of X and Y, and the relationship among them. Under the belief-function framework, this knowledge is expressed in terms of m-values. For example, knowledge about X and Y, based on the corresponding evidence, can be expressed in terms of m-values[3], m_X at X, and m_Y at Y, as: $m_X(x) = 0.6$, $m_X(\sim x) = 0.2$, $m_X(\{x,\sim x\}) = 0.2$, and $m_Y(y) = 0.7$, $m_Y(\sim y) = 0$, $m_Y(\{y,\sim y\}) = 0.3$. The first set of m-values suggests that the evidence relevant to X provides 0.6 level of support that X is true, i.e., $m_X(x) = 0.6$, 0.2 level of support that X is not true, i.e., $m_X(\sim x) = 0.2$, and 0.2 level of support undecided, i.e., $m_X(\{x,\sim x\}) = 0.2$. One can provide a similar interpretation of the m-values for Y. The 'AND' relationship between X and Y, and Z can be expressed in terms of the following m-values: $m(\{xyz, x\sim y\sim z, \sim xy\sim z, \sim x\sim y\sim z\}) = 1.0$. This relationship implies that z is true if and only if x is true and y is true, and it is false when either x is true and ~y is true, ~x is true and y is true, or ~x and ~y are true.

Based on the knowledge about X and Y above and the relationship of Z with X and Y, we can now make inferences about Z. This process consists of three steps which are described in Appendix C in detail. Basically, Step 1 involves propagating[4] beliefs or m-values from X and Y variables to the relational node 'AND' through vacuous[5] extension. This process yields two sets of m-values at 'AND', one from X and the other from Y: $m_{AND \leftarrow X}$ and $m_{AND \leftarrow Y}$ (See Table 2 for definitions of symbols). Also, we already have one set of m-values, m_{AND}, at the relational node 'AND'. Step 2 involves combining the three sets of m-values at the 'AND' node using Demspter's rule. Step 3 involves propagating the resulting m-values from the 'AND' node to variable Z by marginalization[6]. This process yields $m_{Z \leftarrow AND}$. These m-values are then combined with the m-values at Z, m_Z, obtained from the evidence pertaining to Z. The resultant m-values will provide the belief values whether Z is true or not true. As mentioned earlier, he details of the propagation process[7] are discussed in Appendix C through a numerical example.

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[3] The argument of m-function represents the state for which the value is assigned and the subscript describes the evidence from which the value is derived. For example, $m_X(x) = 0.6$ represents 0.6 level of support for 'x' from an item of evidence pertaining to the variable X.

[4] Propagation is the process by which m-values on a variable or a set of variables are moved (mapped) to another variable or a set of variables. For example, m-values from variable X in Figure 1 can be propagated to the relational variable 'AND' that consist of three variables, X, Y, and Z.

[5] Vacuous extension is the process through which m-values on a smaller frame are extended to a larger frame. For example, m(x) when vacuously extended to the joint space of X and Y, i.e., the frame {xy, x~y, ~xy, ~x~y}, yields m(x) = m({xy, x~y}).

[6] Marginalization of m-values is opposite to the vacuous extension. This process is similar to marginalization in probability theory; it involves eliminating all the unwanted variables by summing the m-values over the unwanted variables. For example, assume that we have the following m-values on the joint space of X and Y, $\Theta_{X,Y}$ = {xy, x~y, ~xy, ~x~y}: m({xy}) = 0.1, m({xy, x~y}) = 0.6, and m({xy, x~y, ~xy, ~x~y}) = 0.3. The marginalized m-values onto the space of X variable are: m({x}) = 0.1 + 0.6 = 0.7, and m({x, ~x}) = 0.3. Similarly, the marginalized m-values onto the Y space are: m({y}) = 0.1, m({y, ~y}) = 0.9.

[7] Through this example we are illustrating the details of the propagation process of beliefs or m-values through a tree of variables as this is what is needed in our model of IT job satisfaction obtained through the RCM process. A discussion on the details of the propagation of beliefs through a network of variables is beyond the scope of this chapter. Interested readers should see Srivastava (1995) and Shenoy and Shafer (1990) for this kind of propagation.

Table 2: List of Symbols related to m-values used in the Propagation Process in Figure 1.

Symbol	Description
x, y, and z	These symbols, respectively, represent that the variables X, Y, and Z, are true.
~x, ~y, and ~z	These symbols, respectively, represent that the variables X, Y, and Z, are not true.
Θ_X={x,~x}	The frame of X which represents all the possible values of X.
Θ_Y={y,~y}	The frame of Y which represents all the possible values of Y.
Θ_{AND}= {xyz, x~y~z, ~xy~z, ~x~y~z}	The frame of 'AND' relationship. The elements in the frame are the only possible values under the logical 'AND' relationship between Z, and X and Y.
m_X({.})	m-value for the element or the set of elements {x,~x} in the argument for variable X.
m_Y({.})	m-value for the element or the set of elements {y,~y} in the argument for variable Y.
m_{AND}({.})	m-value for the elements in the argument for the 'AND' relationship.
$m_{AND\leftarrow X}$({.})	m-value for the element or elements in the argument propagated to 'AND' relationship from variable X.
$m_{AND\leftarrow Y}$({.})	m-value for the element or elements in the argument propagated to 'AND' relationship from variable Y.
$m_{Z\leftarrow AND}$({.})	m-values propagated from 'AND' to variable Z in Figure 1.

Modeling Uncertain Relationships among Variables

Srivastava and Lu (2002) have discussed a general approach to modeling various relationships under belief functions. We will use their approach to model the assumed relationships among various variables in Figure 4. As given earlier, the 'AND' relationship among X, Y and Z, under belief functions can be expressed in terms of the following m-value:

$$m_{AND}(\{xyz, x{\sim}y{\sim}z, {\sim}xy{\sim}z, {\sim}x{\sim}y{\sim}z\}) = 1.0.$$

The argument of m-value above determines the possible states of the joint space defining the 'AND' relationship. Similarly, the 'OR' relationship can be expressed as:

$$m_{OR}(\{xyz, x{\sim}yz, {\sim}xyz, {\sim}x{\sim}y{\sim}z\}) = 1.0.$$

A relationship representing 60% of 'AND' and 40% of 'OR' can be expressed as:

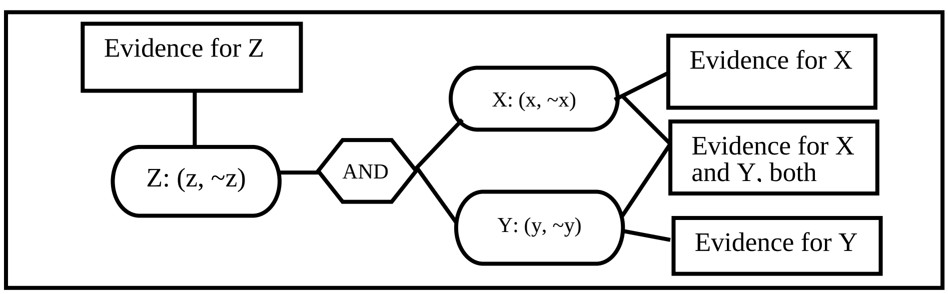
$$m_R(\{xyz, x{\sim}y{\sim}z, {\sim}xy{\sim}z, {\sim}x{\sim}y{\sim}z\}) = 0.6, \text{ and } m_R(\{xyz, x{\sim}yz, {\sim}xyz, {\sim}x{\sim}y{\sim}z\}) = 0.4,$$

where the subscript R stands for the relationship.

Propagation of Beliefs in a Network of Variables

The evidential diagram becomes a network if one item of evidence pertains to two or more variables in the diagram. Such a diagram is depicted in Figure 2 for a simple case of three variables. Even though the evidential diagram of IT Job Satisfaction model obtained through the RCM approach in the current study is not a network (see Figure 4), we describe the approach of propagating beliefs or m-values through a network of variables for completeness. The propagation of m-values through a network is much more complex and thus we will not go into the details of the propagation process in this chapter. Instead, we will briefly describe the process and advise interested readers to refer to Shenoy and Shafer (1990) for the details. Also, Srivastava (1995) provides a step-by-step description of the process by discussing an auditing example.

Figure 2: Evidential Diagram as a Network



Basically, the propagation of m-values (i.e., beliefs) through a network of variables involves the following steps. First, the decision maker draws the evidential diagram with all the pertinent variables and their interrelationships in the problem along with the related items of evidence. This step is similar to creating an evidential diagram for the case of a tree type diagram. Second, the decision maker identifies the clusters of variables over which m-values are either obtained from the items of evidence in the evidential diagram or defined from the assumed relationships among the variables. For example, in Figure 2, the four items of evidence yield the following clusters of variables: {X}, {Y}, {Z}, {X,Y}, and the 'AND' relationship defines m-value for the cluster {X,Y,Z}. Thus, in Figure 2, we have the following clusters of variables over which m-values are defined: {X}, {Y}, {Z}, {X,Y}, and {X,Y,Z}.

The third step in the propagation process in a network is to draw a Markov[8] tree based on the identified clusters of variables as above. This step is not needed for a tree type evidential diagram. One can propagate m-values through a tree type evidential diagram without converting the diagram to a Markov tree. The fourth step is to propagate m-values through the Markov tree by vacuously extending and marginalizing the m-values from all the nodes in the Markov tree to the node of interest. The basic approach to vacuous extension and marginalization remains the same as described earlier through footnotes 5 and 6.

Since the process of propagating m-values in a network becomes computationally quite complex, several software packages have been developed to facilitate this process (see e.g., Shafer, et al., 1988; Zarley, et al., 1988; and Saffiotti & Umkehrer, 1991). The software developed by Zarley et al (1988) and Saffiotti & Umkehrer (1991) require programming the evidential diagram in LISP. Also, these software programs do not provide a friendly user interfaces. On the other hand the software, 'Auditor Assistant', developed by Shafer et al. (1988) has a friendly user interface and does not require any programming language to draw the evidential diagram. In fact, one can draw the evidential diagram using the graphic capabilities of the software. The evidential diagram drawn by using 'Auditor Assistant' looks very similar to

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[8] A Markov tree is characterized by a set of nodes $\mathcal{N}$ and a set of edges $\mathcal{E}$ where each edge is a two-element subset of $\mathcal{N}$ such that (Srivastava, 1995; see also, Shenoy, 1991):

- $(\mathcal{N},\mathcal{E})$ is a tree.
- If N and N' are two distinct nodes in $\mathcal{N}$, and {N, N'} is an edge, i.e., $\{\mathcal{N},\mathcal{N}'\} \in \mathcal{E}$, then $N \cap N' \neq \emptyset$.
- If N and N' are distinct nodes of N, and X is a variable in both N and N', then X is in every node on the path from N to N'.

the one drawn by hand. The internal engine of the program converts this diagram into a Markov tree and propagates m-values once they are entered in the program. The program can be instructed to evaluate the network which then provides the aggregated m-values at each cluster of variables in the network. One can then analyze how one variable impacts another variable by making changes in the input m-values in the network.

Since the evidential diagram in our case is a simple tree, it is pretty straight forward to propagate m-values through such a tree as described Appendix C. In order to analyze the model in Figure 4, we develop a spreadsheet program that combines different m-values at each variable and then propagates them through the tree to the desired variable. This process is elaborated in Section VI.

V. An Illustration of Evidential Reasoning: Causal Map of IT Job Satisfaction

Job satisfaction of information technology (IT) workers has been the focus of several information systems studies (e.g., Igbaria & Guimaraes, 1993; Gupta, et al., 1992; Thatcher, et al., 2003). Organizations want to retain their best IT workers as long as they possess the skills necessary to accomplish the job. However, there is growing concern that many long term IT employees no longer fit the needs of their employers.

The general consensus from the research is that job satisfaction is negatively related to turnover intention (e.g., Thatcher, et al. 2003). In other words, workers who are highly satisfied with their jobs are less likely to contemplate seeking other employment and many unsatisfied workers enter the job market. In the current environment of radical role changes (Darais, et al., 2003) and selectivity in hiring, IT workers within firms are experiencing anxiety and frustration, wondering what skills they will need to remain marketable in the future. The current trend with offshoring many IT jobs has exacerbated this problem for many workers. IT workers with traditionally secure positions are not immune to the pressures of this dynamic job environment.

In the present study, the IT Professional Job Satisfaction Model was developed based on 83 discovery interviews with IT workers in various job positions including systems analysts, programmers, technical specialists, and systems project managers. Table 3 shows the demographics for the interview sample.

Table 3. Interview Sample Demographics.

Demographic	Mean (n=83)	SD or Percent
Number of years experience with current project	5.80	6.10
Tenure (# of years with the organization)	10.77	8.61
Age (years)	41.25	9.16
Gender		
Female	35	42%
Male	48	58%
Education:		
High School	13	15.7%
Associates Degree	14	16.9%
BA/BS	40	48.2%
MA/MS/MBA	14	16.8%
Post-Graduate Degree	2	2.4%

These workers were from eight different corporations in a variety of industries (e.g., banking and insurance, manufacturing, education, state and local government). They voluntarily discussed their opinions on a number of job-related issues, generally focusing on their feelings of uncertainty regarding their personal contributions and job security (see the Interview Protocol in Appendix B). Interviews were generally 30 – 45 minutes in length and tape recorded, with the consent of the participant. Then, the interviews were transcribed and the causal statements were highlighted and analyzed according to the RCM technique described in Section II of this chapter. The causal map (Figure 3) was created based on the concepts represented in the transcripts.

In analyzing the data, one clear finding is that most of the IT personnel interviewed had difficulty describing how they fit within the corporate structure. They acknowledged that their contributions were important, but they felt they were personally expendable. Several persons similarly stated, “I’m just a cog in the wheel.” As many researchers and practitioners have noted (e.g., Darais, et al., 2003), in order to survive in the IT field, workers must continue to retrain and learn new skills. Therefore, acknowledgement of the need to change is depicted as the first node in the IT Professional Job Satisfaction Model (see Figure 3, Item 1). The interviewees indicated that skills stagnation often threatened job security. This realistic fear of job loss (Figure 3, Item 2) is a powerful motivator in pursuing necessary training.

IT workers in the interviews discussed the importance of seeking out training opportunities (Figure 3, Item 3), whether offered by the corporation as in-house training, enrollment in formal college courses, or on-line computer-aided learning. These courses might entail attaining certification credentials, college credit, or practical experience. According to a majority of interviewees, if training is available at the place of work, and offered during work hours, employees are more likely to take advantage of the instruction. In contrast, off-hours training, to be completed outside of work on one’s personal time, was less attractive to these employees. However, there is no guarantee that participation in training courses produces adequate knowledge for accomplishing new tasks.

Beyond merely gaining new knowledge and skills (Figure 3, Item 4), interviewees stressed that they must also be able to practice and apply the new skills in a meaningful way (Figure 3, Item 5). In other words, they believe that their training must be utilized on work projects in order for the new skills to become part of workers’ permanent skill sets.

Unfortunately, technical skills are often lost if they are not used soon after the course is completed (Radding, 1997).

Some of the relevant elements of job satisfaction (Figure 3, Item 9) that emerged from this study were perceived feedback from supervisors and co-workers (Figure 3, Item 6), participation in challenging projects (Figure 3, Item 7), and autonomy within the work setting (Figure 3, Item 8). Many IT projects involve teams working together to accomplish defined objectives. Direct feedback obtained from supervisors and co-workers (Figure 3, Item 6) increases job satisfaction because there is less ambiguity about perceived performance. For instance, the interviewees stated that they like to receive continuous feedback in order to determine whether they have adequately satisfied the user requirements and specifications during systems development.

Next, challenging projects (Figure 3, Item 7) provide intrinsic motivation for IT workers. Interviewees remarked that they were anxious to tackle difficult problems for the basic joy of simply discovering new solutions. But, beyond the initial pleasure of design development is the pride of successful implementation and user adoption of their creative solutions. These accomplishments instill job satisfaction at a deep level for IT problem-solvers.

Finally, the level of autonomy (Figure 3, Item 8) positively affects job satisfaction because most IT employees prefer freedom and independence in determining relevant job-related decisions (Ang & Slaughter, 2001; Hackman & Oldham, 1976). According to the interviewees, they derive positive affect from exercising autonomy in project completion, resulting in increased job satisfaction.

Table 4 shows evidence used to support the construct measures. For this study, evidence was obtained from survey data. The survey was developed as an extension of a study in which the RCM technique was used to develop a model of work identity for IT professionals (Buche, 2003). Other possible examples of evidence would be additional interviews, observation, evaluation of documentation, and reviewing physical artifacts. Some of the elements could be gathered from supervisors and secondary sources, triangulating the evidence to analyze the model and to predict job satisfaction of IT professionals.

Figure 3. Information Technology Professional Job Satisfaction Model.

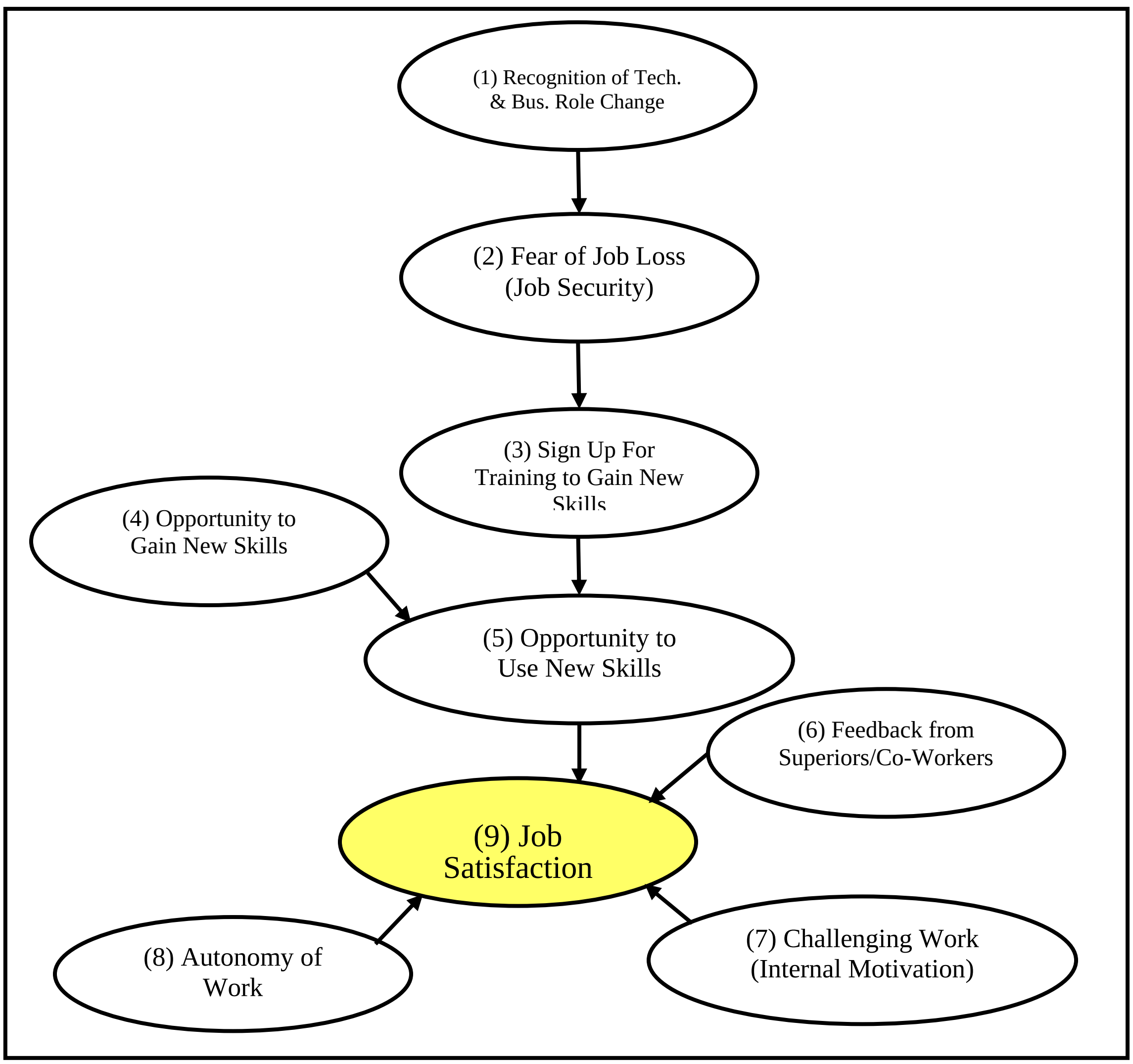

Table 4. Variables, Symbols and Respective Sources of Evidence.

Variable (from RCM)	Symbol	Possible Values	Evidence	Source (Survey Data)
Recognition of Role Change	RR	{yes_{RR}, no_{RR}}	E1	• In my role I am most valued for my technical abilities. • My business knowledge is my most important contribution to the organization. • In my organization, I am perceived to be a technical expert. • I could not be successful this job without broad knowledge of the business domain.
Fear of Job Loss (Job Security)	JT	{yes_{JT}, no_{JT}}	E2.1 E2.2	• Actual layoffs reported in the firm, industry, media • Job security.
Sign Up For Training	ST	{yes_{ST}, no_{ST}}	E3	• Availability of training to learn new skills.
Opportunity to Gain New Skills	GS	{yes_{GS}, no_{GS}}	E4	• Opportunities to learn new things from my work.
Opportunity to Use New Skills	US	{yes_{US}, no_{US}}	E5	• Opportunities to apply new skills in my work.
Feedback from Superiors/Co-workers	FS	{yes_{FS}, no_{FS}}	E6	• My managers or co-workers often let me know how well I'm doing on my job. • I'm frustrated by the fact that my supervisor and co-workers almost never give me any feedback about how well I am doing my work. • My supervisor gives me specific inputs on how well I am performing my responsibilities.
Challenging Work	CW	{yes_{CW}, no_{CW}}	E7	• Stimulating and challenging work.
Autonomy of Work	AW	{yes_{AW}, no_{AW}}	E8	• I have a lot of autonomy in my job. That, is, I decide how to go about doing my projects. • The job denies me any chance to use my personal initiative or judgment in carrying out the work. • My job gives me considerable opportunity for independence and freedom in how I do my work.

VI. Conversion of Revealed Causal Map into Evidential Diagram and Belief Propagation

In this section we first discuss how a revealed causal map can be converted to a belief function evidential diagram and then discuss how beliefs can be propagated through this evidential diagram. Our example is displayed in Figure 3.

Conversion of Revealed Causal Map into Evidential Diagram

The conversion process of revealed causal map into evidential diagram can be described in the following five steps:

1. Identify the main variables (i.e., constructs) in the revealed causal map.
2. Determine the possible values of these variables (such as, 'true/false', or 'high/medium/low').
3. Determine the relationships among the variables (see the details below).
4. Connect the variables through the corresponding relationships.
5. Identify potential items of evidence pertaining to the variables in the diagram and connect these items of evidence to the relevant variables.

The above approach yields the desired evidential diagram for belief-function analysis. In Steps 1 and 2, we have identified nine variables (i.e., constructs; see Figure 3) and their corresponding categorical values (Table 4).

Step 3 (Determining the relationships among various variables) is a somewhat difficult process. Expert judgments about these relationships must be rendered. For example, the relationship R1 defining the relationship between 'Role Recognition (RR)' and 'Job Security (JT)' was extremely difficult to model. In this case, the survey data provided only information on whether the subjects recognize their changing role on the job and did not specify any details on how this knowledge might influence 'Job Security'. For IT personnel, 'Role Recognition' might mean that 'yes' there is 'Job Security', but it also may mean that there is no 'Job Security'. Thus, lacking any other information, we assume, for the present discussion, that when 'Role Recognition' is yes, 'Job Security' is 50% 'yes', and 50% 'no'. However, when there is no knowledge about 'Role Recognition', there is no knowledge about 'Job Security'. Such a relationship can be expressed in terms of m-values as given below.

<u>m-values for R1</u>:

$$m_{R1}(\{(yes_{RR}, yes_{JT}), (no_{RR}, yes_{JT}), (no_{RR}, no_{JT})\}) = 05,$$
$$m_{R1}(\{(yes_{RR}, no_{JT}), (no_{RR}, yes_{JT}), (no_{RR}, no_{JT})\}) = 05.$$

The above relationship propagates[9] 50% of $m_{E1}(yes_{RR})$, the belief on 'Role Recognition' being 'yes' from evidence E1 (Figure 4), to 'yes_{JT}' 50% of $m_{E1}(yes_{RR})$ to 'no_{JT}', and 100% of $m_{E1}(no_{RR})$ and $m_{E1}(\{yes_{RR}, no_{RR}\})$ to $(\{yes_{JT}, no_{JT}\})$, as described in the assumed relationship. In other words, the m-values propagated from variable 'Role Recognition (RR)' to variable 'Job security (JT)' are given as:

$$m_{JT\leftarrow RR}(yes_{JT}) = 0.5m_{E1}(yes_{RR}), m_{JT\leftarrow RR}(no_{JT}) = 0.5m_{E1}(yes_{RR}), \text{and}$$
$$m_{JT\leftarrow RR}(\{yes_{JT}, no_{JT}\}) = m_{E1}(no_{RR}) + m_{E1}(\{yes_{RR}, yes_{RR}\})$$

For the relationship R2 we assume the following. On average, a person with the knowledge that there is no job security will sign up for job training with 90% belief and a person with the knowledge that there is no problem with the job security will not sign up for job training with 90% belief. This relationship can be modeled in the following way:

[9] As described in Section IV, in order to propagate m-values from 'RR' to 'JT' through the relationship R1, one needs to vacuously extend the m-values from the space of 'RR', $\{yes_{RR}, no_{RR}\}$, to the space of R1, which is the joint

m-values for R2:

$$m_{R2}(\{(yes_{JT}, no_{ST}), (no_{JT}, yes_{ST})\}) = 0.9, \text{ and}$$

$$m_{R2}(\{(yes_{JT}, yes_{ST}), (yes_{JT}, no_{ST}), (no_{JT}, yes_{ST}), (no_{JT}, no_{ST})\}) = 0.1.$$

Similar to footnote 9, one can easily show the following m-values to be the result of m-values propagated from variable 'Job Security (JT)' to variable 'Sign up for Job Training (ST)' through the relationship R2:

$$m_{ST\leftarrow JT}(yes_{ST}) = 0.9m_{JT}(no_{JT}),\ m_{ST\leftarrow JT}(no_{ST}) = 0.9m_{JT}(yes_{JT}), \text{ and}$$

$$m_{ST\leftarrow JT}(\{yes_{ST}, no_{ST}\}) = 0.1 + 0.9m_{JT}(\{yes_{RR}, yes_{RR}\}).$$

We assumed the following m-values for R3 (see Table 4 for the definitions of the symbols):

m-values for R3:

$$m_{R3}(\{(yes_{ST}, yes_{US}), (no_{ST}, no_{US})\}) = 0.75, \text{ and}$$

$$m_{R3}(\{(yes_{ST}, yes_{US}), (yes_{ST}, no_{US}), (no_{ST}, yes_{US}), (no_{ST}, no_{US})\}) = 0.25.$$

The above relationship implies that if variable 'ST' is 'yes', i.e., a person sings up for training, then variable 'US' will be 'yes', i.e., the person will have the opportunity to use the new skill, with 0.75 belief and the remaining 0.25 belief is assigned to ignorance. Similarly, the relationship implies that if 'ST' is 'no' then 'US' is 'no' with belief 0.75, i.e., if one does not sign up for job training then he/she will not have the use of new skill with belief 0.75. The remaining 0.25 belief represents ignorance.

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space of 'RR' and 'JT', i.e., $\{(yes_{RR}, yes_{JT}), (yes_{RR}, no_{JT}), (no_{RR}, yes_{JT}), (no_{RR}, no_{JT})\}$, combine the m-values at R1, and then marginalize to the space of 'JT', $\{yes_{JT}, no_{JT}\}$.

For the relationship R4, we assume the following m-values:

m-values for R4:

$$m_{R4}(\{(yes_{GS}, yes_{US}), (no_{GS}, no_{US})\}) = 1.0.$$

This relationship implies that if 'GS' is 'yes' then 'US' is 'yes' with 1.0 belief. Also, if 'GS' is 'no' then 'US' is 'no' with 1.0 belief. In other words, if one has the opportunity to gain new skills on the job then there is 1.0 belief that there is opportunity to use the new skills. Similarly, if there is no opportunity to gain new skills on the job then there is no opportunity to use the new skills.

The relationship R5 relates variables 'US', 'FS', 'AW', and 'CW' to the variable 'Job Satisfaction (JS)'. We have assumed the following relative weights, 0.125, 0.125, 0.25, and 0.5, respectively, for 'US', 'FS', 'AW', and 'CW' when propagating information (m-values) to the variable 'JS'.

Step 4 simply represents a diagram with all the variables interconnected through the assumed relationships (see Figure 4). In Step 5, we identify various items of evidence pertaining to different variables and connect them to the corresponding variables. Table 4 provides a list of evidence pertaining to the nine variables in Figure 4. Once these items of evidence are connected to the corresponding variables, we develop the evidential diagram shown in Figure 4 for the analysis.

Propagation of Beliefs through Evidential Diagram

In order to propagate information in terms of m-values from all the variables to the variable of interest, say, 'Job Satisfaction (JS)' in Figure 4, we need to follow the following steps. First, gather all the information (m-values) at 'Role Recognition (RR)', propagate that information (m-values) to variable ‘Job Security (JT)’ through the relationship R1 by first vacuously extending to the space of R1, combining it with the m-values at R1 using Dempster’s rule, and then marginalizing the resulting m-values to the space of ‘JT’. Combine this information (m-values) with the m-values at ‘JT’ obtained from evidence E2.1 and E2.2, again using Dempster’s rule. Next step is to propagate the resulting m-values at ‘JT’ through R2 to the variable ‘Sign up for Training to Gain New Skills (ST)’. This is achieved again by vacuously extending the total m-values at ‘JT’ to the space of R2, combining them with the m-values at R2 using Dempster’s rule, and then marginalizing them to the space of variable ‘ST’. Combine this information (m-values) with the m-values obtained from evidence E3 for ‘ST’. The resulting m-values are then propagated to the variable ‘Opportunity to use New Skills (US)’. Combine these m-values with the m-values obtained from the variable ‘Opportunity to Gain New Skills on the Job (GS)’ and the m-values from evidence E5 for ‘US’.

In the final step, we need to propagate all the m-values from the four variables, ‘US’, ‘FS’, ‘AW’, and ‘CW’ through the relationship R5 to the variable ‘Job Satisfaction (JS)’ by vacuously extending the respective m-values to the space of R5, combine these m-values with the m-values defining R5 and then marginalize them to the space of ‘Job Satisfaction’. The marginalized m-values on ‘Job Satisfaction (JS)’ can be written as:

$$m_{JS}(yes_{JS}) = 0.125m_{US}(yes_{US}) + 0.125m_{FS}(yes_{FS}) + 0.25m_{AW}(yes_{AW}) + 0.5m_{CW}(yes_{CW}).$$

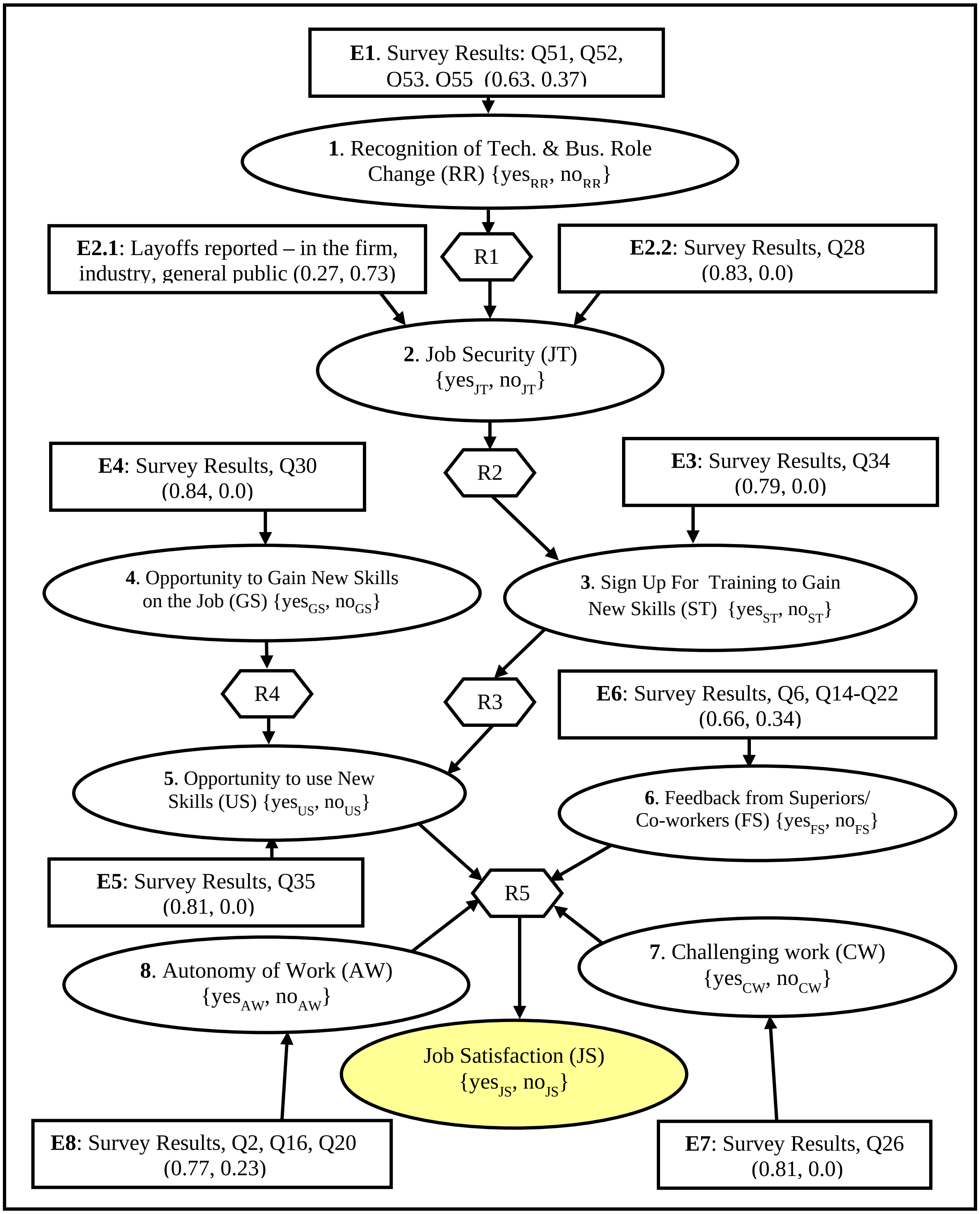
$$m_{JS}(no_{JS}) = 0.125m_{US}(no_{US}) + 0.125m_{FS}(no_{FS}) + 0.25m_{AW}(no_{AW}) + 0.5m_{CW}(no_{CW}).$$

$$m_{JS}(\{yes_{JS}, no_{JS}\}) = 1 - m_{JS}(yes_{JS}) - m_{JS}(no_{JS}).$$

These m-values provide the impact of all the variables in the evidential diagram in Figure 4.

Given that the evidential diagram in Figure 4 is a tree, the propagation of m-values from various variables to the variable of interest, 'Job Satisfaction' is much easier than propagation in a network of variables. We programmed the logic of vacuous extension, marginalization, and Dempster's rule of combination in a spreadsheet program in MS Excel, which then was used to perform various analyses as discussed in the next section.

Figure 4: Evidential Diagram* of the Causal Map in Figure 3



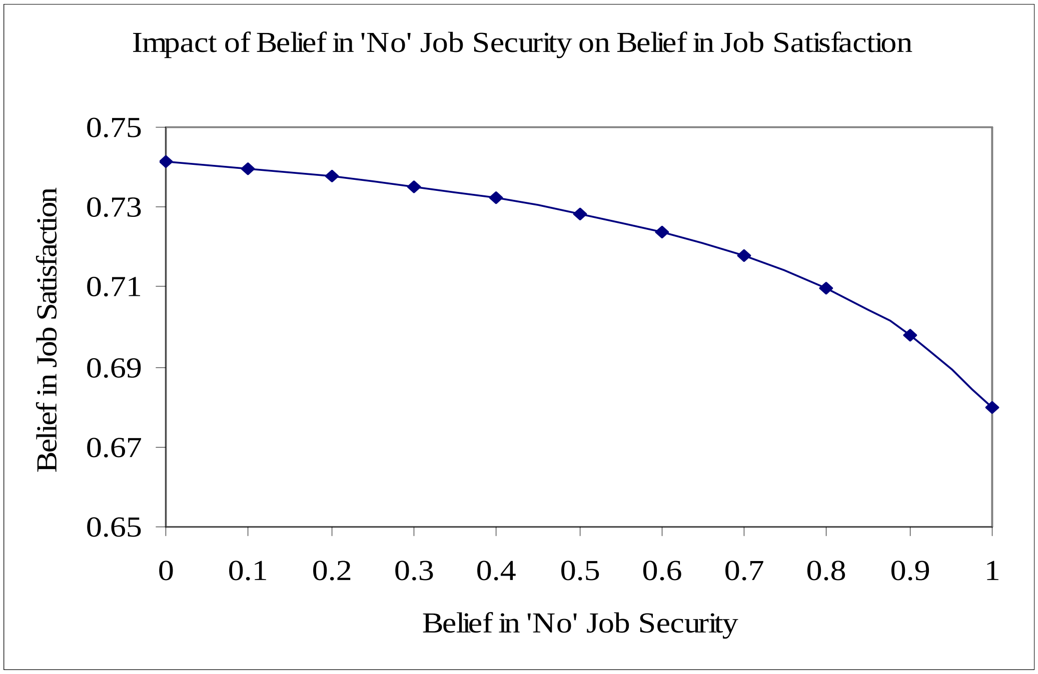
*The oval shaped boxes represent variables and the rectangular boxes represent items of evidence. The numbers in a rectangular box represents the level of support for and against the variable it is connected to. These numbers were determined from the Survey Results except for E2.1 which was determined from the industry data.

VII. Decision Analysis of Causal Map Using Belief Functions

In this section, we discuss how one can analyze the impact of one variable on the other variables in the network given in Figure 4. Such an analysis allows the decision maker to isolate an independent variable while holding the rest of the variables in the model constant. In this example, the overall belief in job satisfaction is 0.803 given the inputs from the Survey Results and industry data. The above value implies that based on the subjects responses, on average, employees are satisfied with their jobs in the environment surveyed with 0.803 level of belief. In order to investigate the impact of a number of variables on the level of job satisfaction, we use a range of possible responses (0 to 1.0) for the variables while keeping the inputs from other items of evidence fixed at values obtained from the survey as given in the respective figures.

First, we investigate the impact of 'Job Security' on 'Job Satisfaction'. We vary the input belief from evidence E2.2 for the negation of 'Job Security' from 0 to 1.0, keeping the rest of inputs fixed. As seen in Figure 5, the impact of 'No Job Security' is pretty severe. As the belief in no job security increases the belief in job satisfaction decreases with increasing rate. In other words, if an employee sees strong evidence in support of 'no job security' then he/she will have very low job satisfaction.

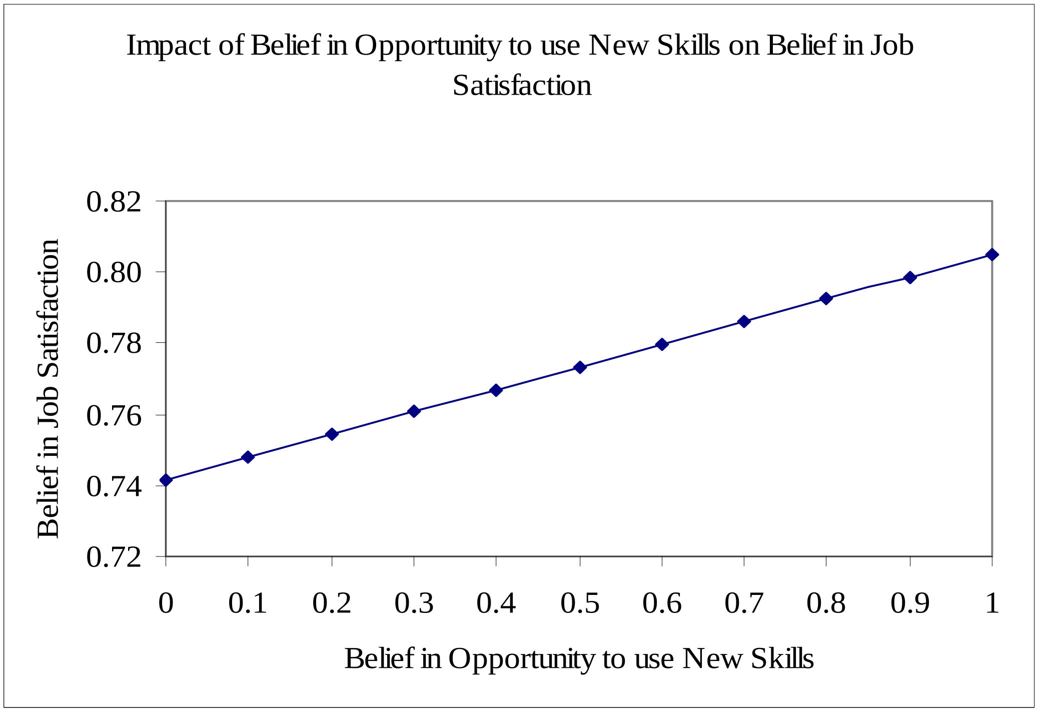
Figure 5: Belief in Job Satisfaction versus Belief in No Job Security*.



*The following input m-values in Figure 4 were used for the graph: $m_{E1}(yes_{RR})=0$, $m_{E1}(no_{RR})=0$, $m_{E2.1}(yes_{JT})=0.27$, $m_{E2.1}(no_{JT})=0.73$, $m_{E2.2}(yes_{JT})=0$, $m_{E2.2}(no_{JT})$ varied from 0 - 1, $m_{E3}(yes_{ST})=0$, $m_{E3}(no_{ST})=0$, $m_{E4}(yes_{GS})=0$, $m_{E4}(no_{GS})=0$, $m_{E5}(yes_{US})=0$,, $m_{E5}(no_{US})=0$, $m_{E6}(yes_{FS})=0.66$, $m_{E6}(no_{FS})=0.34$, $m_{E7}(yes_{CW})=0.81$, $m_{E7}(no_{CW})=0$, $m_{E8}(yes_{AW})=0.77$, $m_{E8}(no_{AW})=0.23$.

The second sensitivity analysis is conducted on the impact of having an opportunity to use new skills on the job. This analysis reveals that the opportunity to use new skills has a significant positive impact on ‘Job Satisfaction’ as seen from Figure 6. As the belief in opportunity to use new skills increases, the belief in job satisfaction increases. We find an 8.5% increase in job satisfaction over the range from 0 – 1.0 for belief in opportunity to use new skills. This impact is linear, unlike the previous case.

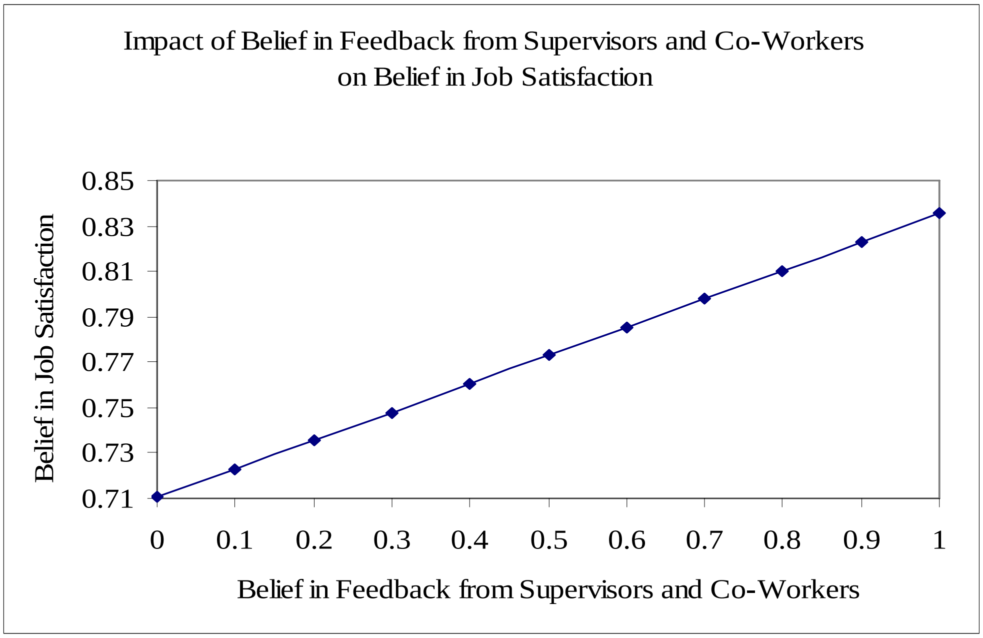
Figure 6: Belief in Job Satisfaction versus Belief in Opportunity to use New Skills*.



*The following input m-values in Figure 4 were used for the graph: $m_{E1}(yes_{RR})=0$, $m_{E1}(no_{RR})=0$, $m_{E2.1}(yes_{JT})=0.27$, $m_{E2.1}(no_{JT})=0.73$, $m_{E2.2}(yes_{JT})=0$, $m_{E2.2}(no_{JT})=0$, $m_{E3}(yes_{ST})=0$, $m_{E3}(no_{ST})=0$, $m_{E4}(yes_{GS})=0$, $m_{E4}(no_{GS})=0$, $m_{E5}(yes_{US})$ varied from 0 - 1, $m_{E5}(no_{US})=0$, $m_{E6}(yes_{FS})=0.66$, $m_{E6}(no_{FS})=0.34$, $m_{E7}(yes_{CW})=0.81$, $m_{E7}(no_{CW})=0$, $m_{E8}(yes_{AW})=0.77$, $m_{E8}(no_{AW})=0.23$.

The third variable analyzed is 'Feedback from Supervisors/Co-workers'. As shown in Figure 7, the results demonstrate a substantial positive impact of feedback on the job satisfaction. In particular, job satisfaction increases about 19% as we progress from the lower to higher levels of perceived feedback. It is obvious that feedback is a powerful variable in predicting job satisfaction.

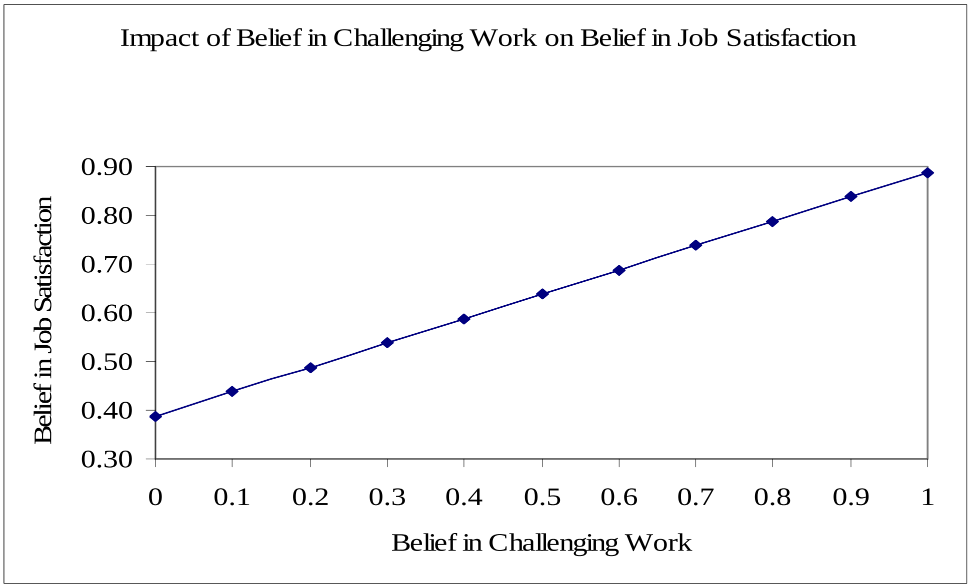
Figure 7: Belief in Job Satisfaction versus Belief in Feedback from Supervisors and Co-workers*.



*The following input m-values in Figure 4 were used for the graph: $m_{E1}(yes_{RR})=0$, $m_{E1}(no_{RR})=0$, $m_{E2.1}(yes_{JT})=0.27$, $m_{E2.1}(no_{JT})=0.73$, $m_{E2.2}(yes_{JT})=0$, $m_{E2.2}(no_{JT})=0$, $m_{E3}(yes_{ST})=0$, $m_{E3}(no_{ST})=0$, $m_{E4}(yes_{GS})=0$, $m_{E4}(no_{GS})=0$, $m_{E5}(yes_{US})=0.81$, $m_{E5}(no_{US})=0$, $m_{E6}(yes_{FS})$ varied from 0 - 1, $m_{E6}(no_{FS})=0$, $m_{E7}(yes_{CW})=0.81$, $m_{E7}(no_{CW})=0$, $m_{E8}(yes_{AW})=0.77$, $m_{E8}(no_{AW})=0.23$.

Next, we conduct a sensitivity analysis with the independent variable, ‘Challenging Work’. ‘Job Satisfaction’ was extremely sensitive to increases in the perceived level of challenging work. From no belief that the job is challenging to the higher range of belief, 1.0, the model indicates that the belief in job satisfaction moves from 0.388 to 0.838; a 129% increase as seen in Figure 8. These results indicate that challenging work is the most powerful variable in the model in the prediction of job satisfaction.

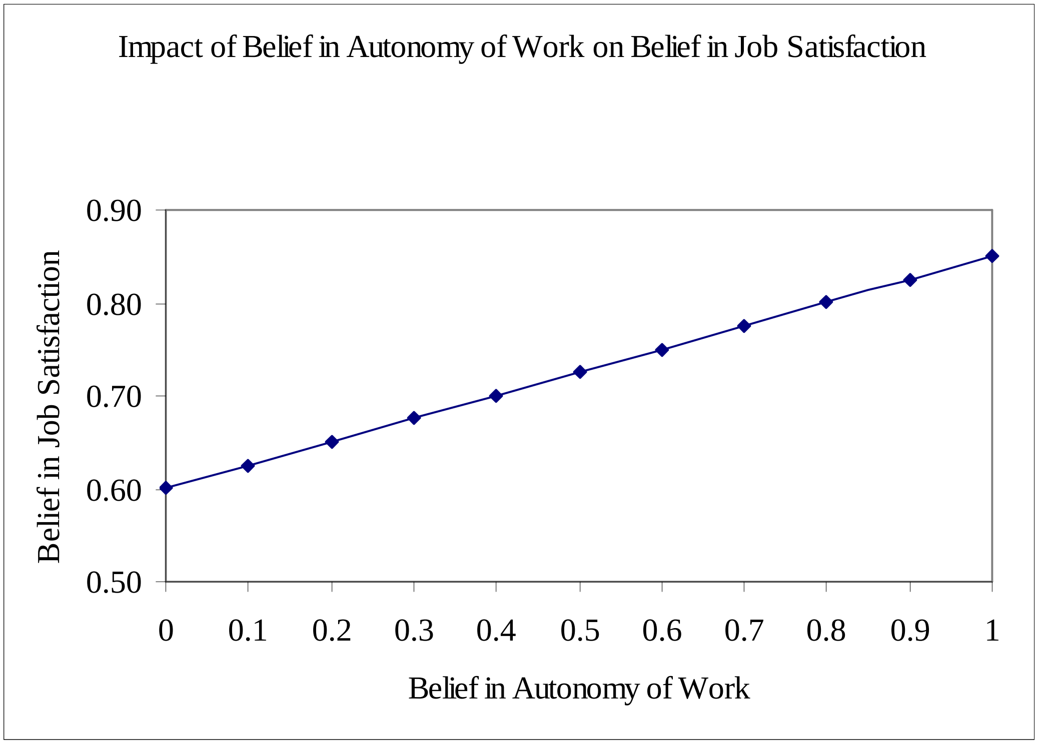
Figure 8: Belief in Job Satisfaction versus Belief in Challenging Work*.



*The following input m-values in Figure 4 were used for the graph: $m_{E1}(yes_{RR})=0$, $m_{E1}(no_{RR})=0$, $m_{E2.1}(yes_{JT})=0.27$, $m_{E2.1}(no_{JT})=0.73$, $m_{E2.2}(yes_{JT})=0$, $m_{E2.2}(no_{JT})=0$, $m_{E3}(yes_{ST})=0$, $m_{E3}(no_{ST})=0$, $m_{E4}(yes_{GS})=0$, $m_{E4}(no_{GS})=0$, $m_{E5}(yes_{US})=0.81$, $m_{E5}(no_{US})=0$, $m_{E6}(yes_{FS})=0.66$, $m_{E6}(no_{FS})=0.34$, $m_{E7}(yes_{CW})$ varied from 0 - 1, $m_{E7}(no_{CW})=0$, $m_{E8}(yes_{AW})=0.77$, $m_{E8}(no_{AW})=0.23$.

Finally, a sensitivity analysis was conducted on ‘Autonomy of Work’. The results indicate that ‘Autonomy of Work’ has a significant impact on the dependent variable, ‘Job Satisfaction’. Job satisfaction was found to be very sensitive to autonomy. As the perceived autonomy increases from 0 to 1.0, job satisfaction improves from 60% to 85%, an increase of 41.6%. These results are presented in Figure 9.

Figure 9: Belief in Job Satisfaction versus Belief in Autonomy of Work*.



*The following input m-values in Figure 4 were used for the graph: $m_{E1}(yes_{RR})=0$, $m_{E1}(no_{RR})=0$, $m_{E2.1}(yes_{JT})=0.27$, $m_{E2.1}(no_{JT})=0.73$, $m_{E2.2}(yes_{JT})=0$, $m_{E2.2}(no_{JT})=0$, $m_{E3}(yes_{ST})=0$, $m_{E3}(no_{ST})=0$, $m_{E4}(yes_{GS})=0$, $m_{E4}(no_{GS})=0$, $m_{E5}(yes_{US})=0.81$, $m_{E5}(no_{US})=0$, $m_{E6}(yes_{FS})=0.66$, $m_{E6}(no_{FS})=0.34$, $m_{E7}(yes_{CW})=0.81$, $m_{E7}(no_{CW})=0$, $m_{E8}(yes_{AW})$ varied from 0 - 1, $m_{E8}(no_{AW})=0$.

These sensitivity analyses have shown the impact on job satisfaction from a broad range of variables and their corresponding beliefs. However, we do want to point out that the interrelationships among the intermediate variables and the relative weights assigned to 'Opportunity to use New Skills', 'Feedback from Supervisors and Co-Workers, 'Challenging Work', and 'Autonomy of Work', have direct impact on the results for the dependent variable, 'Job Satisfaction'.

In summary, the above analysis provides an example of how an evidential reasoning approach under Dempster-Shafer theory of belief functions can be used to determine the impact on a given construct or constructs of other constructs in a revealed causal map. It should be noted that a revealed causal map of a decision problem is only a static model while an evidential diagram of a revealed causal map provides a dynamic model for analyzing the behaviors of various constructs under different conditions.

VIII. Conclusions and Future Directions for Research

In this chapter we have demonstrated the use of evidential reasoning approach under Dempster-Shafer (D-S) theory of belief functions to analyze revealed causal maps. As an example, we used a simplified causal map obtained through a Revealed Causal Mapping (RCM) technique where the participants were from information technology (IT) organizations who provided the concepts to describe the target phenomenon of 'Job Satisfaction'. They also identified the associations between the concepts. After creating the causal map of the problem being investigated, we developed an evidential diagram. This diagram consists of the variables or constructs of the causal map, interconnected to the other variables with some relationships. These relationships were defined by the decision maker based on experience. Various items of evidence were identified that pertained to different variables. Estimates of the beliefs in terms of m-values in support of, or negation of, the variables were made for each item of evidence using survey questions (Buche, 2003, particularly Appendix C). These m-values were then propagated through the evidential network to obtain the overall belief of 'Job Satisfaction'.

To illustrate the usefulness of the evidential reasoning approach under Dempster-Shafer theory of belief functions, we performed various sensitivity analyses to determine the impact of different variables on 'Job Satisfaction'. This technique enables researchers to predict the level of job satisfaction when given evidence for the other variables in the model. As further validation for our findings, our results are directly in line with previous literature on job satisfaction for workers in general. IT personnel are very similar to other professions and vocations. An evidential diagram similar to the one discussed here would be useful in predicting whether a specific work environment would be more or less satisfactory to an employee before joining the job.

In this chapter we have explained the steps necessary to convert revealed causal maps into evidential diagrams. The analysis of the transformed diagram is useful in forming predictions about human behavior. This technique incorporates the existence of uncertainty in the level of belief associated with the evidence. Therefore, the researcher is able to include in the diagram personal intuition and confidence based on direct experience. Another advantage of the evidential reasoning approach over a revealed causal map is that the former provides a dynamic model of a decision problem while the later provides only a static model. As a limitation, the evidential reasoning approach may become quite complex especially when variables or constructs in the diagram are highly integrated. For ease of instruction, the example discussed herein was fairly simplistic, with primarily linear associations.

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APPENDIX A. Concept dictionary with examples.

Construct	Description	Example
Role not valued	Company no longer needs certain skill sets to support certain roles.	Generalists such as myself…don't see that role being valued much
Role change	Expectations of workers experience transition.	I got into the analyst role, being the leader and doing the coordination
Fear of job loss	Lack of job security.	Anyone would be worried about their career
Sign up for training	Training is provided by a company for workers to develop new skills.	We just look at the classes, sign up for them
Opportunity to gain new skills	Workers are taught new skills in classroom or self-paced training.	Once you learn programming, and you have that skill
Opportunity to use new skills	The job environment provides the opportunities for workers to practice the skills learned during training.	Using new skills to make the company more competitive
Feedback form superiors and co-workers	Direct reaction obtained from supervisors and co-workers that reduces ambiguity about perceived performance.	The users let me know if the system meets their needs
Challenging projects	Work assignments provide an intrinsic motivation because the problem-solving aspect takes effort.	Technical challenges of the job
Autonomy of Work	Workers have freedom and independence in determining relevant job-related decisions.	Nobody really tells me what to do or how to do it
Job satisfaction	Affective response to the current job environment.	Pleasant work environment

APPENDIX B. Interview Protocol[10].

1. What motivates you to come to work here every day?
2. What is the best thing about your current work environment?
3. What is the worst thing about your current work environment?
4. What is the most important thing you contribute to this organization?
5. What could you contribute to your organization that you currently are unable to contribute?
6. What barriers keep you from making these (this) contribution?
7. Where do you realistically see yourself professionally in five years?
8. Where would you ideally like to see yourself professionally in five years?
9. What barriers might keep you from your ideal situation?
10. How much do you like change?
11. How much do you think the IT field, in general, is changing?
12. How much do you think the IT field at your company is changing?
13. How do you feel about this level of change?
14. How is your organization supporting you in personally making these changes?
15. What barriers do you see in making these changes?
16. What is your primary, one year professional goal?
17. How can your organization help you achieve you goals?
18. In summary, how do you see yourself fitting into the organization's "big picture"?
19. Would you like to add any further comments or observations?

[10] This semi-structured interview guide was also part of NSF grant proposal and Transition Study research project (Nelson, 2000; Buche, 2003).

APPENDIX C

Propagation Illustration in Figure 1

In this appendix we describe in detail the three steps involved in the propagation of m-values from variables X and Y in Figure 1 to variable Z.

Step 1: Propagation of m-values from X and Y to 'AND' node:

In order to propagate m-values from variable X, a smaller node with one variable and the frame Θ_X={x,~x}, to the 'AND' node, a larger node consisting of three variable X, Y, and Z with the frame Θ_{AND}= {xyz, x~y~z, ~xy~z, ~x~y~z}, we vacuously extend the m-values at X to the space {xyz, x~y~z, ~xy~z, ~x~y~z} defined by the 'AND' node . This process yields the following non-zero m-values from X to the 'AND' node:

$$m_{AND\leftarrow X}(\{xyz, x\sim y\sim z\}) = m_X(x) = 0.6,$$

$$m_{AND\leftarrow X}(\{\sim xy\sim z, \sim x\sim y\sim z\}) = m_X(\sim x) = 0.2,$$

$$m_{AND\leftarrow X}(\{xyz, x\sim y\sim z, \sim xy\sim z, \sim x\sim y\sim z\}) = m_X(\{x, \sim x\}) = 0.2.$$

Similarly, we obtain the following non-zero m-values at the 'AND' node when the m-values from Y are propagated to the 'AND' node:

$$m_{AND\leftarrow Y}(\{xyz, \sim xy\sim z\}) = m_Y(y) = 0.7$$

$$m_{AND\leftarrow Y}(\{xyz, x\sim y\sim z, \sim xy\sim z, \sim x\sim y\sim z\}) = m_Y(\{y,\sim y\}) = 0.3$$

Step 2: Combine m-values from X and Y with the m-values at 'AND'

We have the following set of m-values at the 'AND' node; one from X, one from Y, and one at the 'AND' node defining the relationship.

<u>m-values from X</u>:

$$m_{AND\leftarrow X}(\{xyz, x\sim y\sim z\}) = 0.6,\ m_{AND\leftarrow X}(\{\sim xy\sim z, \sim x\sim y\sim z\}) = 0.2, \text{ and}$$

$$m_{AND \leftarrow X}(\{xyz, x{\sim}y{\sim}z, {\sim}xy{\sim}z, {\sim}x{\sim}y{\sim}z\}) = 0.2.$$

<u>m-values from Y</u>:

$$m_{AND \leftarrow Y}(\{xyz, {\sim}xy{\sim}z\}) = 0.7,$$

$$m_{AND \leftarrow Y}(\{xyz, x{\sim}y{\sim}z, {\sim}xy{\sim}z, {\sim}x{\sim}y{\sim}z\}) = 0.3.$$

<u>m-values at the 'AND' node</u>:

$$m_{AND}(\{xyz, x{\sim}y{\sim}z, {\sim}xy{\sim}z, {\sim}x{\sim}y{\sim}z\}) = 1.0.$$

After we combine the above m-values using Dempster's rule, we obtain the following m-values:

$$m(\{xyz\}) = m_{AND \leftarrow X}(\{xyz, x{\sim}y{\sim}z\}).m_{AND \leftarrow Y}(\{xyz, {\sim}xy{\sim}z\}).\ m_{AND}(\{xyz, x{\sim}y{\sim}z, {\sim}xy{\sim}z, {\sim}x{\sim}y{\sim}z\})$$
$$= 0.6x0.7x1.0 = 0.42,$$

$$m(\{xyz, x{\sim}y{\sim}z\}) = m_{AND \leftarrow X}(\{xyz, x{\sim}y{\sim}z\}).m_{AND \leftarrow Y}(\{xyz, x{\sim}y{\sim}z, {\sim}xy{\sim}z, {\sim}x{\sim}y{\sim}z\}).m_{AND}(\{xyz, x{\sim}y{\sim}z, {\sim}xy{\sim}z, {\sim}x{\sim}y{\sim}z\}) = 0.6x0.3x1.0 = 0.18,$$

$$m(\{{\sim}xy{\sim}z\}) = m_{AND \leftarrow X}(\{{\sim}xy{\sim}z, {\sim}x{\sim}y{\sim}z\}).m_{AND \leftarrow Y}(\{xyz, {\sim}xy{\sim}z\}).m_{AND}(\{xyz, x{\sim}y{\sim}z, {\sim}xy{\sim}z, {\sim}x{\sim}y{\sim}z\})$$
$$= 0.2x0.7x1.0 = 0.14,$$

$$m(\{{\sim}xy{\sim}z, {\sim}x{\sim}y{\sim}z\}) = m_{AND \leftarrow X}(\{{\sim}xy{\sim}z, {\sim}x{\sim}y{\sim}z\}).m_{AND \leftarrow Y}(\{xyz, x{\sim}y{\sim}z, {\sim}xy{\sim}z, {\sim}x{\sim}y{\sim}z\}).m_{AND}(\{xyz, x{\sim}y{\sim}z, {\sim}xy{\sim}z, {\sim}x{\sim}y{\sim}z\}) = 0.2x0.3x1.0 = 0.06,$$

$$m(\{xyz, {\sim}xy{\sim}z\}) = m_{AND \leftarrow X}(\{xyz, x{\sim}y{\sim}z, {\sim}xy{\sim}z, {\sim}x{\sim}y{\sim}z\}).m_{AND \leftarrow Y}(\{xyz, {\sim}xy{\sim}z\}).m_{AND}(\{xyz, x{\sim}y{\sim}z, {\sim}xy{\sim}z, {\sim}x{\sim}y{\sim}z\}) = 0.2x0.7x1.0 = 0.14,$$

$$m(\{xyz, x{\sim}y{\sim}z, {\sim}xy{\sim}z, {\sim}x{\sim}y{\sim}z\}) = m_{AND \leftarrow X}(\{xyz, x{\sim}y{\sim}z, {\sim}xy{\sim}z, {\sim}x{\sim}y{\sim}z\}).m_{AND \leftarrow Y}(\{xyz, x{\sim}y{\sim}z, {\sim}xy{\sim}z, {\sim}x{\sim}y{\sim}z\}).m_{AND}(\{xyz, x{\sim}y{\sim}z, {\sim}xy{\sim}z, {\sim}x{\sim}y{\sim}z\}) = 0.2x0.3x1.0 = 0.06.$$

The above m-values are propagated to variable Z by marginalizing them to Z as described below.

Step 3: Propagate m-values from 'AND' node to Z

The third step deals with propagating beliefs or m-values from 'AND' node to variable Z. Since the "AND' is a bigger node consisting of three variables, X, Y, and Z, the m-values have to be marginalized to variable Z. As discussed in Footnote 6, marginalization of belief functions or m-values is similar to marginalization of probabilities; the unwanted variables are eliminated by summing the m-values over the variables. We obtain the following m-values on variable Z as a result of propagation of m-values from X and Y through the relationship 'AND' by marginalization of m-values at the 'AND' node:

$$m_{Z\leftarrow AND}(\{z\}) = m(\{xyz\}) = 0.42,$$

$$m_{Z\leftarrow AND}(\{\sim z\}) = m(\{\sim xy\sim z\}) + m(\{\sim xy\sim z, \sim x\sim y\sim z\}) = 0.14 + 0.06 = 0.20,$$

$$m_{Z\leftarrow AND}(\{z,\sim z\}) = m(\{xyz, x\sim y\sim z\}) + m(\{xyz, \sim xy\sim z\}) + m(\{xyz, x\sim y\sim z, \sim xy\sim z, \sim x\sim y\sim z\})$$

$$= 0.18 + 0.14 + 0.06 = 0.38.$$

This completes the process. We now know that belief that Z is true is 0.42 (i.e., Bel(z) = 0.42), given that we know that X is true with belief 0.6 and Y is true with belief 0.7. Similarly, we know that Z is not true with belief 0.20, i.e., Bel(~z) = 0.20, given the knowledge about X and Y expressed in terms of the following m-values: $m_X(x) = 0.6$, $m_X(\sim x) = 0.2$, $m_X(\{x,\sim x\}) = 0.2$, and $m_Y(y) = 0.7$, $m_Y(\sim y) = 0$, $m_Y(\{y,\sim y\}) = 0.3$.