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A Belief-Function Perspective to Default Risk Assessments

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1. Introduction

Risk assessment in any field is a challenging problem whether it deals with assessing the potential loss of personal properties due to flood, the presence of material misstatements or fraud in the financial statements of a company, assessing the likelihood of firm survival or loan default. There are two important general concepts related to risk assessment. One deals with the potential loss due to the undesirable event, such as loan default or corporate bankruptcy. The other deals with the uncertainty associated with the event, whether the event will occur or will not occur. There are two kinds of uncertainties. One kind arises purely because of the random nature of the event. For random events, there exist stable frequencies in repeated trials under fixed conditions.

For such random events, one can use the knowledge of the stable frequencies to predict the probability of occurrence of the event. For example, tossing a fair coin is purely a random process with stable frequencies in repeated trials where one would expect to get head 50% of the times and tail 50% of the times. This kind of uncertainty has been the subject of several previous chapters in this volume which has dealt with various statistical approaches to modeling default risk and corporate bankruptcy prediction.

The other kind of uncertainty arises because of the lack of knowledge of the true state of nature where we not only lack the knowledge of a stable frequency, but also we lack the means to specify fully the fixed conditions under which repetitions can be performed.

The present chapter provides a theoretical framework to deal with such situations. In the context of default risk, credit ratings and bankruptcy models, many statistical models attempt to exploit the regularities in empirical data in order to formulate probability estimates of the likelihood of such events in the future. Belief functions postulate that repetitions under fixed

conditions which underlie many statistical models of default risk and bankruptcy prediction are often impossible, particularly where we even lack the means to specify fully the fixed conditions under which we would like to have repetitions. For example, the credit ratings issued by many large rating agencies, such as Standard and Poor's (S&P) are based on a raft of macroeconomic, industry and firm specific information which are to a large extent unique to each particular firm being rated. For instance, S&P ratings (and revision of ratings) are usually based on detailed ongoing interviews with management, where dialogue with management tends to be more frequent in response to significant industry events, material announcements by the company or plans by the company to pursue new financings. Firm specific information used by S&P include the use of current and future oriented financial information (both at the time of the rating and on an ongoing basis), assessments of the quality of management, the adequacy of corporate governance arrangements, the relationships with key suppliers and lenders, including a variety of confidential information (not disclosed to the public when a firm is rated), such as budgets and forecasts, financial statements on a stand-alone basis, internal capital allocation schedules, contingent risks analyses and information relating to new financings, acquisitions, dispositions and restructurings.¹

This information is unique to each firm. What may happen in one firm cannot be generalized to another, hence there may be no observable empirical regularities in which statistical generalizations can be inferred.

This second kind of uncertainty is relevant in assessing default risk in many real world situations. For example, consider a credit ratings agency which uses financial statement data

¹See for example the testimony of the Standard and Poor's rating agency in the public hearings before the US Securities and Exchange Commission, November 15, 2002 on the Role and Function of Credit Rating Agencies in the U.S. Securities Markets (<http://www.sec.gov/news/extra/credrate/standardpoors.htm>).

(along with other factors) in assessing whether to upgrade or downgrade a firm's credit rating. Standard and Poor's is highly dependent on the integrity and quality of a company's financial disclosures, particularly as its stated ratings methodology is not designed to 'second guess' the auditor's report or replace the work of the auditor.² What is the risk that the audited financial statements of a company fail to accurately portray the true going concern status of the firm, or even contain fraudulent and misleading information? The answers to these questions are not easy because the possibility of the presence of, say, fraud or misstatement in the financial statements of a S&P rated company does not depend on the frequency of the presence of fraud or misstatement in the financial statements of other companies or industries. That is, it does not depend on the prior probability of fraud or misstatement. Rather, it depends on the unique characteristics of the company such as the management incentives to commit fraud, whether management has opportunities to perpetrate fraud, and whether management has compromised integrity to justify committing fraud. It is important to emphasize that the presence of fraud in the financial statements of a company is not a random process with a stable frequency in repeated trials under fixed conditions as argued by Shafer and Srivastava (1990). Thus, treating the presence of fraud or misstatement in the financial statements of a company as a random process and trying to estimate the risk of its presence by assessing the prior probability may not be appropriate.

In this chapter, our objective is to demonstrate the use of Dempster-Shafer theory of belief functions for assessing default and bankruptcy risk in situations where the event can not be treated as a random variable, however, evidence exists which provides support for the presence or absence of such a variable.

²See testimony to the SEC, November 15, 2002. S&P also acknowledged that, in the context of recent corporate failures, it was misled by fraudulent and misleading financial statements which subsequently led to improvements

This chapter is divided into six sections. Section 2 discusses problems with probability framework in modeling uncertainties. Section 3 presents an introduction to Dempster-Shafer theory of belief functions. Section 4 demonstrates application of the belief functions in deriving a rudimentary default risk model based on some specified conditions which are used to derive the model. Section 5 discusses an approach to decision making under Dempster-Shafer theory of belief functions and provides an economic analysis of cost and benefit to a credit rating agency in the presence of default risk. Section 6 provides a conclusion of the chapter.

2. Problems with Probability Frameworks

We want to introduce readers to some very fundamental problems of probability theory in representing uncertainty judgments in common situations of decision-making, including one example in default risk and credit ratings.

2.1. Credit Ratings

In deciding to issue a credit rating to a company (or to revise an existing rating), a credit rating agency (such as S&P) must assess a wide range of information, including macroeconomic, industry and firm-specific factors. Firm specific factors might include the current and expectations of future financial performance, management style and ability, the existence of effective strategic plans, the adequacy of corporate governance structures including a variety of confidential firm information used in the ratings process but not disclosed to the public. Suppose that all these factors lead to a positive decision from the credit rating agency towards issuing a favorable rating (say a rating of A or above using the standard S&P rating scale). The credit ratings agency wants to express this judgment and assigns a low level of support, say 0.2 on a scale of 0 - 1 that the overall firm specific performance factors under consideration are adequate

in the methodology of the agency's ratings process.

and no support that they are deficient. If we had to represent this judgment in terms of a probability then we would say that the credit rating agency has 0.8 (complement of 0.2) degree of support that there is deficiency in one or more of the firm specific performance factors under review. But this interpretation suggests that the evidence is negative i.e., 0.8 level of support that there are deficiencies. This would not make sense to the credit rating agency because it has no evidence in support of the fact that such a deficiency exists. The above evidence is easily expressed in terms of belief functions, as 0.2 level of support that the firm specific performance factors are adequate to issue a positive or favorable credit rating, no belief that the firm specific performance factors are deficient, and 0.8 degree of belief remaining uncommitted (see Section 3 for more details).

Just to illustrate a point, let us consider another case. Assume that the crediting rating agency thinks that all the environmental factors lead to a very strong positive support toward the firm specific performance factors being adequate. Further, assume that the credit rating agency expresses this judgment by assigning a value of, say 0.5 to the rating that it is correct. However, based on this assessment the rating agency still has some risk that the rating may have been issued incorrectly, i.e., $R = 0.5$. This value when interpreted in probability theory implies that the rating agency does not know what it is doing, i.e., the rating agency is ignorant because there is a 50 – 50 chance that the firm specific performance factors are adequate or deficient. However, as it will be further elaborated in Section 4, under belief functions, $R = 0.5$ means that the rating agency has 0.5 level of support for the firm specific performance factors being adequate, no support for the presence of deficiencies, and 0.5 level of support uncommitted.

2.2. Nature of Evidence

Under a probability framework, one can define either a positive piece of evidence or a negative piece of evidence using the likelihood ratio (see Edwards 1984 and Dutta and Srivastava 1993, 1996). If the likelihood ratio is greater than one, then it is positive evidence. If the likelihood ratio is positive and less than one, then it is negative evidence. In both situations, by definition, the evidence is mixed, i.e., if the evidence is positive in support of a hypothesis, then it is also negative in support of the negation of the hypothesis. This implies that all items of evidence modeled under probability theory will always be mixed. However, it is quite common in the real world to find pure positive evidence or pure negative evidence. For example, suppose the rating agency finds that the company's management has exceptional ability and the financial statements have a high level of integrity based on the last several years of experience. The firm has never had any problems with these issues in the past. Also, the economic environment under which this firm's business operates is sound. Based on all these items of evidence the ratings agency believes it can trust the accuracy of the financial statements and supporting statements by the managers with some level of comfort; a low level of support, say 0.1, on a scale of 0-1. In the rating agency's judgment, it has no evidence to support a view the financial statements are deficient (or the statements by the managers in the interview processes are untrustworthy in anyway). This type of evidence can be easily expressed under belief functions in terms of the basic belief masses (See Section 3 for more details). However, if we use probability theory to model it, then we run into problems. For the above example, we will have the probability of trustworthy financial statements arrangements to be 0.1 and thus by definition, we will have 0.9 probability that the financial statements are deficient in some way. This is not what the rating

agency's judgment was; it had no information about the financial statements and accompanying management statements being inadequate.

2.3. Representation of Ignorance

Representation of ignorance in probability theory, with “n” mutually exclusive and collectively exhaustive states of nature, is to assign a probability of $1/n$ to each state of nature. If there are only two possible outcomes, then we assign 0.5 and 0.5 to each state to represent ignorance. However, this representation leads to a problem. Let us consider an example. A local infrastructure company under ratings review has started new operations in locations in the middle East and China. It is exposed to two kinds of economic risk in two separate jurisdictions (let us say the risk of civil unrest and economic disruption in the middle East location and the risk of an economic slow down in China, both of which can impact on the financial performance and ultimately the creditworthiness of the company). Let us assume the rating agency starts with no knowledge whether risk in any of the jurisdictions in which the entity operates is particularly high or low, but the risks are nevertheless present. The probability representation of this ignorance about the associated risks, say at just two locations mentioned above (middle East and China), would be $P(f_1) = P(\sim f_1) = 0.5$, and $P(f_2) = P(\sim f_2) = 0.5$, where f_1 and f_2 represent the economic risks being high in jurisdictions 1 and 2, respectively, and $\sim f_1$ and $\sim f_2$, represent the economic risks being low in both jurisdictions.

Consider that we want to determine the probability that the economic risks in both jurisdictions 1 and 2 are low. We know in this case the ratings agency will only consider the combined risks in the two jurisdictions to be adequate only when both locations are judged to be low risk. In other words, the combined risks would still be regarded as unacceptable high if the risks in one jurisdiction alone is too high. This yields a probability of 0.25 for the combined

economic risks being low ($P(f_1 \cap f_2) = 0.5 \times 0.5 = 0.25$), and a probability of 0.75 for the combined economic risks being high. This result is counter-intuitive. We started with ignorance of the particular economic risks in each location, but when we combine the economic risks of both locations we seem to have gained some knowledge; 0.75 chances are that the combined economic risks are high. The probability result looks even worse if we consider a few more locations where economic risk is present for the company.

2.4. Knowledge versus No Knowledge

Consider two urns, Urn 1 and Urn 2, each containing 100 balls. Urn 1 contains 100 balls of red and black colors, but we do not know the proportion. All may be red, all may be black, or in any proportion. Thus, Urn 1 is an urn where we have no knowledge about the proportion of red and black balls. Urn 2 contains 50 red and 50 black balls, i.e., we have complete knowledge about Urn 2. Einhorn and Hogarth (1985, 1986) conducted an experiment where they asked subjects to choose an urn from which they would pick a given color ball, say red, and win \$100, and get nothing if they pick the wrong color ball. The subjects overwhelmingly chose to pick from Urn 2, the urn with complete knowledge. When we model the above situation using probability framework, then we get into a paradox. Einhorn and Hogarth refer to this paradox as Ellsberg (1961) paradox. As discussed by them, the preference of Urn 2 over Urn 1 for picking a ball of either color would mean the following probability inequalities:

$$P(\text{Red from Urn 2}) > P(\text{Red from Urn 1}) = 0.5,$$

$$P(\text{Black from Urn 2}) > P(\text{Black from Urn 1}) = 0.5,$$

or

$$P(\text{Red from Urn 2}) = 0.5 > P(\text{Red from Urn 1}),$$

$$P(\text{Black from Urn 2}) = 0.5 > P(\text{Black from Urn 1}).$$

The first condition implies that the probabilities for Urn 2 add to more than one and the second condition implies that the probabilities for Urn 1 add to less than one. This is the paradox. Einhorn and Hogarth (1986) define these conditions, respectively, as "superadditivity" and "subadditivity." They further state that (Einhorn and Hogarth 1986, p. S228):

... either urn 2 has complementary probabilities that sum to more than one, or urn 1 has complementary probabilities that sum to less than one. As we will show, the nonadditivity of complementary probabilities is central to judgments under ambiguity.

This paradox stems from the difficulty in distinguishing between the two urns under probability framework. The two urns, Urn 1, and Urn 2, are identical under probability framework, since the probability of picking a red ball from Urn 1 (complete ignorance) is 0.5 and the probability of picking a red ball from Urn 2 (complete knowledge, 50 red and 50 black balls) is also 0.5. However, decision makers clearly perceive the two situations to be different. Srivastava (1997) has shown that *super- or sub-additivity property is not needed to explain the decision maker's behavior*. There is no paradox if the uncertainties were modeled using the belief-function framework. Also, probability is not a physical property that one urn would have super-additive probabilities and the other would have sub-additive probabilities. It is only a language to express uncertainties; it is our creation.

2.5. Rodeo Paradox³

Consider a rodeo show in town. One thousand people go to see the show. However, one person buys the ticket to the show and the rest, 999, force their way into the show without a ticket. Police are called in for help. Police randomly pick up a person and take that person to the city judge for prosecution. We know that the probability of this person having entered the rodeo show without a ticket is 0.999. What should the judge do to this person based on this prior

³ This example is described by Smets (1999),

probability? Should the judge prosecute this person since the prior probability of being guilty is so high? Well, if you use common sense then you feel that there is no evidence to support that this person has entered the show without a ticket. Under belief functions, this situation is represented as having a zero belief that the person is guilty and also a zero belief that the person is not guilty. The judge cannot prosecute this person based solely on the prior probability even though the probability of this person being guilty is 0.999. Under such situations, the belief-function framework becomes useful.

What if we now bring a piece of evidence into the story? If the person shows that he has the stub of the ticket, then what is the belief that he is not guilty? It depends on how he got the stub. Could it be that he had snatched it from the rightful owner of the ticket? What if a witness says that he saw this person purchase a ticket? The level of belief about whether the person in question is guilty or not guilty depends on these items of evidence and their credibility. It is important to note that the judge's decision to prosecute or not to prosecute the person does not depend on the prior probability of being guilty. Rather it depends on the belief that the judge can ascribe to the person the guilt or the lack of guilt through combining several pieces of evidence relevant to the case. A belief function treatment of such problems provides a richer framework for decision-making.

3. Introduction To Dempster-Shafer Theory of Belief Functions

Although the current form of belief functions is based on the work of Dempster during 1960s, Shafer (1976) made it popular through his book *A Mathematical Theory of Evidence*. Several authors have provided a basic introduction to Dempster-Shafer theory of belief functions (e.g., see Srivastava 1993, Srivastava and Mock 2002a, and Yager et. al 1994). However,

Shafer's book (1976) is still the classic reference on this subject. In this section, we provide the basics of belief functions as an introduction.

The Dempster-Shafer theory of belief functions (here after called simply belief functions) is similar to probability theory, however, with one difference. Under probability theory, we assign uncertainty to the state of nature based on the knowledge of frequency of occurrence. However, under belief functions, we assign uncertainty to the state of nature or assertion of interest in an indirect way based on the probability knowledge in another frame by mapping that knowledge onto the frame of interest. This mapping may not necessarily be one-to-one. For example, we may have probability knowledge of someone, say Joe, being honest, say 0.9, and not being honest 0.1, based on the observed behavior over the years. If this person is making a statement that he saw the house on the Northwest corner of Clinton Drive and Inverness Drive in the city on fire this morning, then one would believe, based on him being honest 90% of the time, that the house is on fire, with a belief 0.9. However, Joe being dishonest does not give any evidence that the house is *not* on fire when he is saying that the house is on fire. The knowledge that he is dishonest 10% of the times suggests that he may or may not be truthful in what he is saying, which provides a belief of 0.1 that the house may or may not be on fire.

We can provide further elucidation of the belief function concepts through another illustration. Suppose we have a variable, say A , with n possible mutually exclusive and exhaustive set of values: $a_1, a_2, a_3, \dots, a_n$. These values define the frame, $\Theta = \{ a_1, a_2, a_3, \dots, a_n \}$ of discernment for the variable A . Under probability theory, for such a set, we assign a probability mass, $P(a_i)$, to each state a_i such that the sum of all these probabilities equals one, i.e.,

$\sum_{i=1}^n P(a_i) = 1$. However, under Dempster-Shafer theory of belief functions, uncertainties are

assigned in terms belief masses to not only singletons, but also to all the sub-sets of the frame and to the entire frame Θ . These belief masses add to one similar to probability masses. This will be elaborated further in the next sub-section.

3.1. The Basic Probability Assignment Function (Basic Belief Mass Function)

As mentioned earlier, under belief functions, we assign uncertainties in terms of *belief masses* to all the sub-sets of a frame Θ including the entire frame Θ . These belief masses define a function called the *basic belief mass function* (Shafer, 1976, calls it the *basic probability assignment function*). In mathematical terms, we can write a belief mass assigned to a subset B as $m(B)$, where B could be a single element, or a subset of two, a sub-set of three, and so on or the entire frame, Θ . The sum of such belief masses equals one, i.e., $\sum_{B \subseteq \Theta} m(B) = 1$. Thus, one can see that when the non-zero belief masses are only defined on the singletons, then the belief functions reduces to probability theory. Thus, one can argue that probability theory is a special case of Dempster-Shafer theory of belief functions.

Let us consider an example to illustrate the above concepts. Suppose a credit ratings agency has performed an analysis on the financial statements of a firm to assess whether a ratings downgrade is appropriate. In analyzing a particular variable of interest, say earnings quality over a five year period, the rating agencies finds no significant difference between the recorded value and the predicted value, that is based on this information, the ratings agency thinks that the firm's earnings quality appears reasonable for a healthy entity, given its size, longevity and industry background. However, the rating agency does not want to put too much weight on this evidence given some inherent uncertainties with measuring earnings quality coupled with some known cases of earnings management practices in this particular industry, and so assigns a low level of assurance, say 0.3, on a scale of 0-1, that earnings quality is

accurately represented in the financial statements. The rating agency has no evidence supporting the assertion that earnings quality is materially misstated or does not reflect acceptable industry averages. We can express this judgment in terms of belief masses as: $m(\text{EQ}) = 0.3$, $m(\sim\text{EQ}) = 0$, and $m(\{\text{EQ}, \sim\text{EQ}\}) = 0.7$, where the symbol 'EQ' stands for the quality of earnings being a reasonable representation of reality and ' $\sim\text{EQ}$ ' stands for earnings quality being either materially misstated or not reflecting acceptable industry averages. The belief function interpretation of these belief masses is that the ratings agency has 0.3 level of support to 'EQ', no support for ' $\sim\text{EQ}$ ', and 0.7 level of support remains uncommitted which represents ignorance.

However, if we had to express the above judgment in terms of probabilities, we get into problems, because we will assign $P(\text{EQ}) = 0.3$ and $P(\sim\text{EQ}) = 0.7$ which implies that there is a 70% chance that the earnings quality is materially misstated or does not reflect acceptable industry averages. However this is not what the rating agency's judgment is; it has no information or evidence that earning quality is materially misstated. Simply knowing the fact that the current year's earnings quality appears to be reasonable compared to the predicted values based on the industry average and prior years' performances provide no evidence that the current year's value is materially misstated. It only provides some level of support that the earnings quality is accurately stated.

Thus, we can use the belief masses to express the basic judgment about the level of support or assurance the rating agency obtains from an item of evidence for an assertion. An example of a negative item of evidence which will have a direct support for ' $\sim\text{EQ}$ ' would be the following set of inherent factors: (1) in the prior years earnings quality has been misrepresented, and (2) there are economic reasons for management to misstate earnings. In such a case we can express the rating agency's judgment as $m(\text{EQ}) = 0$, $m(\sim\text{EQ}) = 0.2$, and $m(\{\text{EQ}, \sim\text{EQ}\}) = 0.8$,

assuming that the rating agency estimates a low, say 0.2, level of support for ' \sim EQ'. One can express a mixed-type of evidence in terms of the belief masses without any problems as: $m(\text{EQ}) = 0.4$, $m(\sim\text{EQ}) = 0.1$, and $m(\{\text{EQ}, \sim\text{EQ}\}) = 0.5$, where the judgment is that the evidence provides 0.4 level of support to 'EQ', 0.1 level of support to ' \sim EQ', and 0.5 level of support is uncommitted, i.e., unassigned to any specific element but to the entire set, representing the ignorance. In probability theory, we cannot express such a judgment.

3.2. Belief Functions

The belief in B, $\text{Bel}(B)$, for a subset B of elements of a frame, Θ , represents the total belief in B and is equal to the belief mass, $m(B)$, assigned to B plus sum of all the belief masses assigned to the set of elements that are contained in B. In terms of symbols:

$$\text{Bel}(B) = \sum_{C \subseteq B} m(C).$$

By definition, the belief mass assigned to an empty set is always zero, i.e., $m(\emptyset) = 0$.

In order to illustrate the above definition, let us consider our rating agency example of discussed earlier. Suppose that the ratings agency has made the following judgment about the level of support in terms of belief masses for earnings quality being accurately represented (i.e., not materially misstated) and not accurately represented (i.e., materially misstated): $m(\text{EQ}) = 0.3$, $m(\sim\text{EQ}) = 0$, and $m(\{\text{EQ}, \sim\text{EQ}\}) = 0.7$. Based on analytical procedures alone, the belief that earnings quality is not materially misstated is 0.3, i.e., $\text{Bel}(\text{EQ}) = m(\text{EQ}) = 0.3$, no support that earnings quality is materially misstated, i.e., $\text{Bel}(\sim\text{EQ}) = m(\sim\text{EQ}) = 0$, and the belief in the set $\{\text{EQ}, \sim\text{EQ}\}$ is $\text{Bel}(\{\text{EQ}, \sim\text{EQ}\}) = m(\text{EQ}) + m(\sim\text{EQ}) + m(\{\text{EQ}, \sim\text{EQ}\}) = 0.3 + 0.0 + 0.7 = 1$. In general, a zero level of belief implies that there is no evidence to support the proposition. In other words, a zero level of belief in a proposition represents lack of evidence. In contrast, a zero probability in probability theory means that the proposition cannot be true which represents

impossibility. Also, one finds that beliefs for 'EQ' and '~EQ' do not necessarily add to one, i.e., $Bel(EQ) + Bel(\sim EQ) \leq 1$, whereas in probability, it is always true that $P(EQ) + P(\sim EQ) = 1$.

3.3. Plausibility Functions

Intuitively, the plausibility of B is the degree to which B is plausible given the evidence. In other words, $Pl(B)$ represents the maximum belief that could be assigned to B, given that all the evidence collected in the future support B. In mathematical terms, one can define plausibility of B as: $Pl(B) = \sum_{B \cap C = \emptyset} m(C)$, which can also be expressed as: $Pl(B) = 1 - Bel(\sim B)$, which is the degree to which we do not assign belief to its negation ($\sim B$).

In our example above, we have the following belief masses and beliefs: $m(EQ) = 0.3$, $m(\sim EQ) = 0$, $m(\{EQ, \sim EQ\}) = 0.7$, and $Bel(EQ) = 0.3$, $Bel(\sim EQ) = 0$, and $Bel(\{EQ, \sim EQ\}) = 1$. These values yield the following plausibility values: $Pl(EQ) = 1$, and $Pl(\sim EQ) = 0.7$. $Pl(EQ) = 1$ indicates that 'EQ' is maximally plausible since we have no evidence against it. However, $Pl(\sim EQ) = 0.7$ indicates that if we had no other items of evidence to consider, then the maximum possible assurance that earnings quality is materially misstated would be 0.7, even though we have no evidence that earnings quality is materially misstated, i.e., $Bel(\sim EQ) = 0$. This definition of plausibility that earnings quality is materially misstated represents the measure of risk that earnings quality could be materially misstated, even though there is no belief that earnings quality is materially misstated.

3.4. The Measure of Ambiguity

The belief-function measure of ambiguity in an assertion, say B, is straightforward. It is the difference between plausibility of B and the belief in B (Wong and Wang, 1993).

$$\text{Ambiguity}(B) = Pl(B) - Bel(B).$$

The belief in B represents the direct support for B, while the plausibility of B represents the maximum possible support that could be assigned to B if we were able to collect further evidence in support of B. The difference then represents the unassigned belief that could be assigned to B. This unassigned belief represents the ambiguity in B.

3.5. Dempster's Rule

Dempster's rule (Shafer 1976) is the fundamental rule in belief functions for combining independent items of evidence similar to Bayes' rule in probability theory. In fact, Dempster's rule reduces to Bayes' rule under the condition when all the belief masses defined on the frame are zero except the ones for the singletons. For two independent items of evidence pertaining to a frame of discernment, Θ , we can write the combined belief mass for a sub-set B in Θ using Dempster's rule as:

$$m(B) = \sum_{C1 \cap C2 = B} m_1(C1)m_2(C2)/K, \quad (1)$$

where

$$K = 1 - \sum_{C1 \cap C2 = \emptyset} m_1(C1)m_2(C2). \quad (2)$$

The symbols $m_1(C1)$ and $m_2(C2)$ determine the belief masses of C1 and C2, respectively, from the two independent items of evidence represented by the subscripts. The symbol K represents the renormalization constant. The second term in K represents the conflict between the two items of evidence. The two items of evidence are not combinable if the conflict term is 1.

Let us consider an example to illustrate the details of Dempster's rule. Suppose we have the following sets of belief masses obtained from two independent items of evidence related to the accurate representation of earnings quality:

$$m_1(EQ) = 0.3, m_1(\sim EQ) = 0.0, m_1(\{EQ, \sim EQ\}) = 0.7,$$

$$m_2(\text{EQ}) = 0.6, m_2(\sim\text{EQ}) = 0.1, m_2(\{\text{EQ}, \sim\text{EQ}\}) = 0.3.$$

The renormalization constant for the above case is

$$K = 1 - [m_1(\text{EQ})m_2(\sim\text{EQ}) + m_1(\sim\text{EQ})m_2(\text{EQ})] = 1 - [0.3 \times 0.1 + 0.0 \times 0.6] = 0.97.$$

Using Dempster's rule in (1), the combined belief masses for 'EQ', ' \sim EQ', and $\{\text{EQ}, \sim\text{EQ}\}$ are given by:

$$\begin{aligned} m(\text{EQ}) &= [m_1(\text{EQ})m_2(\text{EQ}) + m_1(\text{EQ})m_2(\{\text{EQ}, \sim\text{EQ}\}) + m_1(\{\text{EQ}, \sim\text{EQ}\})m_2(\text{EQ})]/K \\ &= [0.3 \times 0.6 + 0.3 \times 0.3 + 0.7 \times 0.6]/0.97 = 0.69/0.97 = 0.71134, \end{aligned}$$

$$\begin{aligned} m(\sim\text{EQ}) &= [m_1(\sim\text{EQ})m_2(\sim\text{EQ}) + m_1(\sim\text{EQ})m_2(\{\text{EQ}, \sim\text{EQ}\}) + m_1(\{\text{EQ}, \sim\text{EQ}\})m_2(\sim\text{EQ})]/K \\ &= [0.0 \times 0.1 + 0.0 \times 0.3 + 0.7 \times 0.1]/0.97 = 0.07/0.97 = 0.072165, \end{aligned}$$

$$\begin{aligned} m(\{\text{EQ}, \sim\text{EQ}\}) &= m_1(\{\text{EQ}, \sim\text{EQ}\})m_2(\{\text{EQ}, \sim\text{EQ}\})/K = 0.7 \times 0.3/0.97 = 0.21/0.97 \\ &= 0.216495. \end{aligned}$$

The combined beliefs and plausibilities that earnings quality is not misstated (EQ) and is misstated (\sim EQ) are:

$$\text{Bel}(\text{EQ}) = m(\text{EQ}) = 0.71134, \text{ and } \text{Bel}(\sim\text{EQ}) = m(\sim\text{EQ}) = 0.072165,$$

$$\text{Pl}(\text{EQ}) = 1 - \text{Bel}(\sim\text{EQ}) = 0.927845, \text{ and } \text{Pl}(\sim\text{EQ}) = 1 - \text{Bel}(\text{EQ}) = 0.28866.$$

4. Risk Assessment

This section demonstrates the application of belief functions in assessing risk which can be applied to various situations. As discussed in the introduction, belief functions are appropriate for modeling uncertainties when we have partial knowledge about the state of nature. Also, it is useful for the situation when the event is not a random event with a given stable frequency in repeated trials under fixed conditions. Bankruptcy risk, audit risk, fraud risk, auditor independence risk, information security risk, and business risk are examples of such situations where we do not have stable frequencies in repeated trials under fixed conditions.

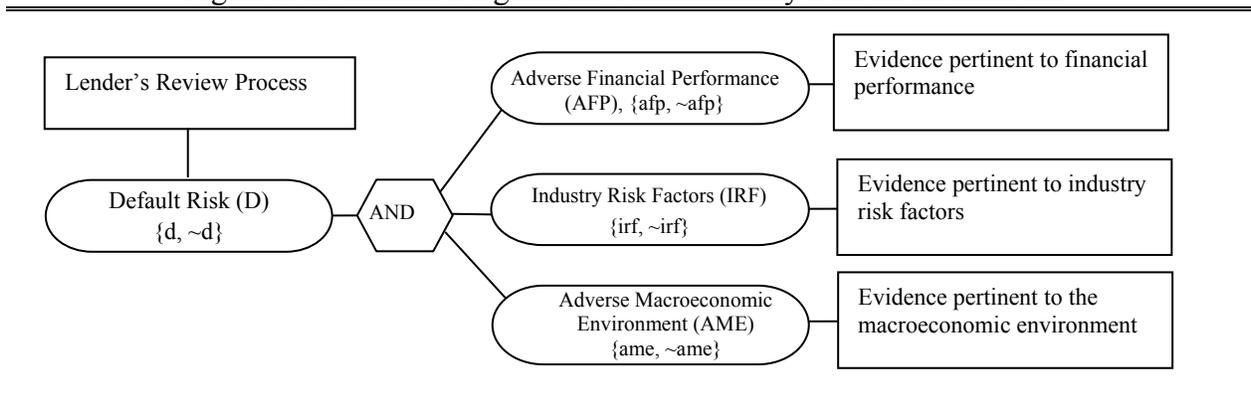
The notion of risk under belief functions is represented in terms of plausibility function. For example, the plausibility of material misstatement in the financial statements is defined as the audit risk by Shafer and Srivastava (1992). The plausibility that fraud is present in the financial statements is defined to be the fraud risk by Srivastava et al (2006). The plausibility of information being insecure is defined as information security risk by Sun et al (2005). Similarly, in this chapter we define the plausibility of loan default or bankruptcies as loan default risk or bankruptcy risk respectively. An illustration follows.

4.1 Default Risk

As an illustration of how a model could be developed, we derive a simple hypothetical default risk formula. Let us suppose a major lender is evaluating the potential risks of a company in its loan portfolio defaulting on a loan. The lender is interested in continually monitoring the financial status of the company (including any relevant industry and economic risk factors) for any signs of deteriorating creditworthiness which may lead to loan default. Loan default can obviously result in economic losses for the lender, hence identifying potential default risk as early as possible may give the lender some valuable lead time to take appropriate

corrective or remedial action (such as increasing the amount of the security for the loan or even calling in the loan). Using belief functions, let us suppose that the following conditions must be present for loan default to occur – in other words, the following three conditions must be true: 1) the company must experience a deterioration in its current or expected future financial performance which will adversely impact its debt servicing capacity, 2) the industry in which the firm operates must experience some change which will adversely affect the risk profile of the company (e.g., a change in governmental regulation or policy that may expose the firm to greater competition), and 3) the firm must experience a significant adverse change in the macroeconomic environment which will adversely affect the risk profile of the company (such as an increase in interest rates which may affect the company’s capacity to repay debt).

Figure 2: Evidential Diagram for a Rudimentary Default Risk Model*



*The rounded boxes represent the variables and the rectangular boxes represent the items of evidence. The hexagonal box represents ‘AND’ relationship between ratings downgrade and the three influencing factors: adverse financial performance (AFP), Industry Risk Factors (IRF), and adverse macroeconomic environment (ME).

Figure 2 represents the evidential diagram of the default risk model where the three factors, adverse financial performance (AFP), industry risk factors (IRF), and adverse macroeconomic environment (AME) are related to the variable default risk (D) through an

‘AND’ relationship. The lower case letters in the rounded boxes represent values that the corresponding variables are present or absent.

For example, ‘AFP’ means that an adverse change in current or expected future financial performance is present and ‘~AFP’ means that this adverse change in current or expected future financial performance is absent. The ‘AND’ relationship implies that loan default will occur if and only if all these three factors or triggers are present. In terms of set notation, we can write $d = \text{afp} \cap \text{irf} \cap \text{ame}$. The evidence labeled ‘lender’s review process’ pertaining to the variable ‘D’ includes all the procedures the lender would perform to assess whether default risk is likely to occur given the presence of AFP, IRF, and AME. Default risk would not occur (or can be avoided) if the lender’s ongoing review procedures are effective.

In Figure 2, we have considered only one item of evidence for each default risk factor and one item of evidence at the default risk level for brevity⁴. However, several sets of evidence could be considered by the lender. For example, for adverse financial performance, the lender might consider the impact of a drop in sales growth, an unexpected increase in operating expenses in one of the business segments, or an analyst downgrade on future EPS estimates for the company. For industry risk factors, the lender might consider evidence relating to changed government policies pertinent to the sector as a whole, such as the removal of subsidies or an increase in a special form of taxation, or a general decline in competitiveness of the industry owing to foreign competition. For macroeconomic effects, the lender may consider the impact of changes in interest rates, foreign currency rates, commodity prices and general inflation rates on the company’s ability to service debt in the longer term.

⁴ One can easily extend the current approach to multiple items of evidence for each variable; simply use Dempster’s rule to combine these multiple items of evidence for each variable and then substitute the combined belief masses in place of the belief masses from the single item of evidence in the current approach.

We want to develop a formula for assessing default risk from the lender's perspective, given what we know about the presence of the three influencing risk factors and that the lender has performed appropriate review processes to assess a firm's default risk. Let us assume that we have the following belief masses (m) at AFP, IRF, AME, and D from the corresponding item of evidence:

$$\begin{aligned}
\text{Adverse Financial Performance (AFP):} & \quad m_{\text{AFP}}(\text{afp}), m_{\text{AFP}}(\sim\text{afp}), m_{\text{AFP}}(\{\text{afp}, \sim\text{afp}\}) \\
\text{Industry Risk Factors (IRF):} & \quad m_{\text{IRF}}(\text{irf}), m_{\text{IRF}}(\sim\text{irf}), m_{\text{IRF}}(\{\text{irf}, \sim\text{irf}\}) \\
\text{Adverse Macroeconomic Factors (AME):} & \quad m_{\text{AME}}(\text{ame}), m_{\text{AME}}(\sim\text{ame}), m_{\text{AME}}(\{\text{ame}, \sim\text{ame}\}) \\
\text{Default Risk Present (D):} & \quad m_{\text{D}}(\text{d}), m_{\text{D}}(\sim\text{d}), m_{\text{D}}(\{\text{d}, \sim\text{d}\}) \tag{5}
\end{aligned}$$

In order to develop the default risk formula, we proceed in two steps. First, we propagate the belief masses from the three default risk factors to D. Next, we combine the belief mass function at D (obtained from the lender's review processes) with the belief mass function (obtained from the three influencing risk factors). For the first step, we use a rudimentary default risk formula⁵, which yields the following belief mass function at variable 'D' as a result of propagating belief masses from AFP, IRF, and AME:

$$\begin{aligned}
m(\text{d}) &= m_{\text{AFP}}(\text{afp})m_{\text{IRF}}(\text{irf})m_{\text{AME}}(\text{ame}), \\
m(\sim\text{d}) &= 1 - (1 - m_{\text{AFP}}(\sim\text{afp}))(1 - m_{\text{IRF}}(\sim\text{irf}))(1 - m_{\text{AME}}(\sim\text{ame})), \\
m(\{\text{d}, \sim\text{d}\}) &= 1 - m(\text{d}) - m(\sim\text{d}). \tag{6}
\end{aligned}$$

Next, we combine the above belief mass function with the belief mass function at D, given in (5) using Dempster's rule. This combination yields the following overall belief mass function at D:

⁵ We use Srivastava, Shenoy and Shafer (1995) to combine the belief masses on AFP, IRF, and AME through 'and' relationship and marginalize them to variable D.

$$M(d) = [m_D(d)m(d) + m_D(d)m(\{d, \sim d\}) + m_D(\{d, \sim d\})m(d)]/K,$$

$$M(\sim d) = [m_D(\sim d)m(\sim d) + m_D(\sim d)m(\{d, \sim d\}) + m_D(\{d, \sim d\})m(\sim d)]/K,$$

$$M(\{d, \sim d\}) = m_D(\{d, \sim d\})m(\{d, \sim d\})/K, \quad (7)$$

$$K = 1 - [m_D(d)m(\sim d) + m_D(\sim d)m(d)]. \quad (8)$$

We obtain the following expression for the plausibility (Pl) of default risk from (7), by replacing the belief mass function defined in (6), and by simplifying:

$$Pl(d) = [m_D(d)+m_D(\{d, \sim d\})][m_{AFP}(afp)+m_{AFP}(\{afp, \sim afp\})][m_{IRF}(irf)+m_{IRF}(\{irf, \sim irf\})][m_{AME}(ame)+m_{AME}(\{ame, \sim ame\})]/K.$$

Using the definition of Plausibility function, we can rewrite the above expression as:

$$Pl(d) = Pl_D(d)Pl_{AFP}(afp)Pl_{IRF}(irf)Pl_{AME}(ame)/K. \quad (9)$$

Since the plausibility of default risk represents the risk of a default (DR), we can express $Pl(d) = DR$. Similarly, the plausibility of the presence of adverse financial performance, $Pl_{AFP}(afp)$, is the risk of the presence of adverse financial performance (RAFP), i.e., $Pl_{AFP}(afp) = RAFP$. The plausibility of the presence of industry risk factors, $Pl_{IRF}(irf)$, is the risk of the presence of industry risk factors (RIRF), i.e., $Pl_{IRF}(irf) = RIRF$. The plausibility of the presence of adverse macroeconomic factors, $Pl_{AME}(ame)$, is the risk of the presence of adverse macroeconomic factors (RME), i.e., $Pl_{AME}(ame) = RAME$. Also, the plausibility, $Pl_D(d)$, of loan default based on the lender's review processes can be expressed as the risk of a default going undetected by the lender ($Pl_D(d) = DPR$) (i.e., the lender's own ongoing review processes failed to detect the default risk). Thus, in terms of these individual risks, we can express the risk of default (DR) as follows:

$$DR = RAFP.RIRF.RAME.DPR/K, \quad (10)$$

where K is a renormalization (see equation 8) constant because of the conflict between the belief mass function from the lender's review procedures and the belief mass function obtained from the three influencing factors. The default risk formula in (10) is logical formula; default risk will exist only when there is risk of adverse current or expected future financial performance, risk of adverse industry factors, the risk of adverse macroeconomic factors, and the risk that the lender's review procedures will fail to detect default risk given the presence of one or more of the three influencing factors on loan default.

We can see from formula in (10) that in situations where we do not have any information about the presence or absence of any of the variables, and also if the lender has not performed the appropriate review procedures on the company, then all the plausibilities would be unity and the default risk will be unity. However, after evaluating the evidence relating to the presence or absence of adverse financial performance, industry risk factors, and macroeconomic events on a firm's overall creditworthiness, one might estimate the risk factors to a medium level, say $RAFP$ to be .6, $RIRF$ to be .7, and $RAME$ to be .5. In this situation, without the lender performing any review procedures, it seems the default risk would still be quite high, about 16.8%. In order to reduce the default risk to an acceptable level, say 2%, it seems necessary that the lender perform effective review procedures with risk of only about 12% of failing to detect loan default problems for the company ($DPR = 0.12$).

One can use belief functions models for assessing various other kinds of risks. Because of shortage of space we do not discuss the other cases of risk assessment formulas. Readers are referred to Srivastava and Shafer (1992) and Srivastava and Mock (2002b) for applications in the audit risk area.

5. Decision Making Under Belief Functions

Traditionally, the utility maximization approach has been used to make decisions under uncertainty, especially when uncertainty is represented in terms of probabilities. However, the traditional approach does not work when uncertainties are not represented in terms of probabilities. There have been several approaches to decision making under belief functions (e.g., see, Jaffray, 1989, 1994; Nguyen and Walker, 1994; Smets, 1990a, 1990b; Strat, 1990, 1994; and Yager, 1990). All these approaches suggest a way to resolve the ambiguities present in the belief function framework and then perform the expected value or utility analysis. We use Strat's approach (1990, 1994, see also Srivastava and Mock, 2000; Sun et al. 2006) because it provides the worst and the best case scenarios of resolving ambiguity. We first discuss Strat's approach, then apply it to an example.

5.1. Strat's Approach

Let us consider the same example of a Carnival Wheel #2 of Strat (1994) where the wheel is divided into ten equal sectors. Each sector is labeled by \$1, \$5, \$10, or \$20. Four sectors are labeled \$1, two sectors \$5, two \$10, one \$20, and one sector's label is masked, i.e., the label is not visible. Also, we are told that there could be any one of the following amounts: \$1, \$5, \$10, and \$20, under the masked label. In order to play the game, you have to pay a \$6 fee. The question is: how will you decide whether to play the game?

Before we answer the above question, let us first express the distribution of labels on the carnival wheel using belief functions as:

$$m(\$1) = 0.4, m(\$5) = 0.2, m(\$10) = 0.2, m(\$20) = 0.1, \text{ and } m(\{\$1, \$5, \$10, \$20\}) = 0.1.$$

The above belief masses imply that we have direct evidence that \$1 appears in four sectors out of ten on the wheel, \$5 appears in two sectors out of ten, and so on. $m(\{\$1, \$5, \$10, \$20\}) = 0.1$

represents the assignment of uncertainty to the masked sector; it may contain any one of the four labels: \$1, \$5, \$10, \$20. It is interesting to note that such a clear assignment of uncertainty under probability framework is not possible.

Based on the above belief masses, we can express the beliefs and plausibilities in the four outcomes as:

$$\text{Bel}(\$1) = 0.4, \text{Bel}(\$5) = 0.2, \text{Bel}(\$10) = 0.2, \text{Bel}(\$20) = 0.1.$$

$$\text{Pl}(\$1) = 0.5, \text{Pl}(\$5) = 0.3, \text{Pl}(\$10) = 0.3, \text{Pl}(\$20) = 0.2.$$

Thus, we have 0.1 degree of ambiguity ($\text{Pl}(A) - \text{Bel}(A)$) in each label.

In order to determine the expected value of the outcomes or the expected value of the utilities of the outcomes, Strat resolves the ambiguity through the choice of a parameter, ρ . This parameter defines the probability of resolving ambiguity as favorably as possible. This implies that $(1 - \rho)$ represents the probability of resolving ambiguity as unfavorably as possible. After resolving the ambiguity, we obtain the following revised belief masses:

$$m'(\$1) = 0.4 + 0.1(1-\rho), m'(\$5) = 0.2, m'(\$10) = 0.2, m'(\$20) = 0.1 + 0.1\rho.$$

The above belief masses are defined only on the single elements and, thus, they are equivalent to probability masses. Hence, we can now determine the expected value of the game using the traditional definition and obtain the following value.

$$E(x) = \$5.5 + 1.9\rho.$$

In order to decide whether to play the game, we need to estimate ρ . If we assume that the labels were put by the carnival hawker, we would be inclined to choose $\rho = 0$, which is the worst case scenario. This choice implies that the decision maker is resolving the ambiguity as unfavorably as possible, i.e., assigns the ambiguity of 0.1 to \$1.0. The expected value for this case is $E(x) =$

\$5.50. Since this amount is less than the fee of \$6, one would not play the game. We can use a similar approach for determining the expected value of utility of the decision maker.

5.2. Economic Analysis of Cost of Credit Ratings versus Reputation Cost

As discussed earlier, predicting the ratings of a credit rating agency for a particular firm can not be made in terms of probability because credit ratings are not a random process that has a stable frequency in repeated trials under fixed conditions. First, credit ratings agencies use a great deal of public and *private* information. Much of the private information is confidential and never released to the public at the time the rating is issued (or revised). As this information is not in the public domain, models to predict the ratings (and changes in ratings) of credit rating agencies tend to rely exclusively on publicly available information. At best, only indirect estimates of the impacts of private information can be inferred from statistical models. Second, the ratings process typically involves a close association between the ratings agency and the particular firm being rated, which involves personal interviews with management on a regular basis, particularly if the company is attempting any new activities or ventures that may impact on the rating. Furthermore, the weights that credit rating agencies assign to different information inputs (particularly private information) is generally not known. It is unlikely that large sample statistical models used to predict ratings and changes in credit ratings can capture such factors in model estimation.

Finally, the assessment of company's level of default risk by a credit ratings agency such as S&P is based on the effectiveness of the review procedures and methodologies and the quality of evidence gathered by the ratings agency. Suppose the rating agency has performed a detailed ratings review and has given a clean bill of health to a company (i.e., issued a positive rating). However, there is always a possibility that the financial statements may contain material

misstatements or even fraud even though the rating agency has not discovered it. This can lead to a spurious rating, which may only come to light if the company later defaults or goes bankrupt. Under belief functions, this risk is defined as the plausibility of the presence of default risk, $Pl(d)$, which is given by Equation (10).

In this section, we want to perform an economic analysis of cost of performing a credit ratings review with the potential reputation cost to the ratings agency if a company's default risk level was not accurately assessed by the ratings agency. Let us consider the following set of overall belief masses for the presence and absence of default risk:

$$M_D(d) = m^+, M_D(\sim d) = m^-, M_D(\{d, \sim d\}) = m^\ominus.$$

The belief that loan default risk is present or absent are, respectively, m^+ and m^- , i.e., $Bel(d) = m^+$ and $Bel(\sim d) = m^-$. The plausibility of default risk being present, i.e., the default (DR), is given by $Pl(d) = DR = m^+ + m^\ominus$. Let us assume that the rating agency has given a clean bill of health for the above case and also consider the following costs and benefits to the ratings agency on conducting the rating review. The rating agency gets 'RF' amount of fee revenue for issuing the rating, incurs 'RC' amount of cost in the conduct of the ratings review, expects to receive future benefits of 'FB' amount if the ratings review is of 'good quality' and there is no default (where a favorable rating has been issued), and incurs a loss of 'LC' as the reputation cost and loses all the future benefits if the rating turns out to be of a bad quality, (i.e., the rating agency did not accurately assess the level of default risk in a company and the company later defaults). In order to determine the expected benefit or loss to the ratings agency given that the agency has given a clean bill of health to a company, we need to use Strat's approach to resolve the ambiguity in the worst case scenario and then determine the expected value of the costs and benefits to the ratings agency.

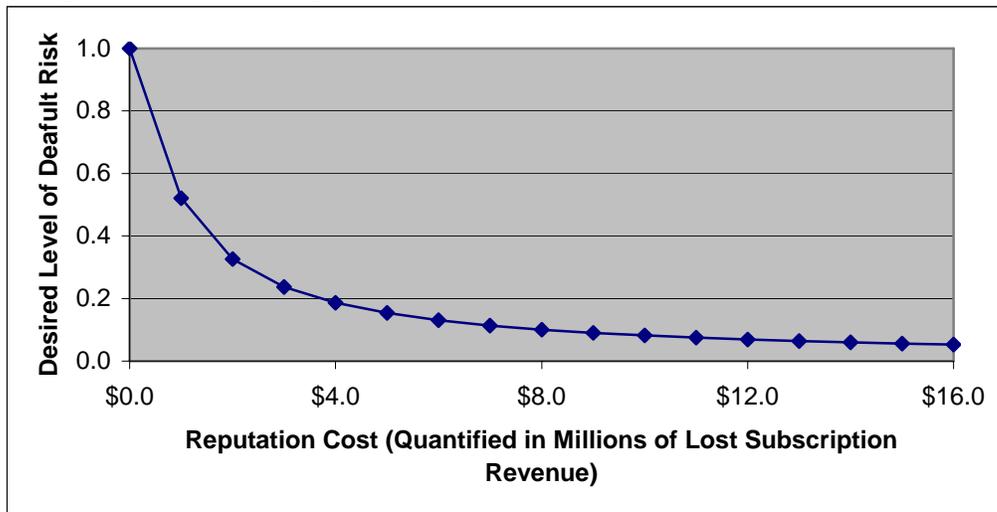
If we resolve the ambiguity of m^\ominus against the ratings agency (worst case scenario), the revised belief masses would be: $m_D'(d) = m^+ + m^\ominus$, and $m_D'(\sim d) = m^-$. In fact, by definition, $m_D'(d) = \text{Pl}(d) = \text{DR}$, and $m_D'(\sim d) = 1 - \text{DR}$. Thus, expected value of cost and benefits to the ratings agency for issuing a favorable rating can be written as:

$$\begin{aligned} \text{Expected Benefit} &= (\text{RF} - \text{RC} + \text{FB}) m_D'(\sim d) + (\text{RF} - \text{RC} - \text{LC}) m_D'(d) \\ &= (\text{RF} - \text{RC} + \text{FB})(1 - \text{DR}) + (\text{RF} - \text{RC} - \text{LC})\text{DR} \\ &= \text{RF} - \text{RC} + \text{FB} - (\text{FB} + \text{LC})\text{DR} \end{aligned}$$

The rating agency will have positive benefit under the following condition:

$$\text{Default Risk (DR)} < (\text{RF} - \text{RC} + \text{FB}) / (\text{FB} + \text{LC}) \quad (11)$$

Figure 3: Effect of Reputation Cost on Desired Level of Default Risk⁺



⁺We have assumed the following costs for this graph: Ratings Fee (RF) = \$1,000,000; Ratings Cost (RC) = \$800,000; Future Benefits (FB) = \$670,000 (net present value of future cash flows of 20% of net income discounted at 15% over five years).

Equation (11) determines the level of desired default risk by the rating agency given the ratings fee, cost of issuing (or revising) a rating, future benefits and the potential loss due to reputation loss for poor quality ratings. In other words, Equation 11 can be interpreted as the acceptable or desired level of default risk needed for a rating agency to issue a favorable rating and be profitable.

To reduce the level of acceptable or desired default risk, a rating agency might choose to avoid rating certain companies where default rates are traditionally higher than industry averages or where there are greater uncertainties or difficulties in assessing information inputs to the rating. Alternatively, the agency might tighten its review processes and take a more conservative approach to ratings for companies and industries where ratings risk is perceived to be high.

For simplicity, we assume that reputation loss is expected to translate (in economic terms) to lost subscription revenues for the ratings agency's products and services (in reality, there may be other costs as well, such as litigation costs). Figure 3 shows a graph of acceptable default risk by the ratings agency versus reputation costs. We assume the following values for the other variables in (11): $RF = \$1,000,000$; $RC = \$800,000$; $FB = \$670,000$ (present value of the cash flow discounted at 15% over five years). It is interesting to see from Figure 3 that the ratings agency will not worry about default risk levels ($RR = 1$) if there is no reputation cost. However, as reputation cost increases, the desired level of default risk by the rating agency decreases as expected. For our example, for a reputation cost of $\$16,000,000$, the ratings agency will perform the rating with 0.05 level of desired default risk. Of course, we have assumed the cost of the rating and rating fee to be fixed in the present calculation, which is not the case in the real world. However, we can analyze such a situation by considering the rating fee and the rating

cost to be a function of the desired default risk; the lower the desired default risk of a company, the higher the cost of rating and, thus, the higher the credit rating fee.

6. Conclusion

This chapter provides a belief function approach to assessing default risk, along with a general introduction to belief functions. The chapter discusses two kinds of uncertainty. One kind arises purely because of the random nature of the event. For random events, there exist stable frequencies in repeated trials under fixed conditions. The other kind of uncertainty arises because of the lack of knowledge of the true state of nature where we not only lack the knowledge of a stable frequency, but also we lack the means to specify fully the fixed conditions under which repetitions can be performed. We have suggested that application of probability theory under these conditions can lead to inconsistent logic, spurious interpretations of evidence and ultimately poor or suboptimal judgments. Belief functions provide a viable quantitatively grounded alternative to probability theory, particularly where statistical generalizations are not possible and/or not appropriate in the circumstances.

To demonstrate the application of belief functions derive a default risk formula in terms of plausibility of loan default risk being present under certain specified conditions. The default formula suggest that if default risk exists, then the only way it could be minimized is for the lender to perform effective ongoing review activities, *ceteris paribus*. Finally, we discuss an approach to decision making under belief functions and apply this to perform an economic analysis of costs and benefits to a ratings agency when default risk is present.

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