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Why We Should Consider Belief Functions in Auditing Research and Practice[1]

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Recent events in the auditing profession [sometimes called the 'accounting' profession] have clearly called for a reconsideration of the paradigms currently in vogue in both auditing research and practice. Although early audit research and standard setting explored more rigorous frameworks for risk assessment such as a specific algebraic definition of audit risk and rigorous guidance concerning statistical sampling, more recently audit practice has moved away from these approaches in favor of judgment sampling and relegating precise formulations of audit risk to providing 'general guidance' for audit planning. As academics we have the responsibility to help improve audit practice and research by exploring alternative, more rigorous frameworks for risk assessment, evidential reasoning and audit planning. In this note we briefly discuss one framework that offers advantages in this quest - The Dempster-Shafer theory of belief functions[2]

Although there is no generally accepted set of criteria that would allow researchers or practitioners to select one framework over another [Smets, 1999][3], there are a number of features of belief functions that argue for their more extensive use in auditing. These are briefly discussed below starting with perhaps the most compelling one, how risk and uncertainty should be treated.

Risk Assessment

The demise of Arthur Andersen, the frequent occurrence of restatements and the many alleged audit failures implies that our current ways of conceptualizing and operationalizing risk

[1] Our thanks to J. Bedard, G. Monroe, P. Shenoy, C, Shenoy, J. Turner, and A. Wright for their suggestions on this note.

[2] For an introduction to belief functions see Srivastava [1993].

[3] This argument also applies to paradigms such as Bayesian inference that are currently in vogue. Although Bayesian inference may be a useful framework for some audit situations, it clearly is not a superior framework for all issues. See our references for detailed discussions of some reasons why.

need reconsideration. Belief functions offer a number of potential advantages in assessing risks in auditing. Concisely stated, belief functions provide a rigorous definition of risk[4] which is appropriately conservative, can be operationalized in any risk assessment setting, and can be expected to be applied consistently by auditors over a wide variety of risk assessment situations. This approach may be applied in typical audit situations, such as the evaluation of fraud risk, where there is limited information and significant uncertainty and ambiguity.

In general, uncertainty deals with situations where one is not certain about the outcome of an event. For example, in a financial audit, suppose the auditor has no evidence whether management fraud is present or not present in the financial statements. In absence of any relevant evidence the auditor is completely ignorant about the state of the event 'fraud' whether it is present or not present. Such a situation is represented by assigning equal probability to both the outcomes under probability framework: P(fraud) = 0.5, P(no fraud) = 0.5. These probability numbers, in general, represent uncertainty in the outcome of the event. Under belief functions, the same situation, where there is no evidence, is represented by assigning zero beliefs to both the outcomes: Bel(fraud) = 0, and Bel(no fraud) = 0. However, under belief functions, plausibility[5] that fraud is present or not present is one, i.e., Pl(fraud) = 1, Pl(no fraud) = 1. The ambiguity in an outcome is defined as the difference between the plausibility and belief in an outcome [see, e.g., Srivastava 1997]. Thus, in the above example, the ambiguity that fraud is present is 1. We cannot represent ambiguity in such a simple way using probabilities. As defined by Srivastava and Shafer [1992], plausibility that there is material misstatement represents the audit risk and plausibility that fraud is present represents the risk of fraud. Thus, in the present

[4] A rigorous, mathematical definition is presented in many of the references to this paper. Space limitations prohibit the presentation of the full mathematical formulations here.

[5] Plausibility in a state 'a', by definition, is equal to one minus the belief in "not a', i.e., Pl(a) = 1 – Bel(not a). See Srivastava and Shafer [1992] for more discussions.

example, the risk of fraud is one under belief functions. However, for the same situation under probability theory, the risk of fraud is only 0.5, as measured by the probability of fraud being present.

Consider another situation where the auditor obtains a piece of evidence, and based on this evidence thinks that there is a low level of support, say 0.3 on a scale of 0-1, that fraud is present. If we represent this judgment using the probability framework then we will have probability of fraud to be 0.3 and probability of no fraud to be 0.7, i.e., P(fraud) = 0.3, P(no fraud) = 0.7, even though there is no evidence that supports 'no fraud'. However, when we use belief functions then, based on the evidence, we have 0.3 level of belief that fraud is present, Bel(fraud) = 0.3, but no belief that fraud is not present, Bel(no fraud) = 0, and 0.7 level of support is undecided, it may pertain to 'fraud' or 'no fraud' depending on the additional evidence. Thus, based on this evidence, the plausibility that fraud is present is 1 and plausibility that fraud is not present is 0.7. For this example, the fraud risk defined by probability framework is 0.3 because of the probability of fraud being 0.3. However, under belief functions, the fraud risk, as defined by plausibility of fraud, is still 1 (0.3 level of belief from the direct evidence, 0.7 level of belief coming from the ambiguity). Thus, the belief function measure of risk is a more conservative estimate of risk than that obtained using the probability framework as elaborated further below.

The belief function notion of risk is defined to be the *plausibility* that the event of interest [e.g. material fraud has occurred or the financial statements are materially misstated] has occurred. From this perspective, both *direct evidence* that the event has occurred [e.g. evidence of fictitious sales] and *ambiguity* as to whether the event has occurred or not is assigned to risk. This definition is operational as long as the auditor can provide an assessment of the strength of

audit evidence in terms of the degree to which it confirms or disconfirms any assertion of interest.[6] More importantly, the belief function notion of risk provides a conservative assessment of risk, which can be argued to be the best approach for the profession. This assessment is conservative in the sense that the highest assessment of risk is used, given both the evidence and ambiguity that exists. Given the litigious environment of the profession and the high penalties the market extracts in cases of failure, underestimation of such risks is clearly not in the best interest of the profession. In such cases, it is the plausibility of risks such as fraud that need to be reduced to miniscule levels. The use of such a conservative approach also is appropriate when assessing audit risk [Srivastava and Shafer, 1992] and the risk of a material weakness in the internal controls over financial reporting. This definition of risk has also been used in several other applied fields. For example, Démotier, Schön and Denoeux [2004] use the same definition in assessing the risk of unhealthful water quality produced by a particular treatment plant. Clearly this is another risk that needs to be reduced to negligible levels.

Further, in support of belief functions, Srivastava and Shafer [1992] argue the interpretations of audit risk and inherent risk under the probability framework do not make sense. For example, during planning, if the auditor sets IR [inherent risk] = 1, how should this be interpreted? Does it mean that the auditor is planning the audit assuming for sure that there is material misstatement in the financial statements? But, this is what a probability based interpretation implies. Under a belief-function interpretation, although setting IR = 1 means there is a 100% plausibility [risk] of a material misstatement, there is also a 100% plausibility of no material misstatement. Which state is correct requires the collection and interpretation of competent audit evidence.

[6] As is the case with any approach to modeling audit risk, the assessment of strength of evidence is difficult and further research is needed as to training, calibration and other aspects.

In addition, suppose the auditor has performed analytical procedures related to accounts receivable and the auditor thinks that the resulting evidence, which is all consistent with auditor expectations, provides support for the account being fairly presented at a low level; say 0.2 on a scale of zero to one. If we interpret this judgment using probability theory then, by definition, we have 0.8 level of support for the account balance being materially misstated. But, as assumed, the auditor actually has no information about the account being materially misstated. Belief functions map this situation correctly: belief that the account is fairly stated is 0.2; belief that the account is materially misstated is zero; and a 0.8 level belief is unassigned representing the ambiguity at that stage of the audit. Subsequent testing would reduce the belief assigned to ambiguity, and correspondingly increase the belief that the account is fairly stated or misstated. Importantly, there is explicit recognition in belief functions of the extent to which currently available information does and does *not* help the auditor determine whether a misstatement exists.

Given that the traditional framework in auditing of modeling risk and uncertainly is based in probability theory and Bayesian inference, several other articles have discussed additional issues one faces in applying the probability framework to audit research or practice. We encourage the interested reader to review these articles when considering belief function or other alternatives[7].

The Proof of the Pudding is in the Eating.

While Srivastava and his co-researchers have begun to explore the applicability of belief functions to audit and assurance services [e.g., Shafer and Srivastava 1990, Srivastava and Shafer 1992, Srivastava 1993, Srivastava and Liu 2003, Srivastava and Mock 1999-2000, and

[7] In particular see Srivastava and Shafer [1990], Srivastava [1993], Smets (1999].

2002], we should also look to other disciplines to assess its potential. Belief functions are being used in variety of real world settings mostly because of its flexibility in modeling uncertainties and assessing risk. Some examples of applications to problems beyond auditing include:

- Remote sensing in agriculture [Cohen and Shoshany, 2004] and in the ocean [Moon, 1990].
- Assessing risk in water treatment: Démotier, Schön and Denoeux [2004]
- Forecasting demand for mobile satellites: McBurney and Parsons [2002]
- Financial Portfolio Management: Shenoy and Shenoy [2002]
- Data mining: Wilkins and Lavington [2002]
- Image processing in radiology: Chen, Lin, and Chen [1992]

Why Consider Belief Functions in Auditing Research and Practice?

In conclusion, audit researchers and practitioners should consider the Dempster-Shafer theory of Belief Functions as it provides a rigorous, operational and appropriately conservative approach to risk assessment and evidential reasoning that can be expected to be applied consistently over a variety of audit situations. It is a flexible framework for managing uncertainties in real world problems where we do not have the necessary knowledge to apply other approaches such as Bayesian inference. As noted, if we do have the requisite knowledge, then the belief-function framework reduces to the probability framework and Dempster's rule of combination reduces to Bayes' theorem.

Clearly, the belief function approach has its own challenges [e.g. see Gillett 1996] and further research is needed concerning issues such as the calibration characteristics of beliefs which measure strength of evidence and the training that would be needed to better utilize the belief function perspective in practice. Some research has begun and has provided evidence that belief functions may best capture auditor judgment. For example, Monroe and Ng [2002] have investigated the degree of consistency between auditors' intuitive assessments of audit risk and audit risk values generated by various models, including the belief-function and Bayesian models. Their research demonstrates that inconsistencies occur with the use of all models except

the belief-function model. Also, the research by Harrison et al. [2002] has shown that 80 percent of the auditors' judgment about the strength of evidence pertaining to an assertion can be modeled under belief functions only and not under the probability framework.

Nonetheless, additional research is needed to see if these results can be replicated and extended to other audit tasks. The significant challenges currently facing the auditing profession and the heightened awareness of the value of belief functions by researchers in other disciplines suggest it is an opportune time for audit researchers to explore the value and limitations of this very promising approach.

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